Analysis of Metaheuristic Solutions to the Response Time Variability Problem

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ABSTRACT

The problem of variation in the response time is known as response time variability problem (RTVP). It is combinatorial NP-hard problem which has a broad range of real-life applications. The RTVP arises whenever events, jobs, clients or products need to be sequenced so as to minimize the variability of the time they wait for their next turn in obtaining the resources they need to advance. In RTVP the concern is to find out near optimal sequence of jobs with objective of minimizing the response time variability. The metaheuristic approaches to solve the RTVP are: Multi-start (MS), Greedy Randomized Adaptive Search Procedure (GRASP) and Particle Swarm Optimization (PSO). In this paper, the computational result of MS and GRASP will be analyzed.

1. INTRODUCTION

The Response Time Variability Problem (RTVP) is a sequence optimization problem [1]. It was first reported in 1994 and first time solved by using a method called lottery scheduling [2]. It occurs in real-life situations in which jobs, clients, products or events need to be sequenced in order to minimize the variability in the time between two successive points at which they receive their necessary resources. The fair sequence concept has emerged from scheduling problem in different environments [1]. The common aim of scheduling problem is to minimize an objective function. The objective is to minimize the response time variability metric value of the solutions. The RTVP has been proved to be NP-hard [1]. Thus, this problem has been mostly solved by means of heuristic and metaheuristic methods [3].

The response time variability problem is formulated in [4]. Let n be the number of symbols (jobs), d_i the number of copies to be scheduled of symbol *i* (*i*=1…n) and D the total number of copies (equal to $\sum_{1\ldots d_i}$)

Consider a sequence $S=S_1S_2\ldots S_D$ of length D where *i* (a client, a product, or a task) occurs exactly d_i times. Such a sequence is called feasible. Here s_j is the copy sequenced in position *j* of sequence *S* and S_1 immediately follows S_D. For any two consecutive occurrences of *i,* we define distance *t* between them as the number of positions that separate them plus 1. So there are d_i distances $t_1^i \ldots t_{d_i}^i$ for i.

So we have, $t_1^i \ldots t_{d_i}^i$ =D.

The average distance $t_i^{'}$ between the i's equals D/d_i

The response time variability for *i* is defined as

$$RTV_i = \sum_{1\le j\le d_i} (t_i^{'} - t_i^{'})^2$$

The total response time variability is defined as

$$RTV = \sum_{i=1}^{n} RTV_i = \sum_{i=1}^{n}\sum_{j=1}^{d_i} (t_j^i - t_i^{'})^2$$

An input to the total response time variability problem is a list of *n* positive integers $d_1 \le d_2 \le d_3 \le \ldots\ldots \le d_n$ (the number of copies of each job). The solution to RTVP is a sequence S of jobs and the objective is to minimize the value of RTV obtained above.

Example:

Let n=3 with symbols A, B, C. Also consider d_A=2, d_B=2 and d_C=4. Thus D=8, t_A^i=4, t_B^i=4 and t_c^i=2. Then the sequence C A C B C B A C is a solution and has

RTV = $((5-4)^2 + (3-4)^2) + ((2-4)^2 + (6-4)^2) + ((2-2)^2 + (3-2)^2) = 12$

2. METAHEURISTIC METHODS

This section discusses the complexity of RTVP and introduces the metaheuristic methods for solving RTVP.

Complexity

The RTVP is difficult to be solved optimally. It has been proved to be NP-hard [5]. Many algorithms are proposed to find the near to optimal solution [6]. Some of the solutions based on metaheuristic procedures are: MS, GRASP and PSO.

Multi –Start (MS) Method

The multi-start is metaheuristic procedure for solving the RTVP [7]. It is a general scheme that consists of two phases. In the first phase obtains an initial solution and in the second phase improves the obtained initial solution by using the local optimization methods and select the best of one. The pseudo code of the adaptation of the multi-start methodis:

Let the value of the best solution found b $\bar{z}$ = ∞.

1. While (actual time<execution time) do:
2. Get a random initial solution X.
3. Apply the local optimization to X and get X'.
4. If value (X') < $\bar{Z}$, then $\bar{Z}$ = value (X').

Random solutions are generated as follows. For each position from 1 to D in the solution, is randomly obtain, which product will be sequenced with a probability equal to the number of units of that type of product that remains to be sequenced divided by the total number of units that remains to be sequenced. A local optimization is performed iteratively in a neighborhood that is generated by interchanging each pair of two consecutive units of the sequence that represent the current solution. The best solution in the neighborhood is chosen, the optimization ends when no neighboring solution is better than current solution.

The Greedy Randomized Adaptive Search Procedure (GRASP) Methods

The GRASP can be considered as a multi-start variant [8]. But the generation of initial solution is obtained by greedy method. In which random steps are added and choice of elements to be included in the sequence is adaptive. The probability of each job is proportional to the value of an associated index. The job to be sequenced is randomly selected from the list with a probability proportion to the value of its Webster index [9].

Let X_{ik} be the number of units of job i, that have been already sequenced in the sequence of length k, k=0, 1,……,d_i the number of units of the job i and D the total number of units; the value of the Webster index of product i to be sequenced position k+1 is $d_i/X_{ik}+\delta$.

Here δ is the Webster's parametric metrics, δ=1/2.

In the Jefferson's sequence the parametric matrices δ=1 used [10]. This parameter affects the relative priority of low decreased jobs and their position in the sequence. When δ is near to 0, low demand jobs will be positioned earlier in the solution but when δ is near to 1, low demand jobs will be positioned later in the solution.

Another form of GRASP algorithm can be obtained by using the insertion sequence as the initial sequence [8]. In insertion sequence, for more than two products, the problem is reduced in to two-product case [6]. Let the demands are d_1 ≤………..≤dn. consider n-1 two case problem.

$$\boldsymbol{p_{n-1}} = (\boldsymbol{d_{n-1}}, \boldsymbol{d_n}), \qquad \boldsymbol{p_{n-2}} = \left(\boldsymbol{d_{n-2}}, \sum_{j=n-1}^{n} \boldsymbol{d_j}\right), \ldots, \boldsymbol{p_1} = \left(\boldsymbol{d_1}, \sum_{j=2}^{n} \boldsymbol{d_j}\right)$$

In each of the problem the first product is the original and second product will be the assumed product, and denoted by the *. Let the sequences s_{n-1}, s_{n-2} , ……….. , s_1. be the optimal solution. For the given problems they can be obtained by using the two case problem. The solution is made up of the product j and *. The sequence of the original problem is built recursively by first replacing * in S_1 by S_2 to obtain s_1'. Next * are replaced by S_3 in s_1' to obtain the solution S_1''.Sequence S_{n-1} replaces all the remaining * and obtain the final solution. This method is called insertion method [11].

3. RESULTS AND DISCUSSION

The metaheuristic algorithms have been run for 882 different instances, which are grouped into four different categories. Formation of category is based on [12]. Category 1 includes 162, category 2 includes 192, category 3 includes 282 and category 4 includes 246 instances. The corresponding instances are same for every category of different algorithms. The instances of first category CATEGORY 1 were generated using a random value of D between 25 and 50, and random value of n between 3 and 15. For the second category CATEGORY 2, D was between 50 and 100 and number of demands n between 3 and 30; for the third category CATEGORY 3 D was from 100 to 200 and n between 3 and 65; and finally for the forth class CATEGORY 4 number of copies are between 200 and 500 and number of demands are between 3 and 150. The instances have been generated by first fixing the total number of copies D and the number of demands n. For all instances and for each type of product i= 1,….., n, a random value of d_i is between 1 and D. The program has been executed to obtain the output of demands among which some of them were executed for several minutes.

The average initial RTV values(AIRTV), the average optimized values (AORTV) and the average number of iterations required to obtained the optimized sequence using in multi-start, $GRASP_{we}$ (GRASP use Wester's sequence as initial solution) and $GRASP_{je}$ (GRASP with use of Jeffersion's sequence as initial solution) metaheuristic algorithms [13]. The computational result is tabulated as:

Table 1: Computational result of MS method.

Category	Average initial RTV	Average optimal RTV	No. of iterations
Global	137515.75	326	1402
CAT 1	890	25	58
CAT 2	4837	57	192
CAT 3	34050	232	781
CAT 4	510286	990	4577

Table 2: Computational result of $GRASP_{we}$ method.

Category	Average initial RTV	Average optimal RTV	No. of iterations
Global	21352.50	292.5	929.25
CAT 1	144	38	22
CAT 2	1056	80	119
CAT 3	5114	315	726
CAT 4	79096	737	2850

Table 3: Computational result of GRASP method.

Category	Average initial RTV	Average optimal RTV	No. of iterations
Global	18698.25	244	629
CAT 1	143	29	21
CAT 2	941	63	95
CAT 3	4537	219	322
CAT 4	69172	665	2078

By analyzing these tables, for all instances in initial value of RTV the $GRASP_{je}$ is 12.43% better than $GRASP_{we}$ and 86.40% better than multi-start. In optimized RTV value $GRASP_{je}$ is 16.58% better than $GRASP_{we}$ and 25.15% better than multi-start. $GRASP_{je}$ also take the less number of iterations for obtaining the optimized solution i.e. it is faster than other two methods. The multi-start algorithm obtains the good averages for small instances (category 1 and category 2) but, poor average results for large instances (category 4). $GRASP_{je}$ and $GRASP_{we}$ gives the better result than MS method for large instances (category 4).

4. CONCLUSION

In the RTVP, the aim is to minimize variability in the distance between any two consecutive copies of the same symbol. i. e. to distribute the symbols as regular as possible. It is an NP-hard problem so metaheuristic methods are needed for solving real life problems. A computational experiment was done and its results show that on average the $GRASP_{je}$ is better than $GRASP_{we}$ and multi-start for small instances, multi-start is better metaheuristic method for solving RTVP. The $GRASP_{je}$ method has a stable behavior for small, medium and large instances**.**

DECLARATION OF COMPLETING INTEREST

The authors declare that they have no known competing interests or personal relationships that could have appeared to influence the work reported in this paper.

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