Quantum squeezing for Dirac fields

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Abstract: Squeezing of Dirac field with expansion of the universe is analysed for its dependence on the scale factor. It is found to peak at a time, and also develops a local minimum for some combinations of k, M and other parameters, indicating possibilities of structure formation. Also, the local minimum could indicate effects that could account for some of the dark matter.

Keywords: FLRW; Space-time; Squeezing; Large-scale structure; NP formalism.

Introduction

Quantum squeezing has gained applications[1] in wide varieties of fields. In gravitational wave detection[2,3], it has helped to improve the sensitivity[4] of gravitational wave detectors like LIGO while it enhances precision of measurements in astronomy enabling to observe the fainter and more distant objects[5,6,7]. Squeezed states not only can enable more secure and efficient transmission of quantum information[8,9,] for quantum communication but also enhance the precision of quantum gates and operations for quantum computing. It can also be used to create more precise sensors contributing towards advancements of various areas including high precision[10] timekeeping, imaging, radar, etc.

Quantum squeezing is the manipulation of Heisenberg's uncertainty principle which states that certain pairs of physical entities, like position and momentum, cannot be precisely measured simultaneously beyond certain limit[11]. In quantum squeezing, the quantum state of a system is prepared such that the uncertainty in one entity is reduced below the limit set by the uncertainty principle while allowing the uncertainty in another to increase.

This paper is our attempt to explore the quantum squeezing for Dirac fields with the objective of gaining further deeper knowledge about role of these fields/particles in Lage-scale structure formation using the basics of our previous work[12]. For the ready references, some necessary mathematical equations from the work have been gathered here which will be used to derive the squeezing in the next section.

The metric for FLRW space-time can be written as[13,14,15]

$$ds^2 = a^2[d\eta^2 - dr^2 - S^2(d\theta^2 + \sin^2\theta \, d\phi^2)], \ldots (1)$$

where a is the scale factor which depends on the conformal time η and related to the co-moving time t by $dt = a\, d\eta$, and

$$S = \frac{\sin\sqrt{k}r}{\sqrt{k}} = \begin{cases} \sin r, & k = 1 \ for\ closed \\ r, & k = 0 \ for\ flat \\ \sinh r, & k = -1 \ for\ open \end{cases} \ \ldots.(2)$$

The temporal part separated from Dirac field equations written in NP-formalism came to be

$$\frac{1}{T_\pm}\left(\frac{\partial}{\partial\eta} \pm iMa\right)T_\pm = ik \quad \ldots.(3)$$

The separation constant is related to the co-moving momentum: $k = pa$

The decoupled equations are of second order and are given by

$$\left(\frac{\partial}{\partial\eta} \pm iMa\right)\left(\frac{\partial}{\partial\eta} \mp iMa\right)T_\pm = -k^2 T_\pm \ldots (4)$$

This can further be written as

$$\left(\frac{\partial^2}{\partial\eta^2} + k^2 + M^2a^2 \mp iMa'\right)T_\pm = 0 \ldots (5)$$

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$$a'^2 = \left(\frac{da}{d\eta}\right)^2 = H_0^2[\Omega_\Lambda a^4 + \Omega_k a^2 + \Omega_M a + \Omega_R] \quad \ldots (6)$$

is given by the Friedmann-Lemaitre equation such that Ω_Λ, Ω_K , Ω_M, and Ω_R are densities contributed by the cosmological constant, curvature, matter and radiation, respectively, in units of the critical density at the time η_0 when the scale factor is normalized to $a_0 = 1$.

The temporal wave functions also satisfy the condition

$$\frac{\partial |T_+|^2}{\partial \eta} = a' \frac{\partial |T_+|^2}{\partial a} = k(T_+^* T_- + T_+ T_-^*) \quad \ldots (7)$$

The equations for the average comoving energy of one Dirac particle and its time evolution are given by

$$\varepsilon a = k(T_+^* T_- + T_+ T_-^*) - Ma(|T_+|^2 - |T_-|^2) \ldots (8)$$

$$\frac{d\varepsilon a}{da} = -M(|T_+|^2 - |T_-|^2) \ldots (9)$$

Quantum squeezing for the Dirac field

Let's write the temporal wavefunction as

$$T_\mp = \begin{cases} \cos\theta \ e^{ix_+} \\ \sin\theta \ e^{ix_-} \end{cases} \quad \ldots (10)$$

Then

$$|T_\mp|^2 = \frac{1}{2}(1 \pm Cos\ 2\theta) \ldots (11)$$

and the normalization condition is also satisfied:

$$|T_+|^2 + |T_-|^2 = 1 \quad \ldots (12)$$

Using Eq. (10) in equations (8) and (9), we get

$$\frac{d\varepsilon a}{dMa} = Cos\ 2\theta\ , \ldots (12)$$

$$\frac{\varepsilon a}{M} - a\frac{d\varepsilon a}{dMa}$$

$$= a^2 \frac{d\varepsilon}{da} = -k\ Cos\ (x_+ - x_-) Sin\ 2\theta \ldots (13)$$

Similarly, from Eq. (7),

$$\frac{\partial |T_\mp|^2}{\partial a} = \pm \frac{ik}{a'}\ Sin\ (x_+ - x_-) Sin\ 2\theta \ldots (14)$$

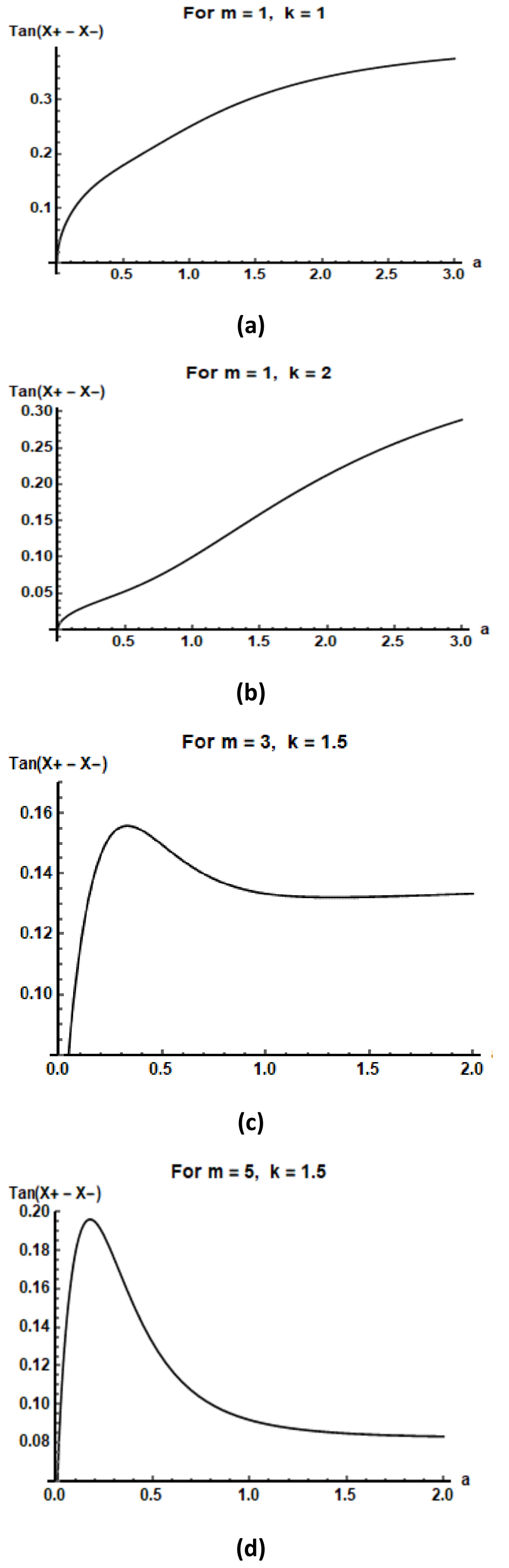


Figure 1: Plots of $Tan\,(x_+ - x_-)$ against the scale-factor '*a*'. The first two are for smaller mass with relatively higher momentum (K) states are found to be ever growing. The last two plots are for larger mass with relatively lower momentum states in which the curve flattens after descending from peak. Peaks have become sharper for larger masses.

Dividing Eq. (14) by Eq. (13) gives $Tan\,(x_- - x_+) = \frac{a\prime}{2M}\frac{\frac{d^2\varepsilon a}{da^2}}{a^2\frac{d\varepsilon}{da}} = \frac{a\prime}{2Ma}\frac{\frac{d^2\varepsilon a}{da^2}}{\frac{d\varepsilon}{da}}$

$$= \frac{a\prime}{2Ma}\frac{d}{da}ln\left(\frac{d\varepsilon}{da}\right) = \frac{a\prime}{2Ma}\frac{d}{da}ln\left(\frac{3\rho}{\rho_{rest}}\right)$$

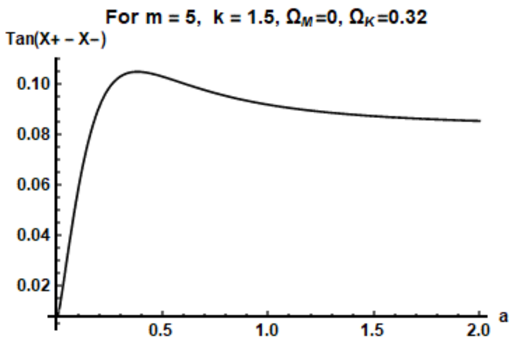
$$= \frac{a\prime}{2Ma}\frac{d}{da}ln(\rho a^4) = -\frac{a\prime M}{2(a\varepsilon)^2} \ldots(15)$$

Using Eq. (6),

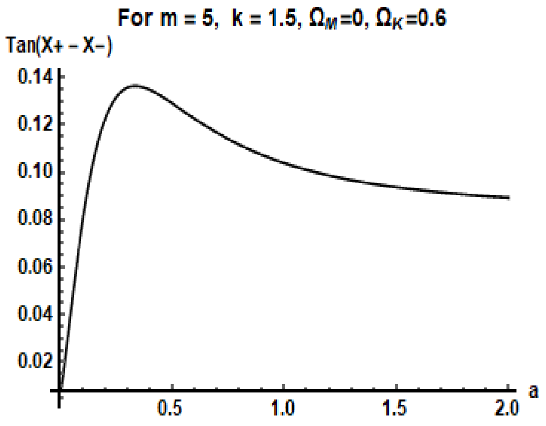
$$Tan\,(x_+ - x_-) = \frac{a\prime M}{2(a\varepsilon)^2}$$

$$= \frac{H_0 M}{2(a\varepsilon)^2}\sqrt{\Omega_\Lambda a^4 + \Omega_K a^2 + \Omega_M a + \Omega_r} \ldots (16)$$

where $(a\varepsilon)^2 = k^2 + (aM)^2$



(a)



(b)

Figure 2: Plots of $Tan\,(x_+ - x_-)$ against the scale-factor 'a' similar to Fig. 1 but Ω_M = 0 compensated by larger proportion of Ω_K. The first one for Ω_K = 0.32 and the second one for 0.60. Peaks have become sharper for larger proportion of curvature in the density.

Some typical plots of this squeezing have been shown in Fig. 1 and Fig. 2.

Discussion and Conclusions

Dirac particles/fields are among the fundamental ingredients of the universe and hence their properties and dynamics provide crucial information about the structure formation in the universe. In this paper, we have tried to analyse how the squeezing of Dirac field evolves with time. The plots in Fig.-1 (c and d) show that the squeezing peaks at some time and then falls down to flatten indicating that the chance of formation of the large scale structures is high at that time. But these are the cases only for low momentum states for relatively larger masses. The plots are ever-growing for higher momenta states with relatively smaller masses as shown in figures 1 (a) and (b). For all of these Ω_Λ, Ω_K , Ω_M, and Ω_R are typically taken to be 0.683, 0.001, 0.316 and 0.00005 respectively such that Ω_{Total} is slightly more than 1. In Fig.2, the plots are for Ω_M= 0 with larger proportion of Ω_K. As the Ω_K increases, the peak becomes sharper. Also, the local minimum could indicate effects that could account for some of the dark matter. These are very preliminary work and will be further extended in future works.

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