Study of Time Correlation and Memory Function for the Derivation of Spectral Density in Plasma

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Highlights

- To study variation of spectral density with temperature for Q- Machine and Fusion reactor
- To study about the fundamental of plasma physics, its parameters and comparison of noise production
- At the level of continuum description, the calculation is based on the equation of hydrodynamics with given values of thermodynamic and transport coefficient

Abstract

This paper is mainly focused on studying time correlation and memory functions in order to determine the spectral density in plasma. The research aims to understand and quantify the noise produced in the plasma as a result of random processes. Noise can arise from various sources, including fluctuations in particle density, temperature, and other plasma parameters. To ensure a fair comparison, the paper appears to keep the cross-sectional area of the plasma which is same for both the Q-Machine and Fusion Reactor. This means that any observed differences in spectral density can be attributed to the inherent characteristics of the two systems rather than differences in plasma volume. It is observed that Fusion Reactor produced more noise in comparison to the Q-Machine. This research seems to contribute to our understanding of plasma physics, specifically in the context of spectral density and noise production, by comparing two different plasma systems. The observed differences in noise levels between the Q-Machine and Fusion Reactor could have important implications for plasma-based technologies and fusion research.

Keywords: *Coulomb Logarithm, Time Correlation Function, Spectral Density, Electric Noise, Fusion Reactor.*

Introduction

Research on spectral density in plasma is a crucial aspect of plasma physics, with applications ranging from nuclear fusion to astrophysics and space science. However, like any scientific field, there are often research gaps or areas where further investigation is needed. Here are a few potential research gaps in the study of spectral density in plasma such as non-linear effect, high temperature plasma, magnetic confinement, instabilities and space plasma. Addressing these research gaps will contribute to a deeper understanding of spectral density in plasma and its implications for various scientific and technological applications. It will also help advance our knowledge of complex plasma phenomena and improve the performance of plasma-based technologies. Plasma, coined by William Crookes in 1879 as the fourth state of matter, is a unique statistical system comprising mobile charged

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particles. When a gaseous medium's temperature surpasses a specific threshold, it undergoes a transformative shift from gas to plasma. In this transition, the thermal energy of its constituent particles rises to a point where the attraction between electrons and atomic nuclei diminishes. Consequently, instead of a heated gas composed of electrically neutral atoms, we find a dynamic mix of two distinct groups of particles: electrons and ionized nuclei [1].

Defying classification as a solid, liquid, or gas, plasma distinguishes itself through its remarkable electrical conductivity. This conductivity is so potent that externally applied electric fields are effectively neutralized by the currents they induce within the plasma's core. Plasma physics draws upon the well-established principles of classical mechanics, electromagnetics, and non-relativistic statistical mechanics. At its core, plasma physics reach into the equilibrium and non-equilibrium characteristics of a statistical system encompassing numerous charged particles. The complexity of this system arises from the multitude of microscopic degrees of freedom resulting from the motion of individual particles [1, 2]. The state of ionized gas (plasma) can be specified by distribution function. As a result, Kinetic theory is the most fundamental description of plasma state and serves as the foundation for the practical and useful fluid approach [3]. Plasma physics involves a rich class of phenomena related to the dynamical processes in statistical mechanics. It is therefore essential to study structure and properties of the basic kinetic equation which govern the dynamical behavior of the plasma. The term "fluid medium plasma" comes from Langmuir's suggestion that the electrons, ions, and neutral atoms in an ionized gas may be conceived of as corpuscular material entrained in some type of fluid medium.

However, in contrast to blood, where a fluid medium conveys the corpuscular material, an ionized gas's ions, electrons, and neutral atoms do not enter a fluid medium [4]. The study of dynamical correlation in many-body systems addresses fundamental issues related to the spectral behavior of thermal fluctuations in gases [5]. Time-dependent correlation functions are well-recognized for their utility in describing deviations from equilibrium in systems with numerous particles. In 1954, L. Van Hove introduced the theoretical framework for general space-time correlation functions [6]. This innovative approach involved the time-dependent extension of the familiar pair distribution function. It aimed to explain scattering theory based on Born's approximation and statistical mechanics.

Subsequently, in 1958, R. Kubo's work on thermodynamic irreversible processes further advanced this field by introducing response functions or relation functions. These functions were derived from Onsager's relations and provided a means to express kinetic coefficients of thermal disturbances without relying on traditional methods of establishing transport equations [7]. These functions are commonly referred to as correlation functions, playing a role to that of partition functions. In the latter half of the 1950s, a novel approach to transport processes emerged as a highly active and productive avenue within non-equilibrium statistical mechanics. They indicated that the phenomenological coefficients, which are explaining different transport phenomena and time-dependent process as well [8].

Theoretical aspects

The theoretical problem of studying time correlation function is basically that of describing the dynamical properties of many body systems. At the level of continuum description, the calculation is based on the equation of hydrodynamics with given values of thermodynamic and transport coefficient. To obtain the time correlation function, we have discussed to solve the equation in their linearized form as well as initial value problem with linearization referring to small deviations in the conserved variables from their equilibrium values. The result of such calculation is available in analytic form and their implications are thoroughly understood.

To make some of forgoing statements more explicit, we consider a typical equation that relates a time correlation function (t) to its memory function (t) is [9],

$$\frac{dC(t)}{dt} = -\int_0^t dt' M(t') c(t-t'),$$
(1)

Where, we have suppressed the position dependence in both functions for simplicity. We will encounter the Integra-differential equation of this type. The utility of this equation is that, for given (t) it can be used to calculate (t). we will call equation (1), the memory function equation because it can be regarded as the definition of $M(t)$ in terms of $C(t)$. This point of view is not useful if the objective is to calculate (t) by an approximation to $M(t)$, on the other hand if $C(t)$ were known from experiment, then (1)

should be inverted to represent the data in terms of $M(t)$. The reason that (t) is called memory function can be seen from equation (1) since time correlation function describe the decay of spontaneous thermal fluctuations. (t) is a function with its maximum value at t=0 which decreases with time.

Equation (1) shows that the decay rate of 'C' at time 't' depends on its value at earlier times and how much it resembles past history is expressed the memory $M(t)$. Suppose (t), is sharply peaked function about t=0, the right hand side of (1) then gives the decay rate that is proportional to (t). In this case the decay spends only on the instantaneous value of $C(t)$, so one can say that there is no memory function on the other hand if $M(t)$ is itself a slowly decaying function in time, then the value of C at different times will be closely correlate a behavior which one can describe as memory effects.

From equation(1) one has the relation $M(0) = -\frac{C(0)}{C(0)}$. Let us suppose we know as often in case the initial value and initial curvature of $C(t)$ even though we do not know $C(t)$,then we can construct a model by writing $M(t) = M(0) = 1 f(t)$,where $f(t)$ is model function with property $f(0) = 1$,whatever we chose for $f(t)$ we see that the approximation will preserve the short time behavior of $C(t)$ at least in terms of its initial value and initial will preserve the short time behavior of $C(t)$ at least in terms of its initial value and initial curvature. This is simplified example of memory function models, which appear in much of the following discussions. Besides modeling (t) by postulating a function (t) one can determine the memory function by expressing its decay in terms of coupling to higher order time correlation function [10].

Spectral density

We may define spectral density as a Fourier transform of correlation function. The most direct way to understand this definition is the time correlation function for the emission or absorption of photon [11].

The Schrodinger equation in time dependent form is

$$H\psi = i\hbar \frac{\partial \psi}{\partial t} \tag{2}$$

The general solution is

$$\psi(t) = e^{-\frac{i}{\hbar}Ht} \psi = e^{\frac{i}{\hbar}Et} \psi \tag{3}$$

Where: $\hbar$ = Planck's constant, E= Energy Eigen value (a constant) ,H = Hamiltonian operator, Ψ = wave function.

Initially, the N-interacting molecule system has i the quantum state.

Consider Hamiltonian system be H_0, with $H_0 \Psi_j = E_j \Psi_j$. If we treat an applied electromagnetic field as a perturbation to the Hamiltonian and now interact with this system. Hamiltonian field is $H_1 = -\mu E$.

The total Hamiltonian field is

$$H = H_0 + H_1 = H_0 - \mu E \tag{4}$$

Where, μ is the electric dipole moment.

We have two states, that is initial state and final state . In fact, the frequency of the radiation is close to $\left(\frac{E_f - E_i}{\hbar}\right)$, the transition of the initial state into other quantum states 'f' will occur as a result of the application of external disturbance.

Using Bohr relations, if the field is monochromatic [12]

$$E(t) = E_0 \varepsilon \cos \omega t = \frac{E_0 \varepsilon}{2}\left[e^{i\omega t} + e^{-i\omega t}\right], \tag{5}$$

Here, E_0 represents the amplitude of the field.

Because the field is uniform or the wavelength is larger in comparison to molecular dimensions, the interaction between them can be expressed as

$$H_1(t) = -M.E(t) \tag{6}$$

Now, from Golden rule of time-dependent quantum mechanical perturbation theory [13], the probability per unit time of a transition from state i to state 'f' is formulated as,

$$P_{i \to f}(\omega) = \frac{\pi E_0^2}{2\hbar^2} |\langle f|\varepsilon.M|i\rangle|^2 [\delta(\omega_{fi}-\omega) + \delta(\omega_{fi}+\omega)] \tag{7}$$

Where, $_i = \omega_f - \omega_i$. Multiplying this by $\hbar\omega_{fi}$ gives the rate of energy loss due to radiation in the transition state from 'i' to 'f'. In total, summarizing all 'f', we get the rate of energy loss in transition from initial state 'i', to any other final state. Multiplying Equation (7) by ρ_i, the probability that the system was in its initial state and summing over all 'i' then get the rate of energy loss to the system from radiation.

$$-\dot{E}_{rad} = \sum_i \sum_f \rho_i \hbar\omega_{fi} P_{i \to f}$$
$$-\dot{E}_{rad} = \frac{\pi E_0^2}{2\hbar} \sum_f \sum_i \rho_i \omega_{fi} |\langle f|\varepsilon.M|i\rangle|^2 [\delta(\omega_{fi}-\omega) + \delta(\omega_{fi}+\omega)] \tag{8}$$

Since the summation and go over all the quantum states of the system, we may interchange these indices in the summation over the second data function such That

$$-\dot{E}_{rad} = \frac{\pi E_0^2}{2\hbar} \sum_f \sum_i \rho_i \omega_{fi} |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) + \rho_f \omega_{if} |\langle i|\varepsilon.M|f\rangle|^2 \delta(\omega_{if}+\omega)] \tag{9}$$

$$-\dot{E}_{rad} = \frac{\pi E_0^2}{2\hbar} \sum_f \sum_i \rho_i \omega_{fi} |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) + \rho_f(-\omega_{fi}) |\langle i|\varepsilon.M|f\rangle|^2 \delta(-\omega_{fi}+\omega)]$$

$$-\dot{E}_{rad} = \frac{\pi E_0^2}{2\hbar} \sum_f \sum_i \rho_i \omega_{fi} |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) - \rho_f(-\omega_{fi}) |\langle i|\varepsilon.M|f\rangle|^2 \delta(\omega_{fi}-\omega)]$$

Since, negative frequency imply nothing so,

$$\omega fi = -\omega fi \text{ and } |\langle f|\varepsilon.M|i\rangle|2 = |\langle i|\varepsilon.M|f\rangle|2$$
$$-\dot{E}_{rad} = \frac{\pi E_0^2}{2\hbar} \sum_f \sum_i \rho_i \omega_{fi} |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) - \rho_f \omega_{fi} |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega)]$$

$$-\dot{E}_{rad} = \frac{\pi E_0}{2\hbar} \sum_f \sum_i \omega_{fi}(\rho_i - \rho_f) |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) \tag{10}$$

If we assume that the system is initially in equilibrium, then

$$\rho_f = \rho_i e^{-\beta\hbar\omega fi} \tag{11}$$

So,

$$\rho_i - \rho_f = \rho_i(1 - e^{-\beta\hbar\omega fi}) \tag{12}$$

If we substitute this into eqn (10), we find that

$$-\dot{E}_{rad} = \frac{\pi E_0^2}{2\hbar}(1 - e^{-\beta\hbar\omega})\omega \sum_f \sum_i \rho_i |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) \tag{13}$$

We have to define the absorption cross-section $\alpha(\omega)$ such that the cross section which is multiplied by the incident flux of radiation, known as the rate of energy loss from the field into the system, this is called pointing vectors [14], having magnitude

$$S = \frac{v}{8\pi}\varepsilon E_0^2 = \frac{nc}{8\pi\mu}E_0^2$$
$$S = \frac{c}{8\pi}nE_0^2$$

Where μ=1, for non-magnetic medium and v be the speed of light in medium and ε be the dielectric constant, c be the speed of light in vacuum, and n be refractive index of the medium.

$$n = \frac{c}{v}(\mu\varepsilon)^{1/2}$$
$$\alpha(\omega) = \frac{4\pi}{\hbar cn}(1 - e^{-\beta\hbar\omega})\omega \sum_f \sum_i \rho_i |\langle f|\varepsilon.M|i\rangle|^2 \delta(\omega_{fi}-\omega) \tag{14}$$

If we divide (12) by S, then we obtain

It is convenient to use this eqn to define an absorption line shape (ω) by

$$I(\omega) = \frac{3\hbar cn a(\omega)}{4\pi^2\left(1-e^{-\beta\hbar\omega}\right)_\omega} = 3\sum_f \sum_i \rho_i \, |\langle f|\mathcal{E}.M|i\rangle|^2 \delta(\omega_{fi} - \omega) \tag{15}$$

This is Schrodinger's representation of spectroscopy as transition between Bohr Stationary states. Since, equation 7 is derived from first order perturbation theory in which the operators are independent with time (t). The wave function of the system varies with time.

Two representation [Schrodinger's representation and Heisenberg's representation] agree for all time if

$((0), (t)(0)) = ((t), (0)(t))$ The time dependent wave function $\chi(t)$ and $\psi(t)$ obeys the Schrodinger's

equation   (16)

$$i\hbar \frac{\partial\psi}{\partial t} = H\psi \tag{17}$$

Whose formal solution for a time independent Hamiltonian operator is

$$\psi(t) = e^{\frac{(-iHt)}{\hbar}}\psi(0) = U(t)\psi(0) \tag{18}$$

With a similar equation for (t)

Thus eqn (16) becomes,

$$((0), (t)(0)) = (\chi(t), A(0)\psi(t)) = (U(t)\chi(0), A(0)U(t)\psi(0) \tag{19}$$

$= ((0), U^*(t)A(0)U(t)\psi(0))$ Now, the time dependent operator is related to a static operator by,

$$A(t) = U^*(t)A(0)U(t) = e^{\frac{iHt}{\hbar}}A(0)e^{-\frac{iHt}{\hbar}}\text{(20)} \tag{20}$$

Equation 20 represents the Heisenberg's operator. Now, we have the Heisenberg's representation in equation 19, naturally leads to the spectroscopic time –correlation function.

We have to convert equation (16) to the Heisenberg picture by introducing the Fourier transform of the Dirac delta function [15],

$$\delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\omega t}dt$$

And using the Einstein relation $\omega_{fi} = \frac{E_f - E_i}{\hbar}$,

The delta function in the intensity expression is,

$$\delta(\omega_{fi} - \omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{\left\{i\left(\frac{E_f-E_i}{\hbar} - \omega\right)t\right\}}dt \tag{21}$$

Giving

$$I(\omega) = \frac{3}{2\pi}\sum_{i,f}\rho_i\langle i|\epsilon.M|f\rangle\langle f|\epsilon.M|i\rangle\int_{-\infty}^{+\infty} e^{\left\{i\left(\frac{E_f-E_i}{\hbar} - \omega\right)t\right\}}dt \tag{22}$$

Now the state $|i\rangle$ and $|f\rangle$ are Eigen states of the system excluding the radiations that

$$e^{\frac{iE_i t}{\hbar}}|i\rangle = e^{\frac{iH_0 t}{\hbar}}|i\rangle \tag{23}$$

And $\langle f|e^{\frac{iE_f t}{\hbar}} = \langle f|e^{\frac{iH_0 t}{\hbar}}$ (24)

Thus eqn (22) can be written as,

$$I(\omega) = \frac{3}{2\pi}\int_{-\infty}^{+\infty} e^{-i\omega t}dt \sum_{i,f}\rho_i\langle i|\epsilon.M|f\rangle\left\langle f\left|e^{\frac{iH_0 t}{\hbar}}\epsilon.Me^{-\frac{iH_0 t}{\hbar}}\right|i\right\rangle$$

$$I(\omega) = \frac{3}{2\pi}\int_{-\infty}^{+\infty} e^{-i\omega t}dt \sum_{i,f}\rho_i\langle i|\epsilon.M|f\rangle\langle f|\epsilon.M(t)|i\rangle \tag{25}$$

Where,

$$M(t) = e^{\frac{iH_0 t}{\hbar}} M e^{-\frac{iH_0 t}{\hbar}} \qquad (26)$$

Using closure relation

$$\sum_f \langle i | \langle f | = 1 \qquad (27)$$

Giving,

$$I(\omega) = \frac{3}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega t} dt \sum_i \rho_i \langle i | \epsilon. M(0)\epsilon. M(t) \rangle | i \rangle \qquad (28)$$

The summation over (initial states) is a weighted Boltzmann average (ρ_i is the Boltzmann weighting factor) so this is nothing more than an equilibrium ensemble average. Thus we have,

$$I(\omega) = \frac{3}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega t} dt \langle \epsilon. M(0)\epsilon. M)t \rangle \qquad (29)$$

$$\because \langle \epsilon. M(0)\epsilon. M(t) \rangle = \frac{1}{3} \langle M(0). M(t) \rangle$$

So, for an isotropic fluid, by averaging ϵ over all directions, it is seen that

$$I(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega t} dt \langle M(0). M(t) \rangle \qquad (30)$$

This is the expected result that is called the line shape function (ω) that has been written as the Fourier transform of the time correlation function of the dipole moment operator of the absorbing molecules if no field is applied.

The line shape (ω) can be used to study the motion of the dipole moments of the molecules by Fourier inversion of itself.

As an introduction, a correlation function represents the mean decay property of the system from its initial value. In this case, the property of the dipole moment, in general, is consider the dipole moment of the entire system(having N interacting dipole moments)c can be expressed as [16]:

$$M(0). M(t) = \left(\sum_{i=1}^{N} \mu(0) \right). \left(\sum_{j=1}^{N} \mu(t) \right) \qquad (31)$$

There are in addition to the terms such as (0). (t), and cross terms between the dipole moments on different molecules (0). (t). Because of these cross terms, one cannot simply intercept $\langle (0). (t) \rangle$ in terms of the re-orientation of single dipole molecule. In the other hand, the dipole molecules are dissolved in some non-polar solvent, the cross terms are negligible and the correlation function can be formulated as

$$\langle M(0). M(t) \rangle = \langle \sum_{j=1}^{N} \mu_j(0). \mu_j(t) \rangle = N \langle \mu(0). \mu(t) \rangle, \qquad (32)$$

Hence, the intensity expression becomes

$$I(\omega) = \frac{N}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega t} dt \langle \mu. \mu(t) \rangle \qquad (33)$$

The time dependent operator (t) plays an important role on the right hand side of this correlation function. Such type of correlation function is known as one sided correlation function. The one sided correlation function $\langle \mu. \mu(t) \rangle$ which follows the fact that the frequency spectrum $I(\omega)$ is a real quantity, that is clear from the definition mentioned in equation (33), Thus letting $C(t) = \langle \mu. (t) \rangle$, we have,

$$I(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} c(t) e^{-i\omega t} dt \qquad (34)$$

Electrical noise in plasma

The fluctuating current I(t) due to random thermal motion of charged particles gives rise to noise across it. The effective fluctuating voltage in the resistor can be expressed as [6]

$$(t) = IR = ne\bar{u}AR \qquad (35)$$

Where R is resistance, A is area of cross section, n be the no. of charged particles per unit volume and $\bar{u}$ be the average velocity component of the charged particles along the length of plasma. The average velocity is given by

$$\bar{u} = \frac{u_1 + u_2 + u_3 + \cdots\ldots\ldots}{total\ no.of\ electrons} = \frac{\sum_i u_i}{nAL} \tag{36}$$

Here sum is over all electrons or ions. From eqn (35) and (36), we have

$$V(t) = IR = neAR\frac{\sum_i u_i}{nAL} = \frac{Re\sum_i u_i}{L} = \sum_i v_i$$

Here, $\frac{Re\sum_i u_i}{L} = \sum_i v_i\ u_i$ and v_i are random variables. Now, charged particles are colliding during thermal motion. Let τ_e is the time between collisions called relaxation time.

The correlation function may be given the form

$$(\tau) = (t)(t + \tau) = \bar{v_i^2}e^{-v t}e \tag{37}$$

From eqn (34), spectral density is given by \

$$I(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} c(\tau)e^{-i\omega\tau}d\tau = \frac{1}{2\pi}.2\int_0^{+\infty}\bar{v_i}^2 e^{-\tau/\tau_e}e^{-i\omega\tau}d\tau$$

$$= \frac{1}{\pi}\int_0^{+\infty}\bar{v_i}^2 e^{-(\tau/\tau_e + i\omega\tau)}d\tau = \frac{1}{\pi}\int_0^{+\infty}\bar{v_i}^2 e^{-(1/\tau_e + i\omega)\tau}d\tau$$

$$= \frac{1}{\pi}\bar{v_i}^2\left[\frac{e^{-(1/\tau_e + i\omega)\tau}}{-(1/\tau_e + i\omega)}\right]_0^{+\infty}$$

$$= \frac{1}{\pi}\bar{v_i}^2\frac{1}{((1/\tau_e + i\omega))}$$

$$= \frac{1}{\pi}\bar{v_i}^2\frac{\tau_e}{((1 + i\omega\tau_e))} = \frac{1}{\pi}\bar{v_i}^2\tau_e \tag{38}$$

Since, $\omega\tau_e \lll \frac{1}{2}$

$$\therefore I(\omega) = \frac{1}{\pi}\left(\frac{Re}{L}\right)^2\bar{u_i}^2\tau_e \tag{39}$$

We know that Electrical conductivity,

$$\sigma = \frac{ne^2\tau_e}{m} \text{ and } R = \frac{L}{\sigma A}\ ,$$

We have,

$$\frac{1}{2}m\bar{u^2} = \frac{1}{2}K_B T \text{ or } \bar{u^2} = \frac{K_B T}{m}$$

$$\therefore I(\omega) = \frac{1}{\pi}\left(\frac{Re}{L}\right)^2 nAL\frac{K_B T}{m}.\frac{mL}{RAne^2}$$

$$= \frac{K_B T R}{\pi} \tag{40}$$

This is relation relating spectral density, (ω) of the fluctuating voltage to the resistance. This relation shows that spectral density is independent upon the frequency.

Putting,

$$\rho = \frac{\pi e^2 m^{1/2}}{(K_B T)^{3/2}}\ln\Lambda, \text{called resistivity of plasma and where coulomb Logarithm} \tag{41}$$

Results and Discussion

The process in which a quantity does not depend in a well-defined way on the independent variable is called random process. Spectral density of a randomly fluctuating quantity is defined as ensemble average of the time average in the power dissipation

per unit frequency band width. The fluctuating current I(t) due to random thermal motion of charged particles in plasma give rise to a random output called noise across it. The spectral density in terms of temperature is given by equation (42)

$$I(\omega) = \frac{e^2 m^{1/2}}{\sqrt{K_B}\sqrt{T}}\frac{L}{A}\ln\Lambda \qquad (42)$$

This relation shows that spectral density is independent with frequency and is dependent on its length, cross section of the plasma and temperature. So spectral density is random process and produces noise across the plasma.

The relevant plot for the variation of the spectral density with temperature for the Q-Machine and Fusion reactor and their comparative study, a combined variation for both the Q-Machine and Fusion Reactor is given in graph below.

The variation of the spectral density in case of the plasma in Q-machine [7] is found to depend upon the temperature of plasma. It is observed that the curve shifts towards the origin as the plasma length is decreased while keeping the cross section fixed.

In each case the increase in temperature causes to decrease in the spectral density, showing a sharp decrease in the initial phase, depicting a parabolic nature. Similar variation is seen for the fusion reactor. And from their combined plot it is seen that for the same values of the parameters, the curve for the fusion reactor was situated farther away from the origin than the curve for the Q-Machine, indicating a more pronounced effect of the variations involved for the fusion reactor than for the Q-machine.
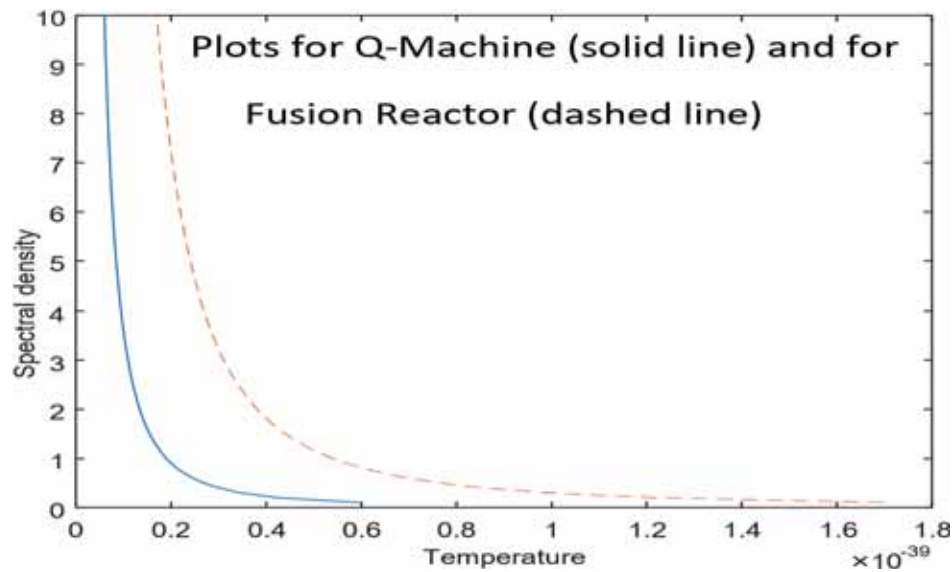

Fig 1. Variation of spectral density with temperature for both Q-Machine and Fusion Reactor a comparative study

Conclusions

In conclusion, this contribution focuses on studying time correlation functions and deriving spectral density. The key finding is that the spectral density exhibits a random, frequency-independent nature, leading to the production of noise in plasma. Comparative analysis between the Q-Machine and Fusion reactor reveals that the Fusion reactor produces more noise. This research paves the way for future investigations into plasma devices like nuclear reactors, Q-Machines, and toroidal devices.

Acknowledgements

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