ESTIMATION OF MASS, COSMOLOGICAL CONSTANT AND KERR PARAMETER OF DIFFERENT GALAXIES USING GEODESIC MOTION IN KERR-DE SITTER SPACE-TIME

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Abstract: In the present work, geodesic trajectories in Kerr-de Sitter geometry is used to fit the rotation curve of galaxies and estimate the values of mass M, Kerr parameter *a*, and cosmological constant Ë. In this work, non-linear least square fitting of rotational velocity is used to estimate the some of the parameters. These estimates are mass in the range of (0.12±0.02) $x10^{10}$ M⊙ to (70.37±11.26) $x10^{10}$ M⊙, the Kerr parameter in the range of (0.7044±0.1127) to (0.9990±0.1598), and the cosmological constant in the range of (1.790±0.286) $x10^{-49}$ km^{-2} to (7.523±1.204) $x10^{-42}$ km^{-2} for positive value of Ë while - (3.983±0.637) $x10^{-41}$ km^{-2} to - (1.860±0.298) $x10^{-42}$ km^{-2} for negative value of Ë. The negative Ë values are found only for some galaxies with large recessional velocity. This suggests that the transition from anti-de-Sitter to de-Sitter phase with the deposition of vacuum energy may have occurred between the formation of these different galaxies.

Keywords: NGC and UGC type galaxies; Geodesic motion in Kerr-de Sitter space-time; Rotational curve data.

INTRODUCTION

Cosmological constant Ë was introduced by Einstein to balance the evolutionary models with repulsion to set a steady state. He abandoned it later as a blunder. When Hubble expansion was seen, it has now returned with a vengeance as the accelerating expansion to contribute 75% of the density of the universe as dark energy. Ë can also be introduced as the vacuum energy that is required to drive inflation (Akcay, 2009). The accelerating expansion may even be interpreted as the continuation of inflation, possibly at a slower rate than in the early universe.

A black hole is supposed to possess three physical properties: mass, angular momentum and charge. The trajectory, of massive and massless particles in various geometries, is described by the geodesics. In particular, the behaviour of massive and massless objects around a spherically symmetric gravitating body is described by Schwarzschild formalism. Schwarzschild space-time is a solution obtained from the Einstein's field equations that is static. When we introduce Ë in Schwarzschild space-time solution, we obtain Schwarzschild-de Sitter space-time. Unlike Schwarzschild solution, Kerr solution is an axi-symmetric solution of Einstein's field equations corresponding to a rotating black hole. A rotating black hole in asymptotically de-Sitter space-time can be described by Kerr-de Sitter geometry.

THE SAMPLE

Database: We have selected 28 galaxies using Sofue et.al (2007) and 2 galaxies (NGC 3379 and NGC 4100) from literature of J. R. Brownstein & J.W.Moffat (2005) (Digitized by software Labfit). A galaxy needs to fulfill the following two selection criteria in order to be selected:

(1) Galaxy rotational curve should be given.

(2) Morphology and distance should be known.

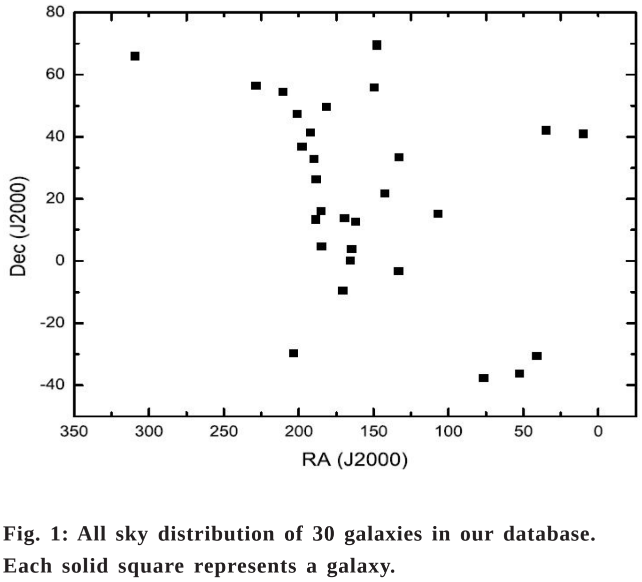

Fig. 1: All sky distribution of 30 galaxies in our database. Each solid square represents a galaxy.

Morphological Distribution: We classified our database according to the morphology of galaxies into nine sub-

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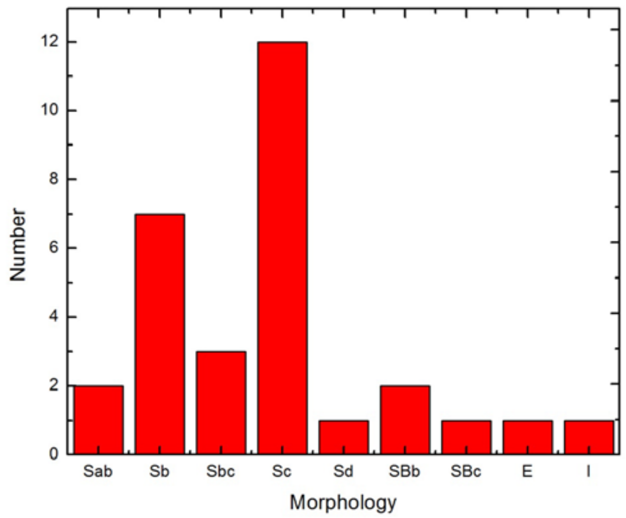

Fig. 2: Morphological distribution of galaxies in our database.

samples. Fig. (2) shows morphological distribution of our sample galaxies. We have considered Hubble's extended classification for the morphological classification of spiral galaxies in our database. The galaxies of morphological type Sa, Sab, Sb and Sbc are considered as early-type spirals whereas Sc, Scd, Sd and Sdm are late-type spirals. There are two barred spiral type galaxies (SBb & SBc) in our database. In addition to these, we have also elliptical and irregular galaxies in our database. In these samples, we can see some galaxies are Irregular, Interacting as well as active type of galaxies. From the morphological distribution, it is seen that there is a majority of Sc type galaxies in our database (12 in number). After Sc type spirals Sb type spirals has dominated our database (6 in number). Barred type spirals (SBb and SBc) as well as I, E & Sd type spirals are equal in numbers, each of them equal to one.

Method of Analysis

In the present work, we derive the time like geodesic motion in Kerr-de Sitter space-time. Taking distance from center as independent parameter and velocity as dependent parameter, we can see mass, angular momentum per unit mass and cosmological constant as constant parameters. Thus, we estimate the values of the mass, cosmological constant and angular momentum per unit mass of different galaxies assuming the circular motion of the galaxies and using their rotation curves. Since this geodesic equation is a non-linear one, determination of the values of its constant parameters is a tedious task. Therefore, we carried out in two steps.

Step I:- Preliminary estimation by trial method : In this initial step we took the rotational velocity of the particle around the central mass in the invariant plane and its value is given by

$$\frac{V_\varphi}{c} = \mp \frac{[(r^2+a^2)\sqrt{a^2-\Delta_r+\frac{r}{2}\Delta_r'} \pm a(r^2+a^2-\Delta_r)]}{(r^2 \mp a\sqrt{a^2-\Delta_r+\frac{r}{2}\Delta_r'})\sqrt{\Delta_r}}$$

Where,

$$\Delta_r = (r^2+a^2)\left(1-\frac{\Lambda r^2}{3}\right) - 2Mr$$

M=Mass of the galaxy

r= distance from the galactic center (radius).

a= rotational Kerr parameter of the black hole.

Ë = cosmological constant.

c = velocity of light.

First we studied the nature of above equation by varying Ë , M and *a* separately. Natures of above equation with the variation of constant parameters are shown in fig. 3 and fig. 4.

Step II:- Graphical Method:

This is the final step in which we used software Matlab7.6. We carried out non-linear least square curve fitting technique. This parametric curve fitting requires initial nearby values for which we supplied data from the first step. By the application of this software, we found precise values of mass distribution of galaxies, cosmological constant and Kerr parameter. Determination of value of *a* is more sensitive than others. Because we saw in fig (4) that the contribution to the curve due to the value of *a* is negligible. However, it affects on goodness of fit statistics. We have used some goodness of fit statistics. These are

- The sum of squares due to error (SSE)
- R-square

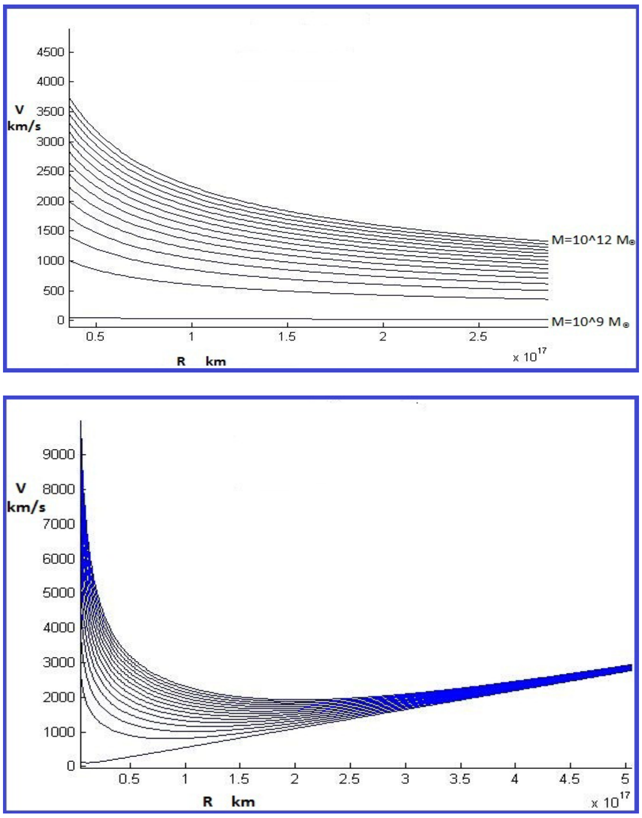

Fig. 3: Variation of mass keeping Ë positive constant and negative constant respectively.

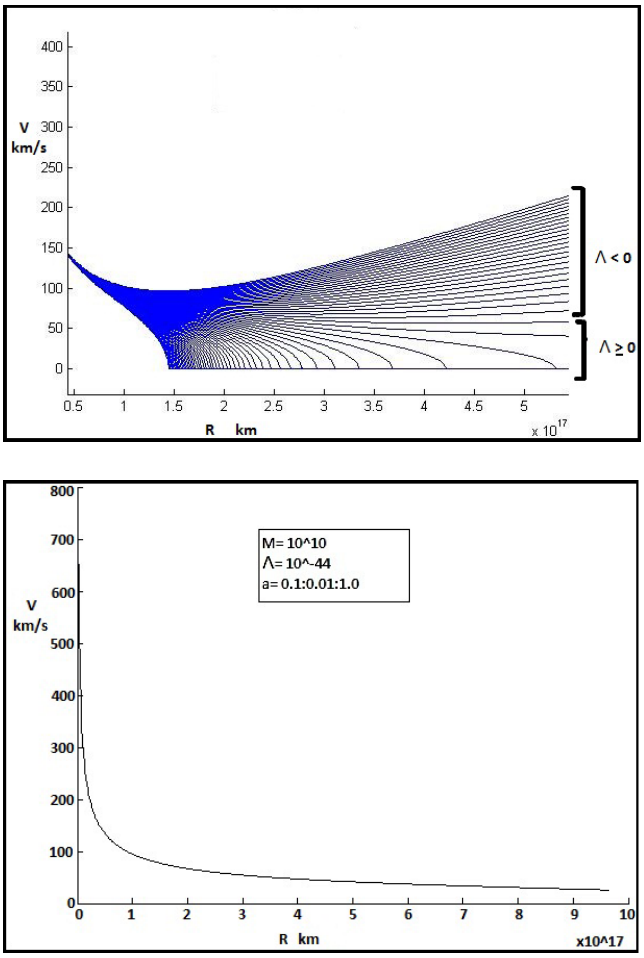

Fig. 4: Variation of Ë and *a* keeping mass constant.

• Adjusted R-square

• Root means squared error (RMSE)

It is obvious that for the best fit the RMSE and SSE can be expected near zero whereas the R-square and Adjusted R-square value near the upper limit (≈1).

RESULT AND DISCUSSION

Estimation of Mass (M), Cosmological Constant (Ë) and Rotational Kerr Parameter (*a*)

Discussion: In the entire work, mainly we focused our interest to the database of galaxies that were well fitted to our problems. For this we took non-linear curve fit statistics in such a way that for which R-square and adjusted R-square errors should be non-negative. We chose 30 galaxies and fitted the values of mass (M), cosmological constant (Ë) and rotational Kerr parameter (*a*).

Out of 30 galaxies, we could fit positive cosmological constant for 23 galaxies while it was negative for 7 galaxies. To find the unknown parameters we analyzed the nature of curve of equation of motion in Kerr-de Sitter space-time and fitted with rotation curves data of galaxies. It is found that the rotational velocity in the flat portion in the rotation curves data of galaxies increases with mass (M) keeping Ë non-negative. Similarly, curvature is found to be dependent on the value of Ë. For non-negative value of Ë velocity decreases as distance increases while for negative value it increases as distance increases keeping mass constant. We found value of *a* has negligible contribution in the curvature but it affects in goodness of fit statistics. Therefore, we minimized the errors associated with it. Thus, we fitted rotation curves data for most appropriate values of mass (M), cosmological constant (Ë) and Kerr parameter (*a*). Detailed values and further discussion will be given later.

The best-fitted graphs of galaxies are shown in respective figures. The detailed best fitted values of mass (M), cosmological constant (Ë), Kerr parameter (*a*) and errors associated with it are given in Table (1).

General Discussion: Since geodesic equation in Kerr-de Sitter is non-linear as well as more complicated than in other space-time, we focused our interest on the nature of time-like trajectory. We found that the rotational velocity in the flat portion in the rotation curves data of galaxies increases with mass (M) keeping cosmological constant non-negative. For non-negative value of Ë '*v*' vs. '*r*' curves do not meet to each other while they meet after a certain point for negative value of Ë. The value of meeting point depends upon the value of Ë. The plots of '*v*' vs. '*r*' curves are shown in fig (3)

Similarly, we found that the curvature of rotation curves data of galaxies is dependent on value of Ë. For non- negative value of Ë velocity decreases as distance increases while velocity increases with distance for negative value of Ë keeping mass constant. The plot showing different curves due to the variation of Ë is shown in fig (4). Since *a* appears as multiplicative factor with mass (M) and lies in the range 0.1 " 1.0, its effect in curvature is found to be negligible. So to find the value of *a* we carried out the less errors associated with goodness of fit statistics in non-linear least square curve fitting. We took the values of M, Ë and *a* so that there would have best fit curves with the rotation curves of galaxies.

The value of cosmological constant is found to lie within the range of $(1.790 \pm 0.286) \times 10^{-49}$ km^{-2} to $(7.523 \pm 1.204)\times 10^{-42}$ km^{-2} for positive value of while $-(3.983\pm 0.637)\times 10^{-41}$ km^{-2} to $-(1.860 \pm 0.298) \times 10\text{-}^{42}$ km^{-2} for negative value of Ë. Among positive values, the least value is found for the galaxy NGC 3521 and maximum is found for the galaxy NGC 3034. Similarly, among negative values maximum is found for NGC 2708 and minimum value is found for NGC 3495. Negative cosmological constants were found to fit for the galaxies that are high red shifted spiral galaxies. Exception to these is NGC 4569 that is high blue shifted having radial velocity equals -235 km/s. This is found to be Sab morphological type and having LINER activity. Galaxies having low value of radial velocities (~ -300 km/s to 1116 km/s) were found to fit with positive cosmological constants. Exceptions to these are NGC 1097, NGC 1365, NGC 4321 and NGC 4565 (~ 1230 km/s to 1636 km/s).

We have found greater value (~ 10Ét ² kmÉ² to 10Ét w kmÉ²) of cosmological constants for galaxies NGC 1808, NGC 3034, NGC 3521, NGC 4736 and NGC 5194. Out of which NGC 1808, NGC 3034 are found to be of active galaxies and NGC 5194 (M51) shows peculiar characteristics (Sofue et al., 1999). While NGC 4736 is found to be of Sab type and having radial velocity equals 606 km/s. In general, we found cosmological constant in the range of 10^{-41} km^{-2} to 10^{-49} km^{-2} for positive value of cosmological constant and in the range of -10^-41 km^-2 to -10Ét ² kmÉ² for negative value of Ë that are in agreement with the values found in other literatures.

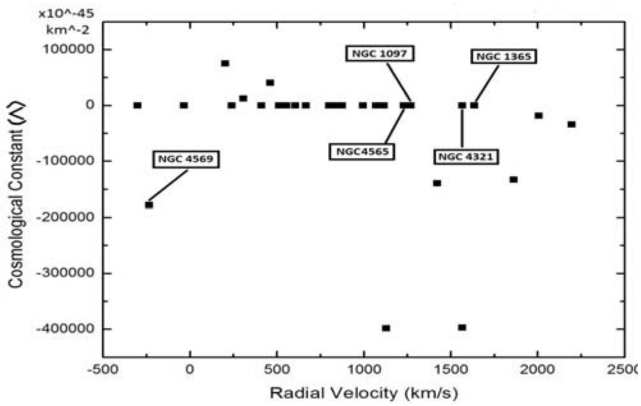

Fig. 5: Plot diagram of Ë vs. Radial Velocity of galaxies. Each square box contains the name of galaxy having exceptional behaviour.

The estimated mass of the galaxies lie in the range of (0.12±0.02)×10¹º M⊙ to (70.37±11.26)×10¹º M⊙ where 1 M⊙ = 1.989 ×10³º kg which are also in good agreement with other estimated values found in literatures. Value of mass is found small (0.73 ± 0.12) × 10¹º M⊙ for NGC 3034 which is Ir II type and have positive value of Ë. While value of mass (0.12 ± 0.02) × 10¹º M⊙ is found for NGC 3495 which is Sd type and have negative cosmological constant. Greater value of mass is found for NGC 1097 that is of SBb type. We had also calculated mass of elliptical galaxy NGC 3379 whose value is found to fit with (1.027±0.164)×10¹¹ M⊙. In general mass is found to be in the range 10y M⊙ to 10¹¹ M⊙ for spiral barred and unbarred galaxies.

Similarly value of Kerr parameter we fitted lie in the range of (0.7044±0.1127) to (0.9990±0.1598). Small value (0.7044±0.1127) of *a* is found for NGC 4736 which is early type barred spiral (Sab) type and it has small mass while greater value (0.9990±0.1598) is found for NGC 3379 which is of elliptical type. Recent measurements of the Kerr parameters *a* for two stellar size black-hole binaries in our Galaxy (Shafee et al. 2006) for GRO J1655-40 and 4U 1543-47 are estimated to fall in the range *a* = 0.65 - 0.75 and *a* = 0.75 - 0.85, respectively. Our estimated values are quite reasonable because we know that spin angular momentum for a collective mass of the galaxy has always-higher value than for a black hole mass. We found most of our estimated massive galaxies (> 50%) are fitted with the higher value of *a*. Since spin of barred and unbarred galaxies not only mass dependent but also depends on their local inner activities such as starbursts as well as globular cause such as its neighbouring galaxies, its position in galaxy cluster etc. These results might be interesting in the future studies.

Table (1) shows the list of database. The first column lists the name of the galaxies according to the catalog. The

Table 1: Parameters and Estimated values of Mass (M), Cosmological Constant (Ë) and Kerr Parameter (*a*) for different Galaxies

Name of Galaxy	Parameters Of Galaxies						Mass (M⊙)	Mass In kg	Cosmological Constant Λ (km⁻²)	Kerr Parameter (*a*)
	Type	RA (deg)	Dec (deg)	a (arcmin)	PA (deg)	RV (km⁻¹)				
NGC 0224	Sb	10.0000	40.9953	190.00	60.0	-300	$(39.05±6.25)×10^{10}$	$(7.767±1.242)×10^{41}$	$(2.339±0.374)×10^{-48}$	(0.9900±0.1584)
NGC 0891	Sb	34.8542	42.1219	13.50	2.5	528	$(18.88±3.02)×10^{10}$	$(3.755±0.601)×10^{41}$	$(2.337±0.374)×10^{-48}$	(0.7800±0.1248)
NGC 1097	SBb	41.0458	-30.4836	9.30	6.3	1271	$(70.37±11.26)×10^{10}$	$(13.996±2.239)×10^{41}$	$(2.331±0.373)×10^{-48}$	(0.8900±0.1424)
NGC 1365	SBb	52.9250	-36.3075	11.20	6.2	1636	$(43.36±6.94)×10^{10}$	$(8.624±1.380)×10^{41}$	$(5.023±0.804)×10^{-49}$	(0.7980±0.1277)
NGC 1808	Sbc	76.4958	-37.5769	6.50	3.9	995	$(9.55±1.53)×10^{10}$	$(1.899±0.304)×10^{41}$	$(2.020±0.323)×10^{-47}$	(0.8809±0.1409)
NGC 2683	Sb	133.1721	33.4219	9.30	44.0	411	$(11.69±1.87)×10^{10}$	$(2.325±0.372)×10^{41}$	$(2.330±0.373)×10^{-48}$	(0.8600±0.1376)
NGC 2903	Sc	142.3333	21.7219	12.60	6.0	556	$(25.04±4.01)×10^{10}$	$(4.980±0.797)×10^{41}$	$(2.278±0.364)×10^{-48}$	(0.7721±0.1235)
NGC 3031	Sb	147.8625	69.3022	26.90	14.1	-34	$(16.06±2.56)×10^{10}$	$(3.194±0.511)×10^{41}$	$(2.337±0.374)×10^{-48}$	(0.9500±0.1520)
NGC 3034	I	147.9333	69.9169	11.20	4.3	203	$(0.73±0.12)×10^{10}$	$(0.145±0.023)×10^{41}$	$(7.523±1.204)×10^{-42}$	(0.9530±0.1525)
NGC 3079	Sc	149.6458	55.9208	7.90	169.0	1116	$(21.76±3.48)×10^{10}$	$(4.328±0.692)×10^{41}$	$(4.100±0.656)×10^{-49}$	(0.9600±0.1536)
NGC 3379	E	161.9542	12.5816	2.50	0	875	$(10.27±1.64)×10^{10}$	$(2.043±0.327)×10^{41}$	$(2.220±0.355)×10^{-48}$	(0.9990±0.1598)
NGC 3521	Sbc	165.8165	0.2364	11.10	166.0	801	$(21.76±3.48)×10^{10}$	$(4.328±0.692)×10^{41}$	$(1.790±0.286)×10^{-49}$	(0.8720±0.1395)
NGC 3628	Sb	169.4542	13.8628	14.80	104.0	843	$(18.65±2.98)×10^{10}$	$(3.709±0.593)×10^{41}$	$(2.221±0.355)×10^{-48}$	(0.8200±0.1312)
NGC 4100	Sc	181.5358	49.5823	1.30	0	1072	$(13.96±2.23)×10^{10}$	$(2.776±0.444)×10^{41}$	$(3.130±0.501)×10^{-48}$	(0.8800±0.1408)
NGC 4321	Sc	185.0958	16.0994	7.40	6.3	1571	$(46.55±7.45)×10^{10}$	$(9.259±1.481)×10^{41}$	$(3.220±0.516)×10^{-48}$	(0.7974±0.1276)
NGC 4565	Sb	188.4667	26.2628	15.90	137.0	1230	$(49.47±7.92)×10^{10}$	$(9.839±1.574)×10^{41}$	$(2.342±0.375)×10^{-48}$	(0.8200±0.1312)
NGC 4631	Sc	189.9167	32.8133	15.50	86.0	606	$(16.12±2.58)×10^{10}$	$(3.206±0.513)×10^{41}$	$(2.338±0.374)×10^{-48}$	(0.7410±0.1186)
NGC 4736	Sab	192.1333	41.3922	11.20	9.1	308	$(6.89±1.10)×10^{10}$	$(1.370±0.219)×10^{41}$	$(1.242±0.198)×10^{-42}$	(0.7044±0.1127)
NGC 5033	Sc	197.7875	36.8586	10.70	179	875	$(61.73±9.88)×10^{10}$	$(12.228±1.956)×10^{41}$	$(7.640±1.222)×10^{-49}$	(0.8600±0.1376)
NGC 5194	Sc	201.0000	47.4561	11.20	6.9	463	$(16.63±2.66)×10^{10}$	$(3.308±0.529)×10^{41}$	$(4.048±0.648)×10^{-42}$	(0.7490±0.1198)
NGC 5236	SBc	203.5500	-29.6117	12.90	11.5	513	$(24.42±3.91)×10^{10}$	$(4.857±0.777)×10^{41}$	$(2.296±0.367)×10^{-48}$	(0.7679±0.1228)
NGC 5457	Sc	210.3583	54.5883	28.80	26.9	241	$(16.70±2.58)×10^{10}$	$(3.321±0.531)×10^{41}$	$(2.180±0.349)×10^{-49}$	(0.7419±0.1187)
NGC 5907	Sc	228.6500	56.5125	12.77	156.0	667	$(41.23±6.59)×10^{10}$	$(8.200±1.312)×10^{41}$	$(2.347±0.375)×10^{-48}$	(0.8600±0.1376)
NGC 2708	Sb	133.4042	-3.1681	2.60	20.0	2008	$(30.76±4.92)×10^{10}$	$(6.118±0.979)×10^{41}$	$-(1.860±0.298)×10^{-42}$	(0.7842±0.1255)
NGC 3495	Sd	164.6708	3.8953	4.90	20.0	1130	$(0.12±0.02)×10^{10}$	$(0.023±0.004)×10^{41}$	$-(3.983±0.637)×10^{-41}$	(0.9100±0.1456)
NGC 3672	Sc	170.6250	-9.5200	4.20	1.9	1862	$(4.08±0.65)×10^{10}$	$(0.811±0.129)×10^{41}$	$-(1.329±0.213)×10^{-41}$	(0.8896±0.1423)
NGC 4303	Sc	184.8417	4.7508	6.50	5.8	1566	$(4.57±0.73)×10^{10}$	$(0.909±0.145)×10^{41}$	$-(3.450±0.552)×10^{-42}$	(0.9900±0.1584)
NGC 4569	Sab	188.5792	13.4378	9.50	23.0	-235	$(3.16±0.50×10^{10}$	$(0.629±0.101)×10^{41}$	$-(3.970±0.635)×10^{-41}$	(0.9860±0.1578)
NGC 6951	Sbc	309.1542	65.9294	3.90	3.2	1424	$(5.04±0.81)×10^{10}$	$(1.002±0.160)×10^{41}$	$-(1.783±0.285)×10^{-41}$	(0.8550±0.137)
UGC 3691	Sc	107.0053	15.1784	10.00	0	2196	$(0.40±0.06)×10^{10}$	$(0.079±0.012)×10^{41}$	$-(1.394±0.223)×10^{-41}$	(0.9129±0.1461)

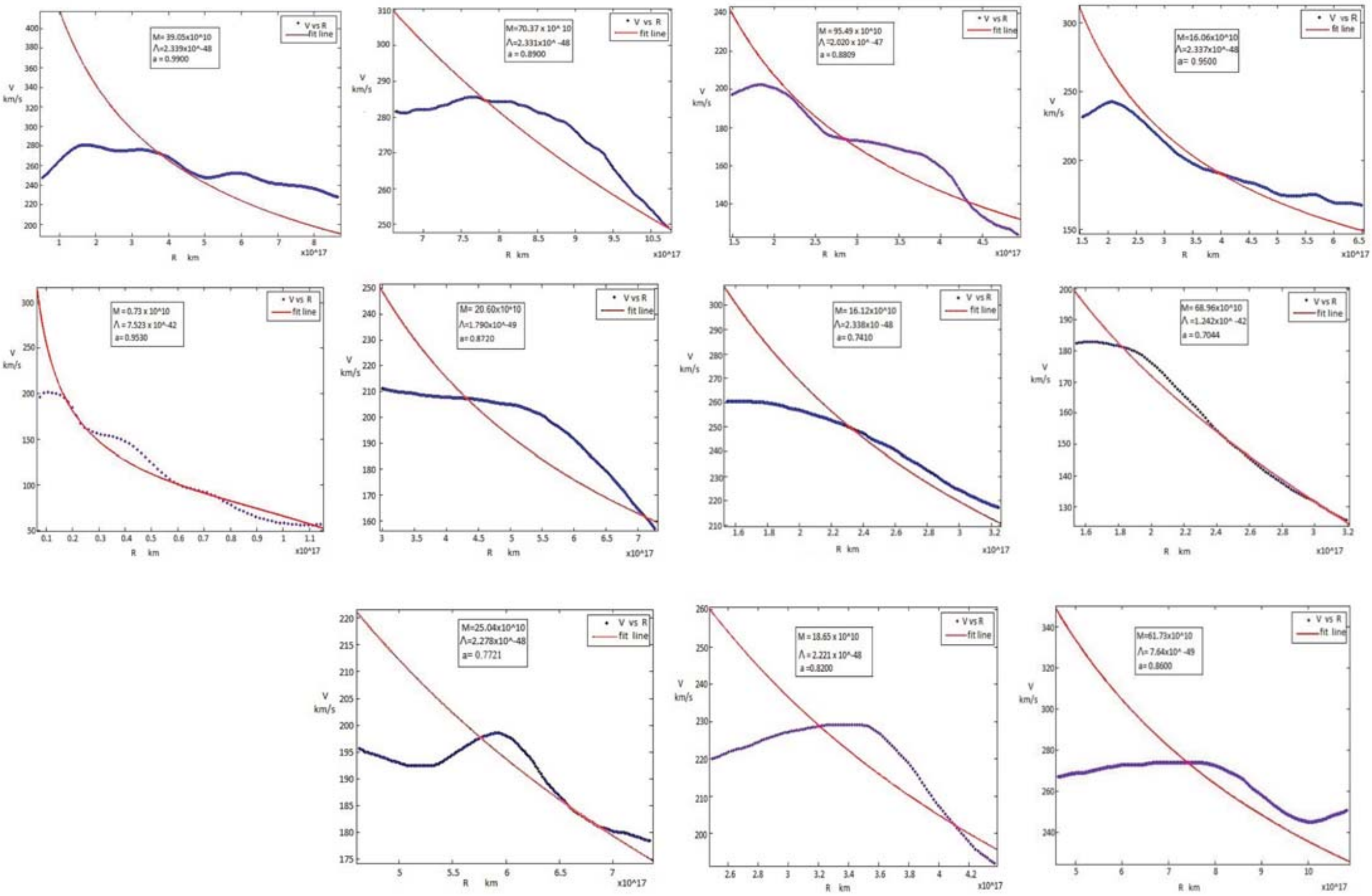

Fig. 6: The observed and fitted rotation curves of galaxies NGC 0224, NGC 1097, NGC 1808, NGC 3031, NGC 3034, NGC 3521, NGC 4631, NGC 4736, NGC 2903, NGC 3628, and NGC 5033. The thick solid and thin solid curve represents the observed and fitted line respectively. The value of M, Ë and *a* is shown inside the square box.

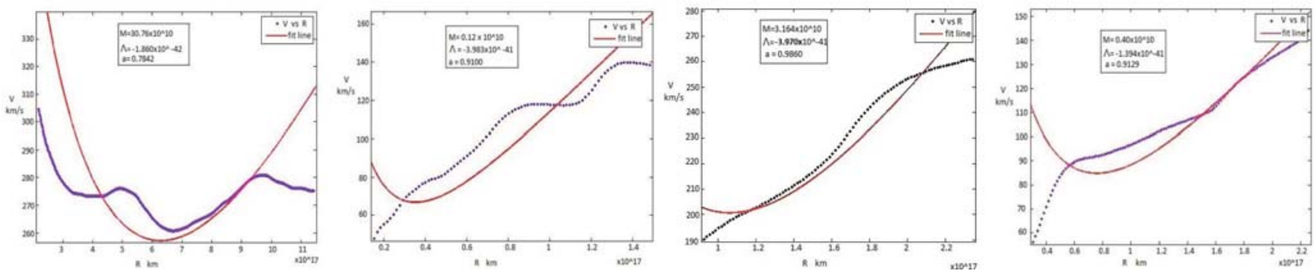

Fig. 7: The observed and fitted rotation curves of galaxies NGC 2708, NGC 3495, NGC 4569, and UGC 3691. The thick solid and thin solid curve represents the observed and fitted line respectively. The value of M, Ë and *a* is shown inside the square box.

second column shows the morphological type. The positions in the equatorial coordinate system are listed in the third and fourth columns. The fifth and sixth columns give the major diameter (in arc min) and position angle (PA) respectively. The seventh column lists the radial velocity in km s^{-1}. The ninth, tenth and last columns list the estimated mass, cosmological constant and Kerr parameter respectively. The observed and fitted rotation curves of some galaxies are shown in above fig.

CONCLUDING REMARKS

From its introduction, the cosmological constant has become one of the most paradoxical quantities. The geodesics motion of massive and massless particles for rotating axisymmetric black hole can be described by Kerr-de Sitter space-time. This contains a finite cosmological constant (Ë). We estimated the values of cosmological constant (Ë), mass (M) and Kerr parameter (*a*) of 30 galaxies by using the formula for rotational velocity of test particle in Kerr-de Sitter space-time using circular orbit and central rotation curves of these galaxies. We used the method of non- linear least square curve fit and fitted for the values of mass (M), cosmological constant (Ë), and Kerr parameter (*a*) for different galaxies. From this method, the results are summarized as follows.

• The value of the rotational velocity in the flat portion in the rotation curves data of galaxies is found to be increased with mass (M) keeping cosmological constant (Ë) non-negative. For a constant negative value of Ë, '*v*' vs. '*r*' curves meets at a point after certain distance with the variation of mass. The meeting point is observed to be dependent on the value of Ë.

• The Curvature of rotation curve data of Galaxies is found to be dependent on the value of Ë keeping mass constant. For non- negative value of Ë, velocity (*v*) decreases as distance from galactic center (*r*) increases while for negative value of Ë velocity (*v*) increases with the distance. Thus, Ë is an essential parameter to fit the curvature of rotation curves data of galaxies.

• Since Kerr parameters lie in the range of 0.1 to 1.0 and appears as coefficient of mass in our equation of motion, it has negligible contribution in '*v*' vs. '*r*' curve. However, it has an effect in goodness of fit statistics, particularly in non-linear least square curve fitting.

• The value of mass (M) of galaxies are estimated in the range of $(0.12 \pm 0.02) \times 10^{10}$ M⊙ to $(70.37 \pm 11.26) \times 10^{10}$ M⊙. NGC 1097 is found to be more massive than others, which is of SBb type. Least mass $(0.12 \pm 0.02) \times 10^{10}$ M⊙ is found for NGC 3495 which is fitted for negative cosmological constant and is of Sd type. Value of mass is found small $(0.73 \pm 0.12) \times 10^{10}$ M⊙ for NGC 3495 which is Ir II type. In general mass is found to lie in the range 10y M⊙ to 10^{11} M⊙ for spiral barred and unbarred galaxies.

• The nature of cosmological constant fitted for galaxies were found to depend upon radial velocities of galaxies. Discarding exceptional cases, for the higher values of radial velocity cosmological constants were found to be negative while positive for small values of RV. These suggest some local phase transition effects at the time when the high redshift galaxies were formed.

• The value of Cosmological Constant is found to fall within the range of $(1.790 \pm 0.286) \times 10^{-49}$ km^{-2} to $(7.523 \pm 1.204) \times 10^{-42}$ km^{-2} for positive value of Ë while $-(3.983 \pm 0.637) \times 10^{-41}$ km^{-2} to $-(1.860 \pm 0.298) \times 10^{-42}$ km^{-2} for negative value of Ë. Most of the galaxies were fitted for the values of Ë in the range of 10^{-49} km^{-2} to 10^{-48} km^{-2}.

• The value of Kerr parameter lie in the range of (0.7044±0.1127) to (0.9990±0.1598). Small value, i.e., (0.7044±0.1127) of *a* is found for NGC 4736 which is Sab type galaxy. It has small mass while greater value (0.9990±0.1598) is found for NGC 3379 that is elliptical type.

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REFERENCES:

1. Akcay, S. The Kerr-de Sitter Universe, (http://www.arXiv:astroph\1011.0479v1). 2009.
2. Bhatta, G. P. Null and Time-Like Geodesics in Kerr-de sitter Space Time, M.sc. Thesis (Physics), Tribhuvan University, Kirtipur. 2001.
3. Brownstein, J.R. & Moffat, J.W. Galaxy Rotation Curves Without Non-Baryonic Dark Matter, arXiv: astroph\0506370v4 (2005).
4. Chandrasekhar, S. The mathematical Theory of Black Holes, Oxford University Press, New York. 1983.
5. Chandrasekhar, S. The mathematical Theory of Black Holes, Oxford University Press, New York. 1999.
6. Goldsmith, D. Einstein's Greatest Blunder: the Cosmological Constant, Havard University Press. 1995.
7. Islam, J. N. An Introduction to Mathematical Cosmology, Cambridge University Press, The Edinburgh Building, Cambridge. 2004.
8. Jones B. & Saha P. The Galaxy, Notes for Lecture Courses ASTM002 and MAS430. Queen Mary University of London 2004.
9. Sahni, R., Bulletin of the American Astronomical Society. **34**, 1142. 2002.
10. Malakar, N.K. & Khanal, U. The null geodesics in Kerr-de Sitter Space-Time. Scientific World, Vol. **3**, No. 3, 2005.
11. Moore, Sir Patrick. Phillip's Atlas of Universe, Phillip's Octopus Publishing Group Ltd. 2005.
12. Narlikar, J. Introduction to Cosmology, Cambridge University Press, Cambridge. 1993.
13. Robinson, L. J. Philip's Astronomy Encyclopedia, Philips Octopus Publishing Group. 2002.
14. Sahani, V. The Physics of the Early Universe, Lect. Notes Phys. 653 (Springer, Berlin Heidelberg), DOI 10.1007/b99562. 2005.
15. Shafee, et al. General Relativity and Quantum Cosmology (gr-qc), (http://www.arXiv:astro-ph\0508302v2). 2006.
16. Sofue, Y., Tutui, Y., Honma, M., Tomita, A., Takamiya, T., Koda, J. & Takeda Y. Central Rotational Curves Of Galaxies, The Astrophysical Journal. **523**:136-146. 1999 September 20.
17. web[1]: http://nasa.gov/dark matter.
18. web[2]: http://nasa.gov/dark energy.
19. web[3]: http://zebu.uoregon.edu/ js/ast123/lectures/lec13.html (3 of 10) [15-02-2002 22:36:10].

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