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MODELING OF WATER POLLUTANT CONCENTRATION USING LAPLACE TRANSFORMATION

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ABSTRACT

This paper introduces a mathematical framework to model water pollution in river channels. The level of pollutants is measured over time using a one-dimensional, unsteady, nonlinear second-order advection-dispersion equation. This model assumes that the additional pollution rate along the river and the pollutants' dispersion coefficient are both constant. Additionally, K, the half-saturation oxygen demand concentration, should be zero. The Laplace transform method is applied to obtain the analytical solution for the concentration of water pollution The variation of pollution concentration is observed for different parametric values, i.e., with the variation of the rate of added pollutants, time, the rate of pollution concentration at the origin, and position. The work is analyzed geometrically. This work will be helpful for those researchers who are involved or want to be involved in the field of water pollution or the motion of contaminants in any medium where the concept of the advection-dispersion equation is used.

Keywords: water pollution, river dynamics, advection - dispersion, pollution, concentration, laplace transform, contaminants

INTRODUCTION

Numerous key challenges faced by humanity in the 21st century are associated with problems related to water availability and quality

(UNESCO, 2009). The fast and unplanned growth of cities, unchecked industries, and crowded urban areas are causing many problems for the natural environment (Goyal, 2011). More specifically, the degradation of the natural environment is due to poor waste management, public transport, excessive use of pesticides in agriculture, deforestation, release of toxic materials from industries, unmanaged drainage, and many more (Goyal, 2011). In some Eastern philosophies, water is often metaphorically linked to human civilization. However, contamination in these water resources, like ponds, rivers, and lakes in the urban areas, poses a significant challenge to the communities, vegetation, and other species that rely on them. Rising river pollution has sparked concern among researchers in various fields, including environmental engineering and mathematics (Paudel, 2021).

Water pollution arises from the discharge of untreated pollutants from both point and non-point sources into rivers. Point source refers to the specifically located source, like industry, from where pollutants are discharged into the river, and non-point source refers to the varying sources that are there along the length of the river from where pollutants are disposed to the river (Manitcharoen, 2020). The ongoing deterioration of water quality in the Bagmati River, revered as a holy river, serves as a compelling motivation for researchers to conduct systematic studies on water pollution. With the rise of environmental engineering and the need to model the transport of pollutants in groundwater and surface water, the advection-dispersion equation gained prominence and was developed during the latter half of the 20th century (Van, 1981). Due to their reliability, these equations have emerged in several fields, including environmental sciences, chemistry, and physics. These equations are incredibly employed for the spreading of pollution, which may take the form of any substance (gas, liquid, or solid) or energy such as radioactivity, heat, sound, or light (Goyal, 2011; Pariyar *et al.,* 2025).

The various reasons of water pollution leads to degradation of its quality and thus rendering it toxic to human and to the environment (Pariyar, 2024). According to World Meteorological Organization (WMO, 2022), Only around 0.5% of Earth's water is accessible and usable as freshwater, and climate change poses a serious threat to this limited supply. As per a recent World Health Organization report, inadequate water quality is responsible for 3.1% of fatalities and contributes to 80% of illnesses (Haseena, 2017). Since water is a common solvent, it can dissolve all the necessary and harmful substances. In recent years, various mathematical models have been created and applied to estimate pollutant levels and dissolved oxygen concentrations in river water. Advection is the bulk movement of solutes and dispersion is spreading of the contaminant plume from high to low concentrated areas.

Figure 1

The Contaminated Bagmati River in Kathmandu, Nepal (Self-taken).

The rising pollution levels in Nepal's Bagmati River are attributed to factors such as ongoing disposal of solid waste, domestic sewage, industrial effluents, inadequate wastewater treatment facilities, rapid urbanization, and a lack of awareness (Fig. 1) (Pal *et al.,* 2019). Indeed, rivers play a crucial role in aquatic ecosystems by transporting water and nutrients to various areas. However, when contaminated with pollutants, these waterways can become hazardous to human health and other species. Consumption of contaminated water or exposure to pollutants can lead to sickness and even death among both humans and wildlife, highlighting the critical importance of protecting water sources from pollution (Alruman, 2016). The motivation for this research stemmed from the elevated Biochemical Oxygen Demand (BOD), Chemical Oxygen Demand (COD), low Dissolved Oxygen (DO), and poor water quality of the Bagmati River in Nepal. This river is vital for drinking, irrigation, small-scale hydroelectric power, industrial use, and serves important recreational, cultural, and religious functions (Khadka *et al.,* 2019).

Different types of mathematical models using Advection-Diffusion equation have been solved by B. Pimpunchat in the time domain from 2007 to 2009 (Van *et al.,* 1981) formulated analytical solutions for chemical transport in porous media, incorporating zero-order production, first-order decay, and linear equilibrium adsorption (Paudel *et al.,* 2021) presented the analytic solution for the unsteady advection-dispersion equation using Heaviside function. In order to solve the advection-dispersion equation for pollutant and dissolved oxygen concentrations analytically (Paudel *et al.,* 2022) looked at steady-state situations with both zero and non-zero dispersion coefficients (Singh, 2015; Khadka *et al.,* 2019; Kannel, 2007) have studied related waste water treatments in Bagmati River, Nepal. A detailed study on wastewater engineering has been given by (George *et al.,* 1991). The unsteady advection-dispersion equations that model pollutant concentrations in one dimension have simple analytical solutions provided by (Wadi, 2014 and Pariyar *et al.,* 2024).

This work aims to solve the one dimensional unsteady pollutant equation using Laplace transforms. Additionally, it aims to evaluate the pollutant concentration over time in a particular direction of river flow and investigate how variations in different parameters affect the changes in pollution levels.

Nomenclature

A	River's cross sectional area (m^2)
D_X	Dispersion coefficient of the pollutant in the x-direction ($m^2.day^{-1}$)
U	The velocity of water in the x-direction ($m.day^{-1}$)
U_o	The water velocity at origin of medium ($m.day^{-1}$)
C	Concentration of pollutant ($kg.m^{-3}$)
k_1	Represents the rate at which pollutants are introduced into the river (day^{-1})
q	Denotes the half-saturation concentration of oxygen required for the breakdown of pollutants ($kg.m^{-1}.day^{-1}$)
k	Refers to the concentration of dissolved oxygen present in the river water ($kg.m^{-3}$)
X	Indicates the degradation rate coefficient of pollutants within the river system ($kg.m^{-3}$)
t	Time (day)

MATHEMATICAL MODEL

There is only one spatial parameter along the x-direction in the one-dimensional model of river flow. Upstream, close to the source, no pollutants are thought to be added, whereas downstream, pollutants are added at a rate of q (Pimpunchat *et al.,* 2007). An unstable movement of the concentration of water pollutants in a single dimension can be expressed as

$$\frac{\partial(\mathrm{AC})}{\partial t}=D_x\frac{\partial^2(\mathrm{AC})}{\partial t^2}-\frac{\partial(UAC)}{\partial x}-k_1\frac{X}{X+k}AC+q;\quad 0\le x<L,\quad t>0 \tag{1}$$

Such that $U = U_0(1+ \in\sin2\pi x)$; U and U_0 denotes velocity in the x-direction and at origin of medium, C is the concentration ($kg.m^{-3}$), D_x is the dispersion along x-direction ($m^2.day^{-1}$), k_1 is the degradation (day^{-1}), q is the rate of pollutant addition ($kg/m.day$), k is the half-saturated decay oxygen demand concentration ($kg.m^{-3}$), X is the concentration dissolved oxygen ($kg.m^{-3}$) and A is the area (m^2) of cross section.

The added pollutants are assumed to be evenly distributed across the river's cross-section but can vary along its length (Pimpunchat, 2009). It is believed that no pollution is added upstream $x < 0$ near the source, whereas pollution is introduced downstream ($0 < x < L$) and the pollutant is added at a rate q (Pimpunchat, 2009). *P*arameters A, q, and k_1 as constants over time and space and assume a small river, which is regarded as a homogeneous system (Li, 2006). Obtaining an exact solution using the Laplace transform is not feasible when the pollutant decay's half-saturated oxygen demand concentration satisfies $k \neq 0$ (Wadi, 2014). In this paper we assume $k = 0$ and used Laplace transform technique to solve Advection Dispersion equation.

For significantly larger pollutants, in this model we take D_x is close to zero (Pimpunchat *et al.,* 2007). By using the above conditions, the equation (1) becomes

$$\frac{\partial(C)}{\partial t}=-\frac{\partial(UC)}{\partial x}-k_1C+\frac{q}{A};\qquad 0\le x<L,\quad t>0 \tag{2}$$

$$\frac{\partial(C)}{\partial t}=-\frac{U\partial C}{\partial x}-k_1C-U_0\in2\pi C cos 2\pi x+\frac{q}{A} \tag{3}$$

Under initial and boundary conditions,

$$C(x,0)=0;\qquad x\le 0 \tag{4}$$

$$C(0,\ t)=p;\qquad t>0 \tag{5}$$

C(*x, t*) represents the pollutant concentration, $D_x = 0$, the initial pollution rate along the river is zero, and the pollution rate at the origin (x = 0) is specified.

Analytical Solution

Equation (6) illustrates the use of the Laplace transform method to derive the analytical solution. The Laplace transform as follows: if *f*(*x*, *t*) is any function on $a \le x \le b$ and $t > 0$, then its Laplace transform t is,

$$L\{f(x,t)\} = F(x,s) = \int_0^{\infty} e^{-st} f(x,t)\, ds, s \succ 0 \tag{6}$$

Where *s* is transform variable (Kumar, 2011; Doetsch, 1970). The inverse Laplace transform is $L^{-1}\{F(x, s)\} = f(x, t)$ and

$$L^{-1}\{F(x,s)\} = f(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} F(x,s)\, e^{-st} f(x,t)\, ds, c \succ 0 \tag{7}$$

We apply the Laplace transform to solve equation (3) and use equations (4) and (5),

$$s\overline{C}(x,s) - C(x,0) = -U\frac{\partial \overline{C}(x,s)}{\partial x} - k_1\overline{C}(x,s) - U_0 \in 2\pi\overline{C}(x,s)\left(\frac{s}{s^2+(2\pi)^2}\right) + \frac{q}{As} \qquad s > 0$$

$$s\overline{C}(x,s) - C(x,0) = -U\frac{\partial \overline{C}(x,s)}{\partial x} - \left(k_1 + U_0 \in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\overline{C}(x,s) + \frac{q}{As} \tag{8}$$

$$\overline{C}(0,s) = \frac{p}{s} \tag{9}$$

By applying (4) to equation (8), we obtain:

$$s\overline{C}(x,s) - U\frac{\partial \overline{C}(x,s)}{\partial x} - \left(k_1 + U_0 \in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\overline{C}(x,s) + \frac{q}{As};$$

When we simplify, we obtain

$$\frac{\partial \overline{C}(x,s)}{\partial x} + \left(s + k_1 + U_0 \in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0(1+\in\sin 2\pi x)}\overline{C}(x,s) = \frac{q}{AsU_0(1+\in\sin 2\pi x)} \tag{10}$$

where $S > 0$ is the Laplace transform variable.

From equation number (10),

$$I.F. = e\int p dx = e^{\int\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0(1+\in\sin 2\pi x)}dx}$$

$$= e^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{\tan\pi x-\in+\sqrt{1+\in}}{\tan\pi x-\in-\sqrt{1+\in}}\right)}$$

Multiply equation (10) by I.F. and integrate, we get

$$\overline{C}(x,s)(I.F.) = \int\left(\frac{q}{As}\right)\frac{1}{U_0(1+\in\sin 2\pi x)}.e^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{\tan\pi x-\in+\sqrt{1+\in}}{\tan\pi x-\in-\sqrt{1+\in}}\right)}dx$$

$$\overline{C}(x,s)=\frac{\left(\dfrac{q}{AsU_0}\right)\displaystyle\int\frac{1}{(1+\in\sin 2\pi x)}\cdot\left(\frac{\tan\pi x-\in+\sqrt{1+\in}}{\tan\pi x-\in-\sqrt{1+\in}}\right)^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}}dx+c}{e^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{\tan\pi x-\in+\sqrt{1+\in}}{\tan\pi x-\in-\sqrt{1+\in}}\right)}}$$

where c is arbitrary constant.

$$\overline{C}(x,s)=\frac{\left(\dfrac{q}{AsU_0}\right)\displaystyle\int\frac{1}{(1+\in 2\sin\pi x.\cos\pi x)}\cdot\left(\frac{\sin\pi x-\left(\in-\sqrt{1+\in}\right)\cos\pi x}{\sin\pi x-\left(\in-\sqrt{1+\in}\right)\cos\pi x}\right)^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}}dx+c}{e^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{\tan\pi x-\in+\sqrt{1+\in}}{\tan\pi x-\in-\sqrt{1+\in}}\right)}}$$

On simplifying, we get

$$\overline{C}(x,s)=\left(\frac{q}{(2.004)AsU_0\pi}\right).\frac{U_0 2\pi\sqrt{1+\in}}{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)}+ce^{-\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{\tan\pi x+0.997}{\tan\pi x-1.007}\right)} \tag{11}$$

Now, applying condition (9) to equation (11) and simplifying, we get

$$\overline{C}(x,s)=\left(\frac{q}{(2.004)As}\right)\left(\frac{2\pi\sqrt{1+\in}}{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right)}\right)$$
$$+\left(\frac{p}{s}+\left(\frac{q}{(2.004)As}\right).\frac{2\pi\sqrt{1+\in}}{s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)}\right)e^{\left(s+k_1+U_0\in 2\pi\left(\frac{s}{s^2+(2\pi)^2}\right)\right).\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{1.007+\tan\pi x}{0.997+\tan\pi x}\right)\left(\frac{0.997}{1.007}\right)} \tag{12}$$

Taking inverse Laplace transform,

$$C(x,t)=\left(\frac{q2\pi\sqrt{1+\in}}{(2.004)A}\right).u(t).\left(\frac{1}{u(t)+k_1+U_0\in 2\pi\cos 2\pi t}\right)+\left(p.u(t)-\left(\frac{q.2\pi\sqrt{1+\in}}{(2.004)As}\right).u(t).\frac{1}{(u(t)+k_1+U_0\in 2\pi(\cos 2\pi t))}\right)$$
$$e^{(t+k_1+U_0\in 2\pi).\frac{1}{U_0 2\pi\sqrt{1+\in}}.\ln\left(\frac{1.007-\tan\pi x}{0.997+\tan\pi x}\right)\left(\frac{0.997}{1.007}\right)}.\cos 2\pi t \tag{13}$$

where u (t) is the unit step function.

RESULTS AND DISCUSSION

The concentration in (10) is dimensional, with parametric values

$t = 1$ *day*,	$t = 2$ *days*,	$t = 3$ *days*	
$q = 0.05$ *kg/m*,	$k_1 = 8.27$ *per day,*	$A = 2100\ m^2$	$\in = 1.0$
$U_0 = 0.11$,	$p = 0.03$,	*0.05,*	0.07

The concentration profile behavior is shown in the dimensional distribution for different scenarios.

Figure 2

Shows C(x, t) as a Function of Space and Time Along the River for Different Values of x Where p and q are Constant. It is Seen that as t Increases, the Value of C(x, t) Increases.

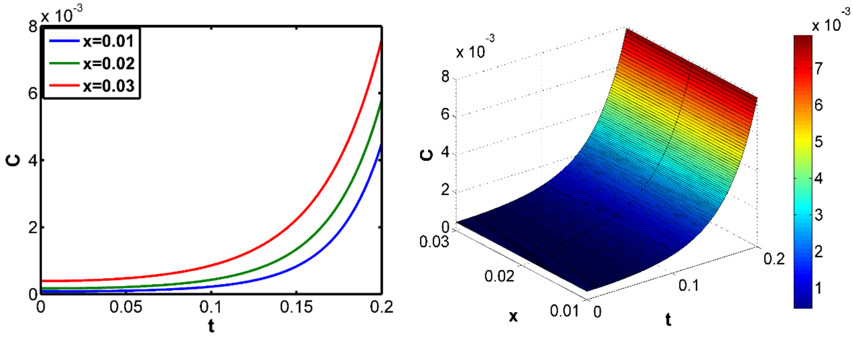


Figure 1 represents the concentration profile against the time ($0 \leq t \leq 1$) for constant values of p and q and different $x = 0.01, 0.02$ *and* 0.03. The effect of position (x) is very small near the origin and dominant away from the origin. For all three positions, the pollution concentration is very less and closed to each other near the initial time (from $t = 0$ to $t = 0.1$) and increases exponentially after certain time (after $t = 0.1$). As the distance increases along the river channel, the pollution concentration also increases slowly near for small time and rapidly for large time. In overall, this figure shows that, pollution concentration *C(x, t)* increases with increase in time as well as distances along the river channel.

Figure 3A

Shows C(x, t) as a Function of Space and Time Along the River for Different Values of q Where p and x are Constants. The Pollution Concentration C(x, t) Increases at Every Time with the Increment of Rate of Added Pollutants q.

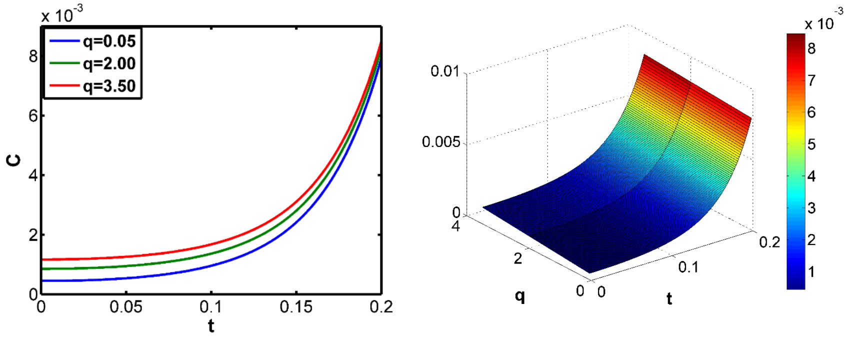


Figure 3A illustrates how the concentration with time ($0 \leq t \leq 1$) for increasing values of rate of added pollutants q taken as *0.05, 2.00 and 3.50* (kg/m^{-3}) and constant p and x. For the initial time i.e., $t = 0$, there is vast difference between pollution concentrations for different values of rate of added pollutants q. As time increases, the curves for these three different values of q gets closer to each other showing that rate of increment of pollution concentration for least value of q is higher than that for most value of q. The concentration increases over time at each point along the river. Very comparable values of $C(x, t)$ is seen at longer times. The effect of rate of added pollutants q is very small at larger time and dominant near the initial time. Overall, As q increases, $C(x,t)$ rises at each river cross-section.

Figure 3B

Shows C(x, t) as a Function of Space and Time Along the River **A:** *The Pollution Concentration C(x, t) Increases as the Value of p Increases.* **B:** *Different Values of Rate of Pollutants at the Origin p, Where q and x are Constants.*

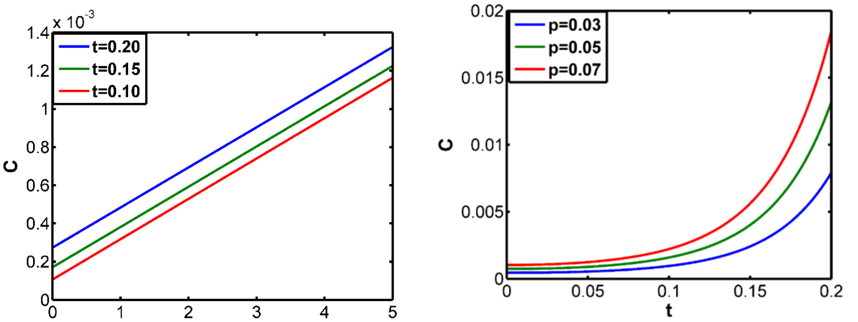
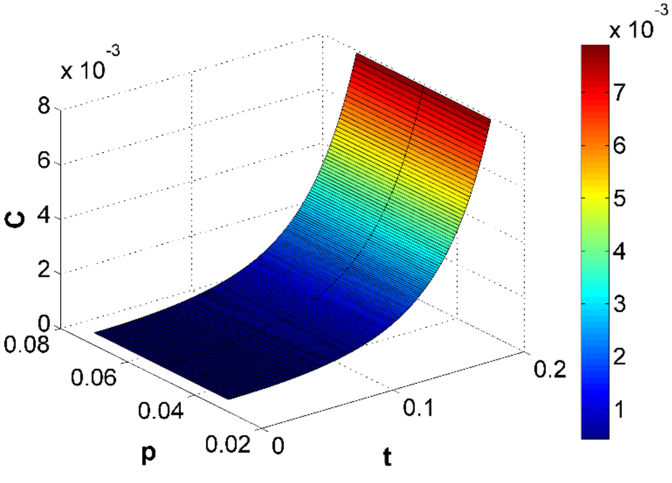


Figure 3B represents the concentration profile against the rate of added pollutants values q ($0 \leq q \leq 5$) for increasing value of time t and constant x and p. The concentration $C(x, t)$ increases uniformly as the rate of added pollutant increases over the length of the river. For a fixed position, the concentration of pollutant is higher for high time than for less time. With increasing time interval, the graph of q gradually coincides. Figure 3B depicts the concentration profile over time ($0 \leq t \leq 0.2$) for a constant value of and varying q and x different values of p. As time progresses, $C(x, t)$ increases at the river channel's cross-section. The value of pollution concentration $C(x, t)$ for various values of p is seen very close and comparable to each other near the initial time. The impact of p is minimal at the start but becomes significant near the end. Though the rate of increment of pollution concentration is comparable near the small time, it is very rapid for most value of rate of pollution concentration at origin p

than that of least value of *p*. As the pollutant rate at the origin *p* increases, the concentration *C(x, t)* rises at all river cross-sections.

Figure 4

Pollutant Concentration Increases Over Time and Distance, Accelerating with High p ans q Rates.



Results show that pollution concentration increases with distance and time, starting slowly and then accelerating. The concentration also increases with the rate of added pollutants *q*, with significant differences at the origin but minimal differences over time. Similarly, the concentration varies with different rates of pollution at the origin *p*, increasing more rapidly over time.

CONCLUSION

We developed a straightforward mathematical model for water pollution in a river channel, utilizing a one-dimensional, nonlinear, second-order advection-dispersion equation to analyze pollutant concentration along a single direction. We derive the analytical solution using the Laplace transform of the equation, assuming no added pollutant rate along the river *q* to be zero and zero dispersion $D_x = 0$. We observed the dynamics of pollution concentration *C*(*x, t*) profile along the time by using various parameters used in the equation. The time is measured in days, and the pollutant concentration *C*(*x, t*) is taken in kg/m^{-3}. The increase in C(x, t) with distance over time reflects the combined effects of advection and diffusion in the model. While dilution reduces concentration locally, continuous pollutant input or limited dispersion mechanisms can lead to higher concentrations downstream. This highlights the importance of addressing pollutant sources rather than relying solely on dilution. The pollution concentration C(x, t) increases with the increase of distance along the time. The concentration increases with very slow rate for the first few time and more rapidly for maximum time. Similarly, the pollution concentration increases with increase in rate of added pollutants q along the time. It has been shown that concentration for all three different values of q at the origin is very different from each other while this difference becomes very less at maximum time, i.e., the rate of added pollutants has less effect at maximum time and more effect near the origin. In the same way, the pollution concentration is also varied with various values of rate of pollution at the origin p. The results indicate that the concentration increases with an increase in p over time The rate of increment is very less for minimum time and that is maximum for maximum time. All the work is shown in two dimensional graphs. This work will help the researchers who are involved or want to involve in the field of pollution or motion of contaminants in any medium.

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