VIRIAL MOMENTS OF LARGE-SCALE NEUTRINO STRUCTURES

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Abstract: In the course of studying the role of massive neutrinos and other particles in the large scale structures formation in the universe, Virial moments of the neutrino structures have been calculated. The Jeans masses calculated this way have been compared with the ones calculated earlier in our previous paper.

Key words: Large-scale structures; Virial equation; Neutrino; Jeans mass.

1. INTRODUCTION:

The formation of large-scale structures normally proceeds from the collapse of particle clouds, of which, neutrinos are supposed to be one of the major components. A lot of works have been done to investigate the evolution of size and eccentricity of spherical and spheroidal / ellipsoidal clusters[1,2,3] in addition to masses. In the majority of these works, Virial theorems / equations and their moments of various orders have been used. In this paper, Jeans masses have been calculated using Virial moments of these masses and are compared with those calculated in previous works[4].

The momentum of a freely moving particle, like neutrinos, in the Friedmann-Robertson-Walker space-time is redshifted by the expansion, i.e., the comoving momentum pa = y remains constant, where p is the momentum and a the scale factor. Using v = p/E for the velocity and the Einstein energy-momentum relation, $a^2E^2= p^2a^2+m^2a^2= y^2+ x^2$, we see that Eva remains constant during expansion, and as the number density scales as a^{-3}, $\rho v a^4$ also remains constant where ρ is the density. As the light neutrinos decouple at the very high temperature of T ≈ 1MeV while still extremely relativistic (ER), they are essentially in free fall since then. So their number density is always distributed as

$$dn = \frac{g}{2\pi^2}\left(\frac{m}{x}\right)^3 \frac{y^2}{e^y+1}dy \qquad (1)$$

where g is the number of spin degeneracy (six, for the three $\upsilon - \bar{\upsilon}$ pairs), and we have used the fact that T ~ 1/a ; Planck units in which G = c = k_B = $\hbar$ = 1 are used throughout this work.

Integrating over y from 0 to ∞ gives the number density $n = \frac{g}{2\pi^2}\Gamma(3)\eta(3)\left(\frac{m}{x}\right)^3$ where Γ and η are the gamma and the Riemann eta functions respectively [5]; also, $\eta(n) = (1-2^{1-n})\zeta(n)$. Thus we can write down the expectation value of any regular function as $\langle f(y)\rangle = \frac{1}{\Gamma(3)\eta(3)}\int_0^\infty dy \frac{y^2}{e^y+1} f(y)$. The particle speed is v = p/E = $y/\sqrt{y^2+x^2}$.

$$v_{mean} = \langle v\rangle = \frac{1}{\Gamma(3)\eta(3)}\int_0^\infty dy \frac{y^2}{e^y+1}\frac{y}{\sqrt{y^2+x^2}}$$

Similarly, the mean density of the a cluster is given by

$$\langle\rho\rangle = \langle m_{rel.} n\rangle = \frac{g}{2\pi^2}\int_0^\infty dy \frac{y^2\sqrt{y^2+x^2}}{e^y+1}\left(\frac{m}{x}\right)^4$$

We had calculated the Jeans mass of neutrino structures by varying different parameters involved in it[4]. The Jeans mass, that is contained within Jeans radius (R_J), $M_J = 4\pi R_J\rho_v/3 = ½ (R^3_J / R^2_v)$, has the momentum dependence:

$$M_J = c\frac{x^2y^3}{\left(y^2+x^2\right)^{7/4}}, \qquad (2)$$

where the constant $c = \frac{\pi^{7/2}m_{pl}^3}{6\sqrt{g\eta(3)}m^2} = \frac{1}{m_{eV}^2\sqrt{g}}15x10^{18}$

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2. VIRIAL THEOREM AND MOMENTS:

In a system of N particles, gravitational forces tend to pull the system together and the stellar velocities tend to make it fly apart. It is possible to relate kinetic and potential energy of a system through the change of its moment of inertia. In a steady-state system, these tendencies are balanced, which is expressed quantitatively through the Virial Theorem. A system that is not in balance will tend to move towards its virialized state. The Scalar Virial Theorem tells us that the average kinetic and potential energy must be in balance. The tensor Virial Theorem tells us that the kinetic and potential energy must be in balance in each separate direction. The scalar Virial Theorem is useful for estimating global average properties, such as total mass, escape velocity and relaxation time, while the tensor Virial Theorem is useful for relating shapes of systems to their kinematics, e.g. the flatness of elliptical galaxies to their rotational speed.

The Virial Equations of the various orders are, in fact, no more than the moments of the relevant hydrodynamical equations. The scalar Virial Equation for a system is given by

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2K + U \qquad (3)$$

where, I = Moment of Inertia about the origin = $\frac{1}{2}\frac{d^2}{dt^2}\int_V \rho r^2 dV$,

K = Kinetic Energy = $\frac{1}{2}\int_V \rho v^2 dV$ and U = Potential Energy

$= \frac{\rho_c}{2}\int_V \frac{\rho}{r} dV$, where, ρ_c is the core density.

Thus the Virial Equation of the 1st order is given by

$$\frac{1}{2}\frac{d^2}{dt^2}\int_V \rho r^2 dV = \frac{1}{2}\int_V \rho v^2 dV + \frac{\rho_c}{2}\int_V \frac{\rho}{r} dV \qquad (4)$$

The Virial Equation of the 2nd order is given by just multiplying the integrand by r before integrating:

$$\frac{1}{2}\frac{d^2}{dt^2}\int_V \rho r^3 dV = \frac{1}{2}\int_V \rho v^2 r dV + \frac{\rho_c}{2}\int_V \rho dV \qquad (5)$$

Similarly, the higher order equation may be written.

For the steady state, 2K + U = 0. This gives the virial radius of the spherical cluster: $R_V^2 = \frac{3}{4\pi}\left(\frac{v^2}{\rho}\right)$. Writing ρ and v in terms of x and y, we have $R_V = cons\tan t.\frac{y}{(y^2+x^2)^{3/4}}\left(\frac{x}{m}\right)^2$ (6)

and hence $\langle R_V \rangle = cons\tan t.\int_0^\infty dy \frac{y^3}{(e^y+1)(y^2+x^2)^{3/4}}\left(\frac{x}{m}\right)^2$. Its evolution has been shown in fig-1.

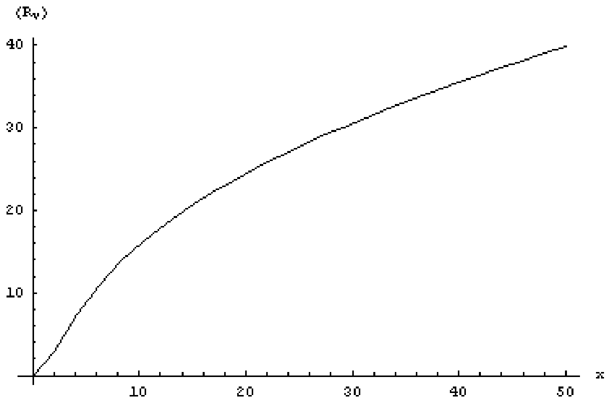

Fig-1: Evolution of Virial Radius <R$_V$> of a cluster.

The Virial Mass of the cluster contained within this radius may be calculated as

$\langle M_V \rangle = Cons\tan t.\langle R_V \rangle^3 \langle \rho \rangle$. Its variation with x is as expected (fig-2), but peaks at x = 3.73 in contrast to the position of peaks of Jeans masses calculated in previous papers, where the peaks have occurred at x = 1.9, 2.1, 4.2 and 5.0.[4,6].

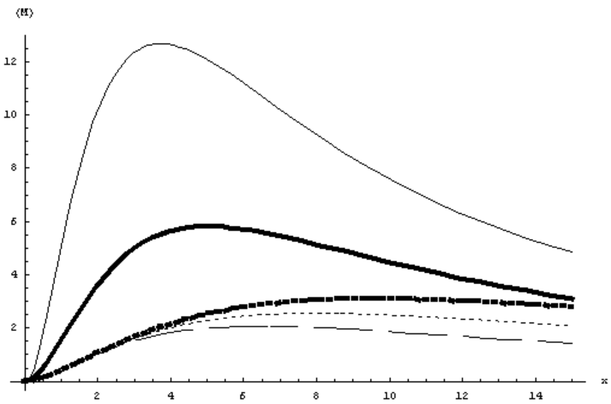

Fig-2: The average cluster masses (Jeans and Virial) against x. The normal curve is for virial mass, thick continuous curve for Jeans mass, the normal dashed curve for the Jeans mass calculated from the 1st moment of the mass and normal dotted and thick dotted are those from 2nd and 3rd moments respectively.

3. VIRIAL MOMENTS OF JEANS MASS:

As stated earlier, in the virial method, we take the moments of the equation of motion. These equations obviously involve the moments of the distribution of density, pressure, velocity, gravitational potential, etc. Here we are taking the spatial moments of various orders of Jeans mass given by eq.-1: The 1st moment of Jeans mass =

$$\langle M_J R_V \rangle = cons\tan t.\int_0^\infty dy \frac{y^2}{e^y+1}(M_J R_V)$$

$$= cons\tan t.\int_0^\infty dy \frac{y^6 x^4}{(e^y+1)(y^2+x^2)^{5/2}}$$

The 2nd and 3rd moments may be written in similar fashion. The variations of the 1st, 2nd and 3rd moments with x have been shown in fig-3. The 1st and the 2nd are found to peak and at x

The way M_J calculated	x at which the M_J peaks	The temperature (K) corresponding to x for		
		m_ν = 1 eV	m_ν = 0.2 eV	m_ν = 0.01 eV
$\frac{\langle \rho v^2 \rangle^{3/2}}{\langle \rho \rangle^2}$	1.9	6100	1220	61
$\frac{\langle \rho \rangle}{\langle k_J \rangle^3}$	2.1	5519	1104	55
$\langle R_V \rangle^3 \langle \rho \rangle$	3.73	3107	621	31
$M_J(y_{rms})$	4.2	2760	552	28
$<M_J(y)>$	5	2318	464	23
1st moment of M_J	6.48	1789	358	18
2nd moment of M_J	8.01	1447	289	14
3rd moment of M_J	9.58	1210	242	12

= 12.2 and 27.02 respectively, but the 3rd moment appears to plateau off from x ~ 100 to large values of x. Jeans masses may be calculated by dividing these moments by $<R_V>$, $<R^2_V>$ and $<R^3_V>$ respectively. The variation of these masses with x has been shown in figure -2 (For comparison $<M_J>$ and $<M_V>$ have also been plotted). They have peaked at x = 6.48, 8.01 and 9.58. Table 1 gives the corresponding temperatures at which the peaks of the Jeans masses occur for a few neutrino masses.

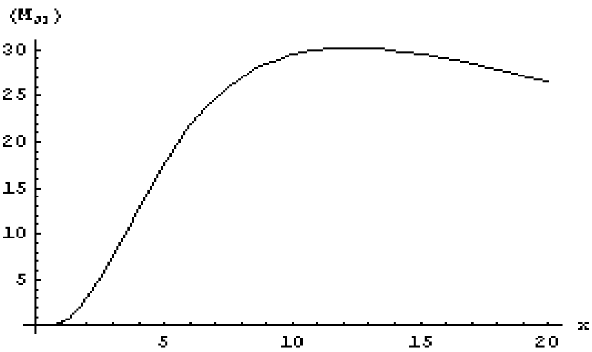

Fig.-3a: Variation of the first virial moment of Jeans mass. The Jeans mass calculated from this peaks at x ~ m_ν /T = 6.48.

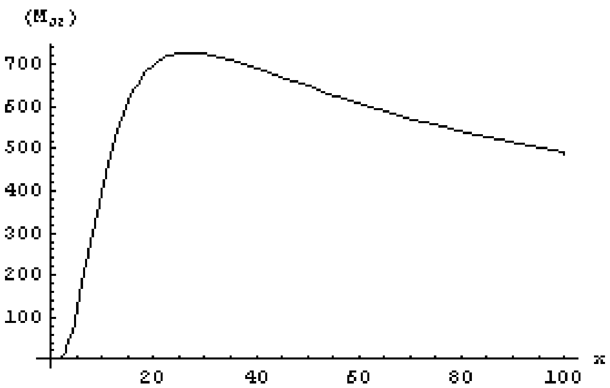

Fig.-3b: Variation of the 2nd virial moment of Jeans mass calculated. The Jeans mass calculated from this peaks at x ~ m_ν /T = 8.01.

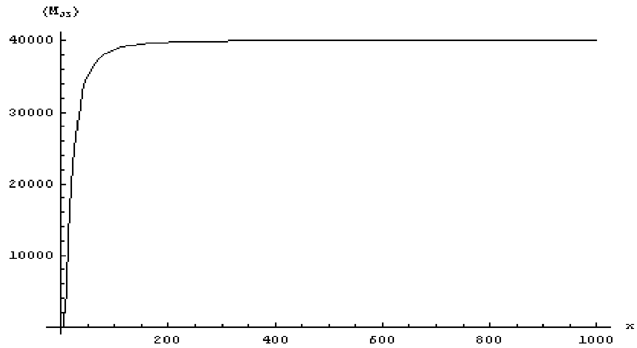

Fig.-3c: Variation of the 3rd virial moment of the Jeans mass. The Jeans mass calculated from this peaks at x ~ m_ν /T = 9.58.

4. CONCLUSIONS:

From the above analysis, it is seen that the large-scale structures of neutrinos of different mass and random velocity distribution can form at different neutrino temperatures, corresponding to different time. The earliest peak that occurs at x = 1.9 corresponds to the time when the neutrino temperature was 1220 K for the 0.2 eV neutrino. Similarly, the latest peak occurs at x = 9.58 corresponding to a temperature of 242 K. In between these two values, it is found that the Jeans mass peaks at a number of different x. Thus it can be interpreted to mean that a distribution of neutrino structures of different masses and of different ages should be in existence. Typical masses of these structures range from 6 x 10^{19} to 4.5 x 10^{20} solar mass. The distribution in size and age of such neutrino structures, and the effect of cold dark matter will be studied in future work.

5. REFERENCES:

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