

## Exploring the Relevance of Advanced Mathematical Knowledge for Secondary-Level Instruction

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### Abstract

*The usefulness of advanced mathematics in teaching is referred to as its relevance for secondary-level mathematics instruction. This paper is based on research that explores teachers' views on the relevancy of advanced mathematics and their expectations from advanced mathematics courses for teacher preparation programs. Ten secondary-level mathematics teachers participated in this study. semi-structured interviews with open-ended questions and focus group discussions were utilised for data collection and thematic for data analysis. The finding indicates that advanced mathematics is relevant for developing mathematical proficiency, mathematical, and logical thinking, knowledge of mathematical context mathematical activities and increased problem-solving skills and professional competencies. Along with advanced mathematical knowledge, technological and research knowledge, and skills are expected from undergraduate teacher preparation courses. Therefore, it is suggested to incorporate the expected form of knowledge in the teacher preparation program.*

**Keywords:** Advanced mathematics, relevance, mathematical knowledge for teaching

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### Introduction

Within university-level Mathematics Teacher preparation programs, a diverse array of advanced mathematics courses is incorporated. In our roles as university teachers, we consistently deliver instruction in subjects such as real analysis, algebra, and advanced calculus. However, an ongoing challenge revolves around the inquiry into the practical significance of studying this complex mathematics. The question of the relevance of such a course prompts us to reflect on why it is essential for being a teacher.

The relevance of advanced mathematics in school-level mathematics instruction pertains to its practicality in teaching and learning processes, as well as its contribution to professional growth. The discussions surrounding the essential knowledge and skills that mathematics teachers need to excel in their profession has been a prominent

focus in the field of mathematics education. Specifically, secondary-level mathematics teachers must possess a strong grasp of both content and pedagogical expertise. Advanced mathematics is the phrase used by university professors to describe the many undergraduate and graduate mathematics courses they teach (Even, 2011). It includes abstract algebra, linear algebra, real analysis, topology, and number theory as a subset of content knowledge of mathematics learned during undergraduate studies (Stockton & Wasserman, 2017; Zazkis & Leikin, 2010). Thus, advanced mathematics is in abstract and complex form.

Mathematics teachers are frequently required to take advanced mathematics courses during their graduation in teacher preparation programs or any other related programs, including mathematics like science and technology. However, many

teachers raise some questions about the relevance of such advanced mathematics courses for their future teaching careers (Zazkis & Leikin, 2010). Additionally, it is noted that the quantity of advanced mathematics preparation doesn't guarantee teaching quality, that is own mathematical understanding doesn't necessarily translate into an ability to enhance the understanding of others (Monk, 1994). However, mathematicians and mathematics educators have alike general perceptions that mathematics teachers should know mathematics more than what they teach (Association of Mathematics Teacher Education, 2017; Conference Board of the Mathematical Sciences, 2013; Wasserman & Stockton, 2013). Therefore, it is often believed that mathematics teachers need to learn more content of advanced mathematics than only what they teach.

Mathematics teacher preparation courses in almost all Universities of the world have introduced advanced mathematics (Schmidt et al., 2013). Different universities of Nepal like TU have introduced advanced mathematics courses in their different programmes (Panthi & Jha, 2016). This course expects to give teachers more potential so that they can offer connections across mathematical domains, mathematical familiarity for the development of problem-solving abilities and increasing epistemological consciousness of the subject matter (Yan et al., 2022). Besides these necessities of advanced mathematics, highly relevant questions, what kind of mathematical knowledge is required for mathematics teachers?, and how can professional specific knowledge of mathematics be characterized have been arising for teacher education designing

programs (Dreher et al., 2018). In this scenario, this paper tries to ascertain the relevance of advanced mathematics and teacher expectations from those mathematics.

Different scholars (eg. Greenstein & Seventko, 2017) have suggested the different structures of mathematics courses to make them relevant for secondary mathematics teachers. The traditional and modern approaches can be found in structuring the relevant course for the prospective teacher. Bishop (1992) pointed out three different traditions, the pedagogue tradition, empirical scientist tradition and scholars' tradition regarding the mathematics preparation program. The discipline-specific teacher knowledge including advanced mathematics was found to be preferred for the preparation of secondary mathematics teachers (Heinze et al., 2016). However, specialized knowledge has received a lot of concentration among researchers in mathematics education and is conceptualized as the foundation for advanced mathematical knowledge for teaching. In general, almost all undergraduate and graduate program of mathematics teacher preparation programs have included advanced mathematics. Besides these scenarios, mathematics teachers and educators (eg. Zazkis & Leikin, 2010) raise questions on the relevancy of advanced mathematics regarding future use: what did mathematic teachers get from this course? Does this course have a connection with secondary mathematics courses? Academic research is essential to address the controversial debate over the relevance of advanced mathematics. This paper aims to explore secondary-level mathematics teachers' views on the relevance of advanced mathematics for secondary-level mathematics instruction.

The research was conducted to answer the

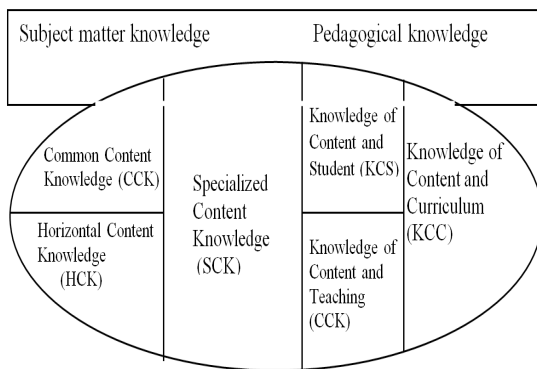
following two research questions:

1. How do secondary-level mathematics teachers perceive the relevancy of advanced mathematics?
2. What opportunities do secondary-level teachers expect from advanced mathematics and mathematics teacher preparation courses?

**Theoretical Framework**

The practice-based theory of mathematical knowledge for teachers developed by Ball et al. (2008) and Shulman (1986) served as the foundation for the development of the theoretical framework for mathematical knowledge for teaching. According to this approach, figure 1 illustrates the six different types of knowledge needed to teach mathematics (Ball et al., 2008, p. 377). These components are considered as fundamentals of the teacher preparation programs.

**Fig. 1**  
*Mathematical Knowledge for Teaching*



( Source: Ball et al.,2008, p. 377).

According to this theory, knowledge of subject matter includes common content knowledge, horizontal content knowledge and specialized content knowledge, and pedagogical knowledge including knowledge

of content and student, knowledge of content and teaching, and knowledge of content and curriculum required for being a mathematics teacher.

Mathematical extensiveness is the main focus of common content knowledge or CCK. The instructor has to be knowledgeable of mathematical theories, terminology, definitions, symbols, rules, and other relevant topics. Being an effective mathematics teacher requires having an understanding of all these factors. These mathematical topics are referred to as common content knowledge (Ball et al., 2008). It falls under the subject matter knowledge.

Specialized content knowledge (SCK) is the higher level of mathematical knowledge by which teachers create connections between how mathematics can be represented and it enables teachers, how mathematics can be applied (Jankvist et al., 2015; O’Meara, 2010). It includes deep and scientific knowledge of related disciplines.

Horizontal content knowledge (HCK) represents the structure of mathematical knowledge which enables us to explore how concepts are hierarchically related to each other and how together, these concepts are from the topic (Koponen et al., 2017). Thus teachers need to know the structure and logical connection of mathematical content from the perspectives of difficulties of content.

Knowledge of content and students (KCS) refers to those requirements for mathematics teachers, which enables them to know how students learn mathematics, and understand, whether students are liable to face challenges in learning mathematics as well as learning difficulties (Hill, Ball, et al., 2008). This type of knowledge enhances the teacher's capacity for the psychology of learning.

The knowledge of content and

teaching (KCT) refers to the skill and knowledge of being a teacher which enables the teachers to plan lessons, communicate and promote interaction in the classroom. The teachers need to be able to change or diversify the teaching method and strategy and organization of learning support for the students and improve their teaching (Ball et al., 2008). This type of knowledge enables the teacher on pedagogical knowledge and skills in mathematics instruction.

The knowledge of content and curriculum (KCC) is another dimension of the mathematics teacher preparation program. Mathematics teachers must have the knowledge and skill of selecting and using appropriate teaching material according to curriculum and content. All these features of knowledge can be encapsulated in terms of knowledge of the content and curriculum required to teach secondary-level mathematics (Jankvist et al., 2015). This knowledge enables the teacher to structure the mathematical knowledge to teach at the secondary level.

The foundation of mathematical knowledge for teaching (MKT) lies within the American context. This framework is based on Shulman's conceptualization (Shulman, 1986). This framework highly spread to all other countries: Finland (Koponen et al., 2017), Malawi (Kazima et al., 2016), Ireland (Delaney et al., 2008) etc. Although MKT is considered a popular framework for describing teacher knowledge. However several questions about what kind of mathematical teaching task is required (Ball et al., 2008) and what is the relationship between common content knowledge and specific content knowledge still require further inquiry.

This framework emphasis on knowledge on teaching method, curriculum and students. However, the perspective

teachers expectation may be more than these domain. The oppertunities of developing ICT competencies is one expectation of teacher in this era (Mumcu et al., 2022). This type of skill is essential for professional development. The teachers' identity in mathematics classroom impacts on students learning (Xenofontos & Andrews, 2023). Moreover, cultivating skills in teachers through positive psychology is essential (Cumhur et al., 2022). Therefore developing formative teacher should be the agenda in teacher preparation program. The instructional approach within university-level mathematics teacher preparation programs should establish a connection between didactic practices and school-level mathematics. This ensures that prospective teachers have opportunities to enhance their understanding and apprenticeship in teaching by actively observing and participating in mathematics instruction (Wasserman et al., 2023) at the university level.

This framework is employed for crafting research instruments and evaluating the expectations of mathematics teachers regarding their teacher preparation programs. It seeks to ascertain whether the elements proposed by theoretical frameworks align with the actual expectations of mathematics teachers and the perceived relevance of introduced advanced mathematical knowledge from this theoretical standpoint.

### **Content Knowledge for Mathematics Instruction**

Researchers are seriously raising the question of what kind of mathematical knowledge is relevant for being a secondary-level mathematics teacher and they have developed different models. All the models of different scholars agreed that mathematics teachers require content knowledge of mathematics and pedagogical content

knowledge. The focus point on the types of knowledge of advanced mathematics varies across the model. Among the models, a group of models viewed common content knowledge and specialized content knowledge required for mathematics teachers (Shulman, 1987; Rowan et al., 2001; Ball et al., 2005). Another group claimed that common content knowledge, horizontal content knowledge and specialized content knowledge are required for mathematics teachers (Hill, et al., 2008; Baumert et al., 2010; Hurrell, 2013). The content knowledge is required to equip the prospective teacher with a productive disposition and procedural skills (Crooks-monastra, 2021). Nevertheless, the common view of these models has focused on knowledge of advanced mathematics for being a secondary-level mathematics teacher.

The teachers' fundamental knowledge of the subject matter they are teaching represents the content knowledge that facilitates in-depth understanding with its logic (Shulman, 1998; Mustafa, 2008; Harel et al., 2006). Content knowledge influences pedagogical knowledge (Krauss et al., 2008; Sorto et al., 2009) and real teaching practices and professional practices (Mustafa, 2008; Arslan, 2017). The knowledge of mathematics beyond what the teacher teaches enables the teacher to design classroom activities and mathematical production that are generated by students and found in resources of teaching (Suzuka et al., 2009; Rivas et al., 2012; Markworth et al., 2009).

The types of mathematical knowledge have evolved from advanced mathematics categorized below.

#### **Axiomatic Knowledge**

The knowledge that describes the roots of the mathematical system comes under

the axiomatic knowledge of mathematics (Sachan et al., 2017). Mathematics teachers need to gain knowledge of the foundation of mathematical facts.

#### **Logical Knowledge**

Logical knowledge is concerned with logical structure and valid rules (Wasserman, 2018b). This type of knowledge has been employed to create proving theorems and solve the mathematical problem. Mathematics teachers require knowledge of logically proving mathematical facts. This knowledge helps to create proof learning opportunities such that the obstacles to learning proof can be minimized (Otten et al., 2021). Thus logical knowledge is required to become a teacher.

#### **Evolutionary Knowledge**

The mathematical ideas are developed based on some mathematical system. The knowledge helps to understand how mathematical propositions evolved or changed (Wasserman, 2018a). This knowledge focuses on how mathematical knowledge is developed and it is essential for mathematics teachers. Among these types of knowledge evolutionary knowledge is similar to mathematical activity including mathematical reasoning, creating and noticing, axiomatic knowledge and logical knowledge are similar to the mathematical activities, mathematical proficiency and mathematical context expressed by Heid & Wilson (2016). Evolutionary knowledge is reflected and associated with mathematical activities: mathematical reasoning, creating and noticing patterns, axiomatic knowledge, and logical knowledge.

#### **Research Methods**

The interpretive inquiry was considered as a research method to conduct the study.



### **Research Site and Participants**

For the researcher's convenience, the Dhaulagiri Birendara Secondary School Resources Centre of the Baglung district of Nepal was selected as a research site. Ten secondary schools of selected resource center were selected randomly. Ten secondary-level mathematics teachers were selected purposively. Among them, three were from private and seven were from government schools. Those who have graduated with at least mathematics as a major subject and who are familiar with each other were selected based on the purpose of delivering the relevancy of advanced mathematics and expected opportunities in advanced mathematics courses for school teaching. Their teaching experience varied from 5 to 17 years and they have studied analysis, algebra, calculus, analytical geometry etc at the undergraduate level.

### **Data Collection Tools**

Semistructure-interview is a useful research tool to extract the experience and views of the respondents through the certation questions (Patton, 2015). Thus the It was used to collect the teacher's views on the relevancy of advanced mathematics for secondary level mathematics instruction. Teachers were provided oppertunities to articulate their perspectives on the relevance of advanced mathematics in their school's teaching practices and their expectations from advanced mathematics and mathematics teacher preparation courses.

The research objective was to explore the mathematics teacher's view on the relevancy of advanced mathematics and the opportunities of advanced mathematics courses. To meet this objective focus group discussion was found to be suitable because it obtains in-depth feelings regarding

participants' attitudes, opinions, perceptions, motivations and behaviour (Patton, 2015; Hennink, 2014). Regarding focus group discussion, the group members should have potential and similarities, have trusting relationships, and have extensive experience on the topic (Morgan & Hoffman, 2015). This group was formed because the group members have a background in mathematics education in secondary school and have at least a bachelor's degree in mathematics or mathematics education. The ideal size of the focus group discussion is 8-10 because a large group limits the detail of responses and a smaller group creates uncomfortable pressure (Krueger & Casey, 2000). Thus only 10 members were included in the focus group discussion. The date and location for the focus group discussion were determined in accordance with participant consensus, and the discussion followed a protocol derived from responses obtained in in-depth interviews.

### **Data Collection and Analysis Procedure**

The primary open-ended queries presented during the semi-structured interview were as follows: (1) How do you perceive the significance of university-level advanced mathematics, such as real analysis, calculus, algebra, and number theory, in the context of teaching secondary mathematics? (2) What insights can be gained from the study of advanced mathematics (real analysis, calculus, algebra, and number theory) when it comes to teaching mathematics at the secondary level? These questions were presented to a group of ten secondary school mathematics teachers who had agreed to participate in the focus group discussion. Subsequently, interview responses were collected, and two follow-up focus group discussion was conducted based on participant agreement. The data was

collected in September 2022.

The thematic analysis process, familiarizing with data, generating codes, searching themes, reviewing themes, defining and naming themes, and generating reports (Braun & Clarke, 2006; Dakduk & Gonzalez, 2019) was utilized to analyze the data. Being guided by these processes, the interview record and focus group discussion record were transcribed in textual form. The transcribed data and responses on the interview records were studied and separated according to similarities as a group. Codes were made from similar groups of views. These codes were grouped and recoded by searching similar patterns of views. Themes were generated from re-coded data and these themes were named for interpretation.

The study upheld the criteria of trustworthiness, confirmability, and validity as outlined by Guba and Lincoln (1989). Furthermore, the study adhered to ethical considerations, including obtaining consent and maintaining a prolonged period of contact with participants. To keep the rigour, both expert and critical feedback from colleagues as noted by Cohen et al. (2007) and Creswell (2015) were considered.

### **Findings and Discussions**

All participants were male and had at least five years of teaching experience. The majority of the teachers got a master's degree in mathematics and mathematics education and the rest got a master's degree in other disciplines. The participant's experience and qualifications are supported to explain the relevancy of advanced mathematics and expectations of the advanced mathematics course for the betterment of the mathematics teacher preparation program. The finding after data analysis was prepared according to

the research questions.

### **Teachers' Voice on Relevance of Advanced Mathematics for Instruction**

To answer the first research question "How do secondary-level mathematics teachers perceive the relevance of advanced mathematics? The recorded responses by the participants and the transcription of the focus group discussion were analyzed. The six major themes were generated from data analysis.

#### ***Advanced mathematical knowledge is Required to Develop Mathematical Proficiency***

All participants' responses in the interview and views expressed in the focus group discussion indicated that advanced mathematics is relevant for developing mathematical proficiency. Without more mathematical knowledge rather than what they are required to teach is suitable for the advancement of their knowledge. A teacher viewed that:

Mathematics teachers need to have more mathematical knowledge than what they teach. Teachers need to have a strong mathematical background with a very wide range to explain, justify and link the contents with context, applying the theorem to solve the mathematical problem (T2, Personal Communication, September 10, 2022).

A similar viewpoint, "To tackle the problem, we need to have a deeper understanding of mathematics. I think that having solid fundamental knowledge facilitates the creation of logical links between the various element of matheamtis (T7, Personal Communication, September 10, 2022 )" This view is supported by T5. A deep understanding of the subject matter

is included in mathematical proficiency. All participants in the focus group discussion agreed that the teacher should possess greater mathematical expertise rather than only imparting it to students to guarantee the teacher's aptitude in the subject. A teacher viewed that "We need to acquire advanced mathematical knowledge to be a mathematics teacher with strong understanding and skill (T8, Personal Communication, September 10, 2022)" These viewpoints stressed the importance of advanced mathematics in helping mathematics teachers become mathematically proficient.

The finding focuses that advanced mathematics is relevant to develop mathematical proficiency which makes it easy to link and justify the mathematics content required to teach in school. This result is supported by the theoretical hypothesis that advanced mathematics is important for the development of specialized and horizontal subject knowledge in mathematics, as outlined by Ball et al. (2008). This finding is consistent with the theoretical meta-analysis by Davis and Simmt (2006), which showed that having a strong background in advanced mathematics is essential for teaching mathematics. Furthermore, the result justifies the claim made by Even (2011) that secondary mathematics training should include advanced mathematics.

#### ***Advanced Mathematics Helps the Teacher to Become a Resourceful and Expert Person***

Through the study of advanced mathematics, the teacher's writing focuses on resourcefulness. Response papers on opinion from 7 teachers out of 10 indicated that being resourceful is important. "School teachers need to be resourceful with subject matters' knowledge, which requires university-

level maths (T1, Personal Communication, September 10, 2022)" and another teacher felt similarly: "To be a resourceful maths teacher, higher maths levels are needed (T6, Personal Communication, September 10, 2022)". During the focus group discussion, T5 expressed, "Without knowing what we teach, how can we be resourceful or experts, so higher mathematics helps us to be resourceful." T2 supported T5 and said "Yes, I agree with you, content of advanced mathematics may be challenging, but it helps us develop and establish ourselves as capable math teachers (Personal Communication, September 10, 2022)." Because of this, the written response and opinions shared during the focus group discussion supported the relevance of advanced mathematics as a knowledgeable and effective mathematics teacher of secondary-level.

The information previously provided indicates that having a strong foundation in advanced mathematics is essential and enhances the expertise of aspiring teachers. The finding supports the theoretical assertion made by Ball et al. (2008). Furthermore, as Shulamn (1998) and Simmt (2011) explain, advanced mathematics is necessary for knowledge enhancement, which supports the requirement that teachers take advanced mathematics courses.

#### ***Knowledge Creation and Justification of Procedure of Knowledge***

The written response of the responders concentrated on university-level mathematics supporting to justify the mathematical knowledge and facts with the procedure of creating mathematical knowledge. According to the written response of the respondent:

In the university course, I have learnt how to derive the formula which we are using in school mathematics



which has always motivated me to further study. Besides the very difficult course, I have learnt the procedure of proof like direct proof, and indirect proof which justified the formulas which we have been using in teaching activities (T8, Personal Communication, September 10, 2022).

During the focus group discussion, a teacher said, "Creating proof and evaluating the developed mathematical knowledge is a remarkable thing gained from the understanding of university-level mathematics (T5, Personal Communication, September 10, 2022)." Another respondent stated, "As teachers, we need to explain how mathematical facts are generated; being able to do that job requires university mathematics (T6, Personal Communication, September 10, 2022)." These opinions show how important advanced mathematics is for developing and supporting mathematical understanding.

#### ***Developing Communication Skill for Learning New Mathematical Concept***

In the search for the relevancy of advanced mathematics, one focus point was "What do you get from the study of advanced mathematics?". All the participants agreed advanced mathematics is relevant for developing mathematical proficiency, being resourceful and expert personnel, and capable of creating and justifying mathematical knowledge. Besides these common voices, three of them indicated communication skills regarding mathematics. One teacher write, "In case of curriculum change sometimes we need to clear and learn new topics regarding the self-study, in this case, the base for learning terminology, symbols and systems is higher mathematics (T3)". During the focus group discussion, participants mentioned that they

learnt those contents at their graduation which guides the teachers to search the sources to solve and clarify any new content which occurs during instruction.

They expressed that they found it easy to understand the new content through the mathematical knowledge from advanced mathematics. In a focus group, discussion T3 said, "In my opinion, different symbols and terminologies learnt at campus level is helping me to link the ideas, the content in the topic transformation the more information I read from the help of bachelors' book" during focus group discussion.

The language of math exists. For maths-related activities, we must learn it. Communication skill here is not simple like talking but it is a complex concept which enables the teacher to comprehend the subject matter and make a mental map about the mathematical object when they read mathematical text. Studying new topics and content from the resources and grasping the mathematical concepts via communication skills.

Participants viewed that they benefited from advanced mathematics courses to develop communication skills across the mathematical discipline. This skill makes it easy to study new emerging mathematical content. Hence, more advanced mathematics is relevant to develop the communication skills required not only to teach but also to study a newly emerging subject matter. The assertion stated by Jankvist et al. (2015) and O'Meara (2010) that sophisticated mathematical understanding ensures that knowledge of how mathematics can be applied supports this finding.

#### ***Mathematical Fluency in Problem Solving***

All participants agreed that a teacher must be proficient in problem-

solving techniques to teach mathematics. They believed that university mathematics contributed to the occurrence of logic and algorithms in memory by observing the problem. This insightfulness capacity is developed due to the mathematical activities performed during the study of advanced mathematics. One teacher wrote "The occurrence of solution of any mathematical problem in our mind is the contribution of the study of advanced mathematics (T1)".

In the focus group discussion, one teacher explained:

I always remember a time that spent solving the problems of calculus and analytical geometry, when I saw a new problem. I feel I solved those hard problems with my bachelor's degree. so, in comparison school mathematics problem is very easy. I think this context contributed to me being a capable teacher of problem-solving (T4, Personal Communication, September 10, 2022).

Another teacher added, "hard problem-solving capacity which was developed all period of learning mathematics, advanced mathematics makes this capacity very strong and made us capable of solving school mathematics problems (T6, Personal Communication, September 10, 2022)". Other teachers agreed with the aforementioned view. Consequently, the ability of a teacher to offer insightful solutions and logical problem-solving during instruction is a valuable skill that is honed, matured, and fortified through the mathematical practices undertaken at the university level. Another significant finding in this study is the relevance of advanced mathematics in equipping school teachers with the logic and insight necessary to solve novel mathematical problems.

Participants agreed that their capacity for problem-solving is sharp by the study of advanced mathematics. Advanced mathematics is relevant to increasing the confidence level, motivational skill and content connecting knowledge and skill required for being a school mathematics teacher. The results of this study are comparable to those of other studies by Thompson (1984); and Zazkis and Leikin (2010) that advanced mathematics contributes to more confidence in mathematics teaching. Furthermore, the results of this study confirm the theoretical claim that secondary-level mathematics teachers require specialized subject knowledge and horizontal content knowledge theorized by Ball et al. (2008). Participants viewed that they required advanced mathematical knowledge to expertise them in the subject matter. This finding supports the finding that the quality of the instruction depends upon teachers' subject matter knowledge (Ball et al., 2008; Knuth, 2002). Hence advanced mathematics develops the capacity for insightfulness.

#### ***Content Connection between Advanced Mathematics and School Mathematics***

Participants expressed some content connection between undergraduate mathematics and school mathematics courses. Matrices and determinants of linear algebra are linked with the topic of optional mathematics at the secondary level. Transformation, sets, and field axioms found in undergraduate mathematics are more theoretical and their concrete application has been practised in school-level. Similarly other content also has direct or indirect connections. A participant viewed that "set, function, matrix, determinant, sequence, series, linear equation etc. are interconnected. Almost contents of optional mathematics is connected with the content of bachelor's mathematics (T1

Personal Communication, September 10, 2022)". During the focus group discussion all the participants agreed with the above-mentioned connection. Hence more or less content connection is an exit between school-level mathematics and advanced mathematics.

In addition to the connections mentioned above, participants also suggested that some topics in advanced mathematics should be changed. One participant argued that "some topics, such as field theory in algebra, more advanced content of group theory like Sylow's theorem, and polynomial rings, are not relevant to secondary school teaching and should therefore rethink about them (T8, Personal Communication, September 10, 2022)".

The participant reported that the course of advanced mathematics required linking the theory with the behavioural and concrete example that is practised in school teaching. This type of view is supported by a framework named Common Content Knowledge (Ball et al. 2008; Shulman, 1986) includes advanced mathematical theories and rules. Wasserman (2018) has identified the connection between advanced mathematics and secondary-level mathematics and pointed out : (1) content connection; (2) disciplinary practice connections; (3) classroom teaching connections; and (4) model instruction connection" p. 6). Among them, the content connection is justified by the finding of this study because participants explored it during focus group discussion and are supported by their views on the interview. However practising the experience of content connection slightly differs from the claim made by Dreher et al., (2016). The connection of top-down -direction is a beneficial connection between advanced and school-level mathematics (Allmendinger, 2016;

Klein, 2016) but this type of connection was found to be very weak.

To sum up, advanced mathematics is relevant for developing mathematical proficiency, problem-solving skills, connecting the mathematics axiom with reality, communication skills, mastery in mathematical activities, and mathematical reasoning and proving the theorem besides the poor connection between the contents.

### **Teacher Expectations of Opportunities from Advanced Mathematics**

To answer the research question, what opportunities do secondary-level teachers expect from advanced mathematics and mathematics teacher preparation courses? semi-structured interviews and focus group discussions were conducted to the data from the teachers. The data analysis resulted in the identification of the following four major themes.

#### ***Content Bridging Teaching Strategy at the Undergraduate Level***

According to the secondary school teachers' experience, advanced mathematics courses at the university level need to be set in such a way that the prospective teacher can easily find the link between mathematical axioms, facts, and theorems taught at the university level to show the applied link with school mathematics. A teacher viewed that "we learnt more theoretical content in university mathematics like field axiom these axioms are applying in school mathematics school level, it needs to give link of theory and practice in advanced mathematics course more clearly during the instruction in undergraduate level (T9, Personal Communication, September 12, 2022)". Other participants agreed with these cases.

The teacher's expectation from advanced mathematics is the theoretical

connection with the concrete example found in a secondary-level mathematics course. For example, field axiom:  $a*b = b*a$ ;  $a*(b*c) = (a*b)*c$  etc. are always used in school-level mathematics teaching. This type of abstract and concrete connection between advanced mathematics and school mathematics is expected by the teachers. This scenario indicates that advanced mathematics courses need to provide the chance of getting such experience for prospective teachers. The content bridging pedagogical practices need to address this type of expectations.

### ***Content Knowledge for Teaching***

"University courses of advanced mathematics required to add those courses which are beneficial to teach secondary courses like the connection between modern geometry and school geometry (T2, Personal Communication, September 12, 2022)". "The advanced mathematics course of teacher preparation program may expand according to the academic norm but some parts should link with mathematics courses of secondary level (T7, Personal Communication, September 12, 2022). These views were given by the teacher in focus group discussions. Thus, some parts of advanced mathematics include those parts which are directly connected to secondary-level mathematics to provide the chance for the prospective teacher. According to teachers, they admitted to sharpening their knowledge of pedagogy and content. They have experienced that the connective courses (the connection between advanced mathematics and school mathematics) are more beneficial for their teaching profession. Therefore, university teachers need to teach advanced mathematics by linking concrete examples and school mathematics courses.

### ***Integrating Technology for Mathematical Modelling***

One of the primary expectations for secondary-level mathematics teachers is the chance to learn new technology skills for modeling mathematical content.

One teacher wrote in his opinion paper "Advanced mathematics courses need to integrate with the technology such that we can understand the mathematics facts in visualisation form and share with students in instructions (T6, Personal communication, September 12, 2022)". During focus group discussion, another teacher shared his expectations, saying, "University courses give the opportunities in their courses to gain the knowledge on how to demonstrate the mathematical facts, without demonstrating the mathematical model, we can't feel and accept the truth of mathematics (T8, Personal communication, September 12, 2022)". Hence, it is imperative for mathematics teacher preparation programs to incorporate technology that visually illustrates mathematical concepts. This expectation justifies the claim made by Mumcu et al. (2022). Therefore, the mathematics teacher preparation program should integrate information communication technology.

### ***Research Skill***

Participants conveyed their need for action research skills as part of their professional development. One participant emphasized the significance of action research in professional practice, stating, "In our professional practices, action research plays a crucial role (T6, Personal communication, September 12, 2022)." Similarly, another participant noted, "I didn't have the chance to acquire specialized action research skills during my undergraduate studies, despite their common use in professional practice. Therefore, undergraduate programs should provide opportunities to develop research-

related skills (T2, Personal communication, September 12, 2022)." In the focus group discussion, all participants unanimously agreed that a fundamental understanding of research is essential for its integration into mathematics education courses. This finding justifies the suggestion provided by Altynbekov et al. ( 2023). Hence research skill is required to solve problem during professional life.

**Purposed Model on Mathematical knowledge for Teaching**

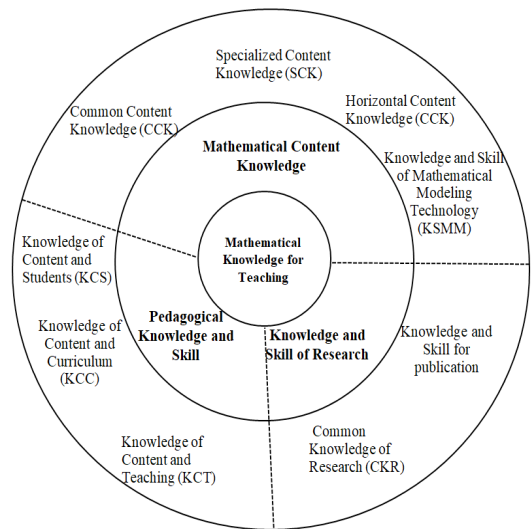
Secondary-level mathematics teachers have specific expectations regarding the content and subject matter knowledge they need for their professional roles. These expectations revolve around the connection between advanced mathematics courses and their teaching practice. To fulfil these expectations, a theoretical framework developed by Ball et al. (2008) is not sufficient therefore new components need to be added to this framework.

The previously developed framework serves as a structured approach or set of principles that guide the design and delivery of advanced mathematics courses, ensuring that they effectively address the content and subject matter knowledge needs of secondary-level mathematics teachers. These framework is common to the other frameworks developed by Shulman (1987); Rowan et al. (2001); Hill, et al., (2008); Baumert et al.(2010); and Hurrell (2013). The above-mentioned model falls short of Nepalese teachers' expectations for an undergraduate degree in advanced mathematics for teacher preparation in the areas of mathematical modelling and research skills on the relevant subject. Therefore a new model is required which adds mathematical modelling and skill in research in the developed model.

**Components of Mathematical Knowledge for Teaching**

From the review of the literature and findings of this study, one framework for components of mathematical knowledge of teaching at the secondary level is proposed. Additionally, it adds new components and provides a new frame which is shown in Figure 2. The resulting model, presented in Figure 2, is an amalgamation of Ball et al.'s (2008) model and the newly developed components for teacher preparation. It encompasses not only the knowledge and skills related to research but also introduces a technological aspect for mathematical modelling in mathematical content knowledge, as proposed by Ball et al. (2008). Hence it is a proposed model for the context of Nepal.

Fig. 2  
*Components of Mathematical Knowledge for Teaching*



**Conclusion and Implication**

The relevancy of advanced mathematics for secondary school teachers is an ongoing agenda of research. Advanced mathematics has proven its relevance in



teacher preparation by contributing to the development of mathematical proficiency. Prospective teachers gain valuable skills during their instruction, such as improved mathematical aptitude, greater competence in mathematical activities, enhanced confidence, stronger belief in their abilities, improved communication skills in the subject matter, heightened problem-solving capabilities, and the ability to connect and demonstrate the axiomatic foundations of mathematical principles.

Furthermore, the study highlights the importance of professional characteristics that mathematics teachers acquire through the study of advanced mathematics. Notably, the ability to incorporate suitable mathematical modeling skills, along with advanced mathematical concepts, and the integration of theoretical knowledge with concrete examples presented during instruction, are key strategies for enhancing mathematics teacher preparation programs.

The relevance of advanced mathematics courses for building mathematical proficiency and fostering fluency in mathematical problem-solving during instruction is emphasized. Secondary-level mathematics teachers have suggested three vital components to integrate into advanced mathematics courses for teacher preparation. The first component involves continuing the advanced mathematics course with a focus on common content knowledge, specific mathematical knowledge, and horizontal content knowledge. The second component emphasizes the knowledge of mathematics that bridges theoretical content with real-life examples. This component can be integrated with the first one.

To meet the expectations of Nepali teachers, two additional components are

proposed to be added to the existing model developed by Ball et al. (2008). The first is the use of mathematical modelling to illustrate mathematical truths and inspire students to appreciate the beauty of mathematics. The final component is the ability to conduct relevant research, which is desirable for inclusion in the new model.

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