Nepal Journal of Science and Technology Vol. 16, No.1 (2015) 79-86

Common Fixed Point Theorems for Mappings of Compatible Type(A) in Dislocated Metric Space

Dinesh Panthi[1] and P. Sumati Kumari[2]
[1]Department of Mathematics, Valmeeki Campus, Nepal Sanskrit University, Kathmandu, Nepal.
[2]Department of Mathematics, K L University, Vaddeswaram, A. P., India.
e-mail: panthid06@gmail.com

Abstract
In this article we establish a common fixed point result for two pairs of mappings of compatible type(A) in dislocated metric space which generalizes and extends the similar results in the literature.

Key words: d-metric space, common fixed point, compatible of type(A), cauchy sequence.

Introduction
In 1922, S. Banach established a fixed point theorem for contraction mapping in metric space. Since then a number of fixed point theorems have been proved by many authors and various generalizations of this theorem have been established. G. Jungck (1976) initiated the concept of commuting maps and generalized it with the concept of compatible maps (Jungck 1986, 1988) and established some important fixed point results. G. Jungck, P. P. Murthy and Y. J. Cho (1993) initiated the concept of compatible mappings of type (A) in metric space.

The study of common fixed point of mappings satisfying contractive type conditions has been a very active field of research activity. S. G. Matthews (1986)introduced the concept of dislocated metric space under the name of metric domains in domain theory. P. Hitzler and A. K. Seda (2000) generalized the famous Banach Contraction Principle in dislocated metric space. The study of dislocated metric plays very important role in topology, logic programming and in electronics engineering.

The purpose of this article is to establish a common fixed point theorem for two pairs of mappings of compatible type (A) in dislocated metric spaces which generalize and improve similar results of fixed point in the literature.

Preliminaries
We start with the following definitions, lemmas and theorems.

Definition 1 *Let X be a non empty set and let $d : X \times X \rightarrow [0,\infty)$ be a function satisfying the following conditions:*

(i) $d(x,y) = d(y,x)$

(ii) $d(x,y) = d(y,x) = 0$ implies $x = y$.

(iii) $d(x, y) \leq d(x, z) + d(z, y)$

for all $x, y, z \in X$.

Then d is called dislocated metric(or simply d-metric) on X.

Definition 2 *A sequence $\{x_n\}$ in a d-metric space (X,d) is called a Cauchy sequence if for given $e > 0$, there corresponds $n_0 \in N$ such that for all $m,n \geq n_0$, we have $d(x_m,x_n) < e$.*

Definition 3 *A sequence in d-metric space converges with respect to d (or in d) if there exists $x \in X$ such that $d(x_n,x) \rightarrow 0$ as $n \rightarrow \infty$.*

In this case, x is called limit of $\{x_n\}$ (in d)and we write $x_n \rightarrow x$.

Definition 4 *A d-metric space* (X,d) *is called complete if every Cauchy sequence in it is convergent with respect to* d.

Definition 5 *Let* (X,d) *be a d-metric space. A map* $T: X \to X$ *is called contraction if there exists a number* l *with* $0 \le l < 1$ *such that*
$d(Tx,Ty) \le l\ d(x,y)$.

We state the following lemmas without proof.

Lemma 1 *Limits in a d-metric space are unique.*

Theorem 1 *Let* (X,d) *be a complete d-metric space and let* $T: X \to X$ *be a contraction mapping, then* T *has a unique fixed point.*

Definition 6 *Let* A *and* S *be two self mappings on a set* X. *Mappings* A *and* S *are said to be commuting if* $ASx = SAx\ \forall x \in X$.

Definition 7 *Let* A *and* S *be two self mappings on a set* X. *If* $Ax = Sx$ *for some* $x \in X$, *then* x *is called coincidence point of* A *and* S.

Definition 8 *Two mappings S and T from a metric space (X, d) into itself are called compatible of type(A) if*
$\lim_{n\to\infty} d(STx_n, TTx_n) = 0$
and
$\lim_{n\to\infty} d(TSx_n, SSx_n) = 0$
whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x \quad for \quad some \quad x \in X$

Proposition 1 *Let S and T be mappings of compatible type(A) from a metric space (X, d) into itself. Suppose that*

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x \quad for \quad some \quad x \in X$$

If S is continuous then $\lim_{n\to\infty} TSx_n = Sx$.

Proof. Since S is continuous, so

$Sx_n \to x \Rightarrow SSx_n \to Sx$

and $Tx_n \to x \Rightarrow STx_n \to Sx$.

Therefore,

$$d(TSx_n, Sx) \le d(TSx_n, SSx_n) + d(SSx_n, STx_n) + d(STx_n, Sx)$$

Now taking limit as $n \to \infty$ and using above relations we get

$$\lim_{n\to\infty} d(TSx_n, Sx) = 0 \Rightarrow \lim_{n\to\infty} TSx_n = Sx.$$

Now we establish the following result

Main Results

Theorem 2 *Let (X, d) be a complete d-metric space. Let* $A, B, S, T: X \to X$ *be mappings satisfying the condition*

$$T(X) \subset A(X) \quad \& \quad S(X) \subset B(X) \quad (1)$$

The pairs (T,B) & (S,A) *are compatible of type*(A) (2)

$$d(Tx,Sy) \le a\left[\frac{d(Ay,Sy)d(Bx,Ay)}{d(Ax,Tx)+d(Sy,Ax)}\right] + b\left[\frac{d(Tx,Ax)d(Ty,By)}{d(Ax,Tx)+d(Sy,Ax)}\right] \quad (3)$$

$$+g\left[\frac{d(Ax,Sx)d(Sy,Ay)}{d(Ax,Tx)+d(Sy,Ax)}\right]+k\left[\frac{d(Bx,Ay)d(Tx,Sy)}{d(Bx,Tx)+d(Bx,Sy)}\right] +d\left[\frac{d(Bx,Tx)d(Ay,Sy)}{d(Ax,Ty)+d(Bx,Ty)}\right]+m\left[\frac{d(Ax,Sx)d(By,Ty)}{d(Bx,Ay)+d(Ax,Ty)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a+b+g+d+k+m < 1$. If any one of A, B, S, T is continuous then A, B, S and T have a unique common fixed point in X.

Proof. Let us define a sequence $\{y_n\} \in X$ such that

$$Tx_{2n+1} = y_{2n+2}, \quad Ax_{2n} = y_{2n}, Sx_{2n+1} = y_{2n+2}, \quad Bx_{2n} = y_{2n} \quad for \quad n = 1,2,3,... \tag{4}$$

Let us consider

$$d(Tx_{2n}, Sx_{2n+1}) \le a\left[\frac{d(Ax_{2n+1}, Sx_{2n+1})d(Bx_{2n}, Ax_{2n+1})}{d(Ax_{2n}, Tx_{2n}) + d(Sx_{2n+1}, Ax_{2n})}\right]$$

$$+b\left[\frac{d(Tx_{2n}, Ax_{2n})d(Tx_{2n+1}, Bx_{2n+1})}{d(Ax_{2n}, Tx_{2n}) + d(Sx_{2n+1}, Ax_{2n})}\right]$$

$$+g\left[\frac{d(Ax_{2n}, Sx_{2n})d(Sx_{2n+1}, Ax_{2n+1})}{d(Ax_{2n}, Tx_{2n}) + d(Sx_{2n+1}, Ax_{2n})}\right]$$

$$+d\left[\frac{d(Bx_{2n}, Tx_{2n})d(Ax_{2n+1}, Sx_{2n+1})}{d(Ax_{2n}, Ax_{2n+1}) + d(Bx_{2n}, Tx_{2n+1})}\right]$$

$$+k\left[\frac{d(Bx_{2n}, Ax_{2n+1})d(Tx_{2n}, Sx_{2n+1})}{d(Bx_{2n}, Tx_{2n}) + d(Bx_{2n}, Sx_{2n+1})}\right]$$

$$+m\left[\frac{d(Ax_{2n}, Sx_{2n})d(Bx_{2n+1}, Tx_{2n+1})}{d(Bx_{2n}, Ax_{2n+1}) + d(Ax_{2n}, Tx_{2n+1})}\right]$$

Now,

$$d(y_{2n+1}, y_{2n+2}) \le$$

$$a\left[\frac{d(y_{2n+1}, y_{2n+2})d(y_{2n}, y_{2n+1})}{d(y_{2n}, y_{2n+1}) + d(y_{2n+2}, y_{2n})}\right]$$

$$+b\left[\frac{d(y_{2n+1}, y_{2n})d(y_{2n+2}, y_{2n+1})}{d(y_{2n}, y_{2n+1}) + d(y_{2n+2}, y_{2n})}\right]$$

$$+g\left[\frac{d(y_{2n}, y_{2n+1})d(y_{2n+2}, y_{2n+1})}{d(y_{2n}, y_{2n+1}) + d(y_{2n+2}, y_{2n})}\right]$$

$$+d\left[\frac{d(y_{2n}, y_{2n+1})d(y_{2n+1}, y_{2n+2})}{d(y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n+2})}\right]$$

$$+k\left[\frac{d(y_{2n}, y_{2n+1})d(y_{2n+1}, y_{2n+2})}{d(y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n+2})}\right]$$

$$+m\left[\frac{d(y_{2n}, y_{2n+1})d(y_{2n+1}, y_{2n+2})}{d(y_{2n}, y_{2n+1}) + d(y_{2n}, y_{2n+2})}\right]$$

therefore

$$d(y_{2n+1}, y_{2n+2}) \le (a + b + g + d + k + m)d(y_{2n}, y_{2n+1})$$

$$Let \quad h = (a + b + g + d + k + m) < 1$$

$$Then \qquad d(y_{2n+1}, y_{2n+2}) \le hd(y_{2n}, y_{2n+1})$$

Similarly we can obtain

$$d(y_{2n}, y_{2n+1}) \le hd(y_{2n-1}, y_{2n})$$

$$Hence \qquad d(y_{2n+1}, y_{2n+2}) \le hd(y_{2n}, y_{2n+1}) \le h^2 d(y_{2n-1}, y_{2n}) \le ... \le h^n d(y_0, y_1)$$

This shows that

$$d(y_{n+1}, y_n) \le hd(y_n, y_{n-1}) \le ... \le h^n d(y_1, y_0)$$

For every integer $q > 0$ we have

$$d(y_{n+q}, y_n) \le d(y_{n+q}, y_{n+q-1}) + ... + d(y_{n+2}, y_{n+1}) + d(y_{n+1}, y_n)$$
$$\le (h^{q-1} + ... + h^2 + h + 1)d(y_{n+1}, y_n)$$
$$\le (h^{q-1} + ... + h^2 + h + 1)h^n d(y_1, y_0)$$

Since $h < 1$, so $h^n \to 0$ as $n \to \infty$.

Therefore $d(y_{n+q}, y_n) \to 0$ as $n \to \infty$.

This implies that $\{y_n\}$ is a cauchy sequence.

Since X is complete, so there exists a po int $z \in X$ such thart $\{y_n\} \to z$. Consequently subsequences

$$\{Ax_{2n}\} = \{Bx_{2n}\} \to z \quad \& \quad \{Sx_{2n+11}\} = \{Tx_{2n+1}\} \to z. \tag{5}$$

Since the pair (T, B) is compatible of type(A) and let T is continuous, then by proposition (1) we have

$$TTx_{2n} \to Tz \quad and \qquad BTx_{2n} \to Tz \quad as \quad n \to \infty. \tag{6}$$

Now, we show that z is the fixed point of T i.e $Tz = z.$ For this,

$$d(TTx_{2n}, Sx_{2n+1}) \leq$$

$$a\left[\frac{d(Ax_{2n+1}, Sx_{2n+1})d(BTx_{2n}, Ax_{2n+1})}{d(ATx_{2n}, TTx_{2n}) + d(Sx_{2n+1}, ATx_{2n})}\right]$$

$$+b\left[\frac{d(TTx_{2n}, ATx_{2n})d(Tx_{2n+1}, Bx_{2n+1})}{d(ATx_{2n}, TTx_{2n}) + d(Sx_{2n+1}, ATx_{2n})}\right]$$

$$+g\left[\frac{d(ATx_{2n}, STx_{2n})d(Sx_{2n+1}, Ax_{2n+1})}{d(ATx_{2n}, TTx_{2n}) + d(Sx_{2n+1}, ATx_{2n})}\right]$$

$$+d\left[\frac{d(BTx_{2n}, TTx_{2n})d(Ax_{2n+1}, Sx_{2n+1})}{d(ATx_{2n}, Ax_{2n}) + d(BTx_{2n}, Tx_{2n+1})}\right]$$

$$+k\left[\frac{d(BTx_{2n}, Ax_{2n+1})d(TTx_{2n}, Sx_{2n+1})}{d(BTx_{2n}, TTx_{2n}) + d(BTx_{2n}, Sx_{2n+1})}\right]$$

$$+m\left[\frac{d(ATx_{2n}, STx_{2n})d(Bx_{2n+1}, Tx_{2n+1})}{d(BTx_{2n}, Ax_{2n+1}) + d(ATx_{2n}, Tx_{2n+1})}\right]$$

Now taking limit as $n \to \infty$ and using relations (5) and (6) we have

$$d(Tz, z) \leq k\left[\frac{d(Tz, z)d(Tz, z)}{d(Tz, Tz) + d(Tz, z)}\right]$$

$$\leq \frac{k}{3}d(Tz, z)$$

which is a contradiction, hence

$$d(Tz, z) = 0 \Rightarrow Tz = z.$$

Again, since the pair (T, B) is compatible of type(A) and B is continuous then

$$BBx_{2n} \to Bz \quad \& \quad TBx_{2n} \to Bz \qquad (7)$$

We show that z is the fixed point of B i.e $Bz = z.$

For this put $x = Bx_{2n}$ & $y = x_{2n+1}$ in the relation (3), we have

$$d(TBx_{2n}, Sx_{2n+1}) \leq$$

$$a\left[\frac{d(Ax_{2n+1}, Sx_{2n+1})d(BBx_{2n}, Ax_{2n+1})}{d(ABx_{2n}, TBx_{2n}) + d(Sx_{2n+1}, ABx_{2n})}\right]$$

$$+b\left[\frac{d(TBx_{2n}, ABx_{2n})d(Tx_{2n+1}, Bx_{2n+1})}{d(ABx_{2n}, TBx_{2n}) + d(Sx_{2n+1}, ABx_{2n})}\right]$$

$$+g\left[\frac{d(ABx_{2n}, SBx_{2n})d(Sx_{2n+1}, Ax_{2n+1})}{d(ABx_{2n}, TBx_{2n}) + d(Sx_{2n+1}, ABx_{2n})}\right]$$

$$+d\left[\frac{d(BBx_{2n}, TBx_{2n})d(Ax_{2n+1}, Sx_{2n+1})}{d(ABx_{2n}, Ax_{2n}) + d(BBx_{2n}, Tx_{2n+1})}\right]$$

$$+k\left[\frac{d(BBx_{2n}, Ax_{2n+1})d(Tbx_{2n}, Sx_{2n+1})}{d(BBx_{2n}, TBx_{2n}) + d(BBx_{2n}, Sx_{2n+1})}\right]$$

$$+m\left[\frac{d(ABx_{2n}, SBx_{2n})d(Bx_{2n+1}, Tx_{2n+1})}{d(BBx_{2n}, Ax_{2n+1}) + d(ABx_{2n}, Tx_{2n+1})}\right]$$

Now taking limit as $n \to \infty$ and using relations (5) and (7) we have

$$d(Bz, z) \leq k\,\frac{d(Bz, z)d(bz, z)}{d(Bz, Bz) + d(Bz, z)}$$

$$\leq k\,\frac{1}{3}d(Bz, z)$$

which is a contradiction, hence

$$d(Bz, z) = 0 \Rightarrow Bz = z$$

Again, Since the pair (S, A) is compatible of type(A) and if S is continuous, then by proposition (1) we have

$SSx_{2n} \to Sz \quad and \quad ASx_{2n} \to Sz$

If we consider $x = x_{2n+1}$ and $y = Sx_{2n}$ in relation (3) and proceed as in above cases, then we obtain

$Sz = z$

Similarly one can show that if the pair (S, A) is compatible of type(A) and A is continuous then

$Az = z$

Therefore from above relations, we have

$Az = Bz = Sz = Tz = z$

Thus z is the common fixed point of the mappings A, B, S & T.

Uniqueness :

Let z and w are two common fixed points of the mappings A, B, S and T. Then

$$d(z,w) = d(Tz, Sw) \le a\left[\frac{d(Aw,Sw)d(Bz,Aw)}{d(Az,Tz)+d(Sw,Az)}\right] + b\left[\frac{d(Tz,Az)d(Tw,Bw)}{d(Az,Tz)+d(Sw,Az)}\right] + g\left[\frac{d(Az,Sz)d(Sw,Aw)}{d(Az,Tz)+d(Sw,Az)}\right] + d\left[\frac{d(Bz,Tz)d(Aw,Sw)}{d(Az,Aw)+d(Bz,Tw)}\right] + k\left[\frac{d(Bz,Aw)d(Tz,Sw)}{d(Bz,Tz)+d(Bz,Sw)}\right] + m\left[\frac{d(Az,Sz)d(Bw,Tw)}{d(Bz,Aw)+d(Az,Tw)}\right]$$

$$= a\left[\frac{d(w,w)d(z,w)}{d(z,z)+d(w,z)}\right] + b\left[\frac{d(z,z)d(w,w)}{d(z,z)+d(w,z)}\right] + g\left[\frac{d(z,z)d(w,w)}{d(z,z)+d(w,z)}\right] + d\left[\frac{d(z,z)d(w,w)}{d(z,w)+d(z,w)}\right] + k\left[\frac{d(z,w)d(z,w)}{d(z,z)+d(z,w)}\right] + m\left[\frac{d(z,z)d(w,w)}{d(z,w)+d(z,w)}\right]$$

$$\le \left(\frac{2a}{3} + \frac{4(b+g)}{3} + 2(d+m) + \frac{k}{3}\right)d(z,w)$$

which is a contradiction.

Therefore $d(z,w) = 0 \Rightarrow z = w$.

Hence A, B, S and T have a unique common fixed point.

Now we have following corollaries.

If we put $B = A$ in theorem (2) , then it is reduced to following corollary

Corrollary 1 *Let (X, d) be a complete d-metric space. Let* $A, S, T : X \to X$ *be mappings satisfying the condition*

$T(X)$ & $S(X) \subset A(X)$

The pairs (T, A) & (S, A) *are compatible of type*(A)

and

$$d(Tx,Sy) \le a\left[\frac{d(Ay,Sy)d(Ax,Ay)}{d(Ax,Tx)+d(Sy,Ax)}\right] + b\left[\frac{d(Tx,Ax)d(Ty,Ay)}{d(Ax,Tx)+d(Sy,Ax)}\right] + g\left[\frac{d(Ax,Sx)d(Sy,Ay)}{d(Ax,Tx)+d(Sy,Ax)}\right] + d\left[\frac{d(Ax,Tx)d(Ay,Sy)}{d(Ax,Ty)+d(Ax,Ty)}\right] + k\left[\frac{d(Ax,Ay)d(Tx,Sy)}{d(Ax,Tx)+d(Ax,Sy)}\right] + m\left[\frac{d(Ax,Sx)d(Ay,Ty)}{d(Ax,Ay)+d(Ax,Ty)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a + b + g + d + k + m < 1$. If any one of A, S, T is continuous then A, S and T have a unique common fixed point in X.

If we put T = S and B = A then the above theorem (2) is reduced to the following corollary

Corrollary 2 *Let (X, d) be a complete d-metric space. Let* $A, S : X \to X$ *be mappings satisfying the condition*

$S(X) \subset A(X)$

The pair (S, A) *is compatible of type*(A)

and

$$d(Sx,Sy) \le a\left[\frac{d(Ay,Sy)d(Ax,Ay)}{d(Ax,Sx)+d(Sy,Ax)}\right]$$

$$+b\left[\frac{d(Sx,Ax)d(Sy,Ay)}{d(Ax,Sx)+d(Sy,Ax)}\right]$$

$$+g\left[\frac{d(Ax,Sx)d(Sy,Ay)}{d(Ax,Sx)+d(Sy,Ax)}\right]$$

$$+d\left[\frac{d(Ax,Sx)d(Ay,Sy)}{d(Ax,Sy)+d(Ax,Sy)}\right]$$

$$+k\left[\frac{d(Ax,Ay)d(Sx,Sy)}{d(Ax,Sx)+d(Ax,Sy)}\right]$$

$$+m\left[\frac{d(Ax,Sx)d(Ay,Sy)}{d(Ax,Ay)+d(Ax,Sy)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a+b+g+d+k+m < 1$.

If any one of A, S is continuous then A and S have a unique common fixed point in X.

If we put A = B = I and T = S in the above theorem (2) then we obtain

Corrollary 3 *Let (X, d) be a complete d-metric space. Let* $S, I : X \to X$ *be mappings satisfying the condition*

$S(X) \subset X$

The pair (S, I) *is compatible of type*(A)

and

$$d(Sx,Sy) \le a\left[\frac{d(y,Sy)d(x,y)}{d(x,Sx)+d(Sy,x)}\right]$$

$$+b\left[\frac{d(Sx,x)d(Sy,y)}{d(x,Sx)+d(Sy,x)}\right]$$

$$+g\left[\frac{d(x,Sx)d(Sy,y)}{d(x,Sx)+d(Sy,x)}\right]$$

$$+d\left[\frac{d(x,Sx)d(y,Sy)}{d(Ax,Sy)+d(Ax,Sy)}\right]$$

$$+k\left[\frac{d(x,y)d(Sx,Sy)}{d(x,Sx)+d(x,Sy)}\right]$$

$$+m\left[\frac{d(x,Sx)d(y,Sy)}{d(x,y)+d(x,Sy)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a+b+g+d+k+m < 1$. If S is continuous then S have a unique common fixed point in X.

If we put A = B = I in the above theorem (2) then we obtain the following corollary

Corrollary 4 *Let (X, d) be a complete d-metric space. Let* $S, T, I : X \to X$ *be mappings satisfying the condition*

$T(X)$ & $S(X) \subset I(X)$

The pairs (T, I) & (S, I) *are compatible of type*(A)

and

$$d(Tx,Sy) \le a\left[\frac{d(y,Sy)d(x,Ay)}{d(x,Tx)+d(Sy,x)}\right]$$

$$+b\left[\frac{d(Tx,x)d(Ty,y)}{d(x,Tx)+d(Sy,x)}\right]$$

$$+g\left[\frac{d(x,Sx)d(Sy,y)}{d(x,Tx)+d(Sy,x)}\right]+d\left[\frac{d(x,Tx)d(y,Sy)}{d(x,Ty)+d(x,Ty)}\right]$$

$$+k\left[\frac{d(x,y)d(Tx,Sy)}{d(x,Tx)+d(x,Sy)}\right]+m\left[\frac{d(x,Sx)d(y,Ty)}{d(x,y)+d(x,Ty)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a+b+g+d+k+m < 1$.

If any one of S and T is continuous then S and T have a unique common fixed point in X.

If we put T = S in the above theorem (2) then we obtain

Corrollary 5 *Let (X, d) be a complete d metric space.*

Let $A, B, S : X \to X$ *be mappings satisfying the condition*

$S(X) \subset A(X)$ & $S(X) \subset B(X)$

The pairs (S, B) & (S, A) *are compatible of type* (A)

and

$$d(Sx, Sy) \le a\left[\frac{d(Ay, Sy)d(Bx, Ay)}{d(Ax, Sx) + d(Sy, Ax)}\right]$$

$$+ b\left[\frac{d(Sx, Ax)d(Sy, By)}{d(Ax, Sx) + d(Sy, Ax)}\right]$$

$$+ g\left[\frac{d(Ax, Sx)d(Sy, Ay)}{d(Ax, Sx) + d(Sy, Ax)}\right]$$

$$+ d\left[\frac{d(Bx, Sx)d(Ay, Sy)}{d(Ax, Sy) + d(Bx, Sy)}\right]$$

$$+ k\left[\frac{d(Bx, Ay)d(Sx, Sy)}{d(Bx, Sx) + d(Bx, Sy)}\right]$$

$$+ m\left[\frac{d(Ax, Sx)d(By, Sy)}{d(Bx, Ay) + d(Ax, Sy)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a + b + g + d + k + m < 1$. If any one of A, B, S is continuous then A, B and S have a unique common fixed point in X.

Now we have the following theorem

Theorem 3 *Let (X, d) be a complete d-metric space. Let* $A, B, S, T : X \to X$. *Suppose that any one of A, B, S, T is continuous and for some positive integers p, q, r, s which satisfy the following conditions*

$T^s(X) \subset A^p(X)$ & $S^r(X) \subset B^q(X)$

The pairs (T, B) & (S, A) *are compatible of type* (A) and

$$d(T^s x, S^r y) \le a\left[\frac{d(A^p y, S^r y)d(B^q x, A^p y)}{d(A^p x, T^s x) + d(S^r y, A^p x)}\right]$$

$$+ b\left[\frac{d(T^s x, A^p x)d(T^s y, B^q y)}{d(A^p x, T^s x) + d(S^r y, A^p x)}\right]$$

$$+ g\left[\frac{d(A^p x, S^r x)d(S^r y, A^p y)}{d(A^p x, T^s x) + d(S^r y, A^p x)}\right]$$

$$+ d\left[\frac{d(B^q x, T^s x)d(A^p y, S^r y)}{d(A^p x, T^s y) + d(B^q x, T^s y)}\right]$$

$$+ k\left[\frac{d(B^q x, A^p y)d(T^s x, S^r y)}{d(B^q x, T^s x) + d(B^q x, S^r y)}\right]$$

$$+ m\left[\frac{d(A^p x, S^r x)d(B^q y, T^s y)}{d(B^q x, A^p y) + d(A^p x, T^s y)}\right]$$

for all $x, y \in X$ and $a, b, g, d, k, m \ge 0$ such that $0 \le a + b + g + d + k + m < 1$ then A, B, S and T have a unique common fixed point in X.

Proof. The proof of this theorem is similar to the above theorem (2).

References

Banach, S. 1922. Sur les operations dans les ensembles abstraits et leur applications aux equations integrales, *fundamental Mathematicae* **3**(7): 133-181.

Hitzler, P. and A. K. Seda. 2000. Dislocated Topologies, *J. Electr. Engg.* **51** (12): 3-7.

Jungck, G. 1976. Commuting mappings and fixed points, *Amer. Math. Monthl.* **83**: 261-263.

Jungck, G. 1986. Compatible mappings and common fixed points, *Int. J. Math. Sci.* **9**: 771-779.

Jungck, G.1988. Compatible mappings and common fixed points(2), *Int. J. Math. Sci*. **11**: 285-288.

Jungck, G., P.P.Murthy and Y. J. Cho. 1993. Compatible mappings of type (A) and common fixed points, *Math. Japon.* **38**(**2**): 381 - 390 .

Matthews, S. G. 1986.*Metric domains for completeness. Technical report 76*, Department of Computer Science, University of Warwick, Coventry, UK.

Panthi D., K. Jha, P.K. Jha and P. S. Kumari. 2015. A common fixed point theorem for twopairs of mappings in dislocated metric space, *American J. Computational Mathematics* **5**: 106-112.