Integration of GNSS, Total Station, and Grid Controls: An Analysis of Combined Effect of Elevation of Topography and Map Projection Distortion to Solve the Distance Discrepancy

Shanker KC[1], Suresh Shrestha[1], Sandesh Upadhyaya[1], Stallin Bhandari[1]
Shankerkc01@gmail.com, Shrestha.suress@gmail.com, sandeshupadhya1@gmail.com, stallin.bhandari@gmail.com
[1]Survey Department

KEYWORDS

Control Survey Methods, Topography, Map Projection Distortion, Distance Discrepancy, GNSS, Total Station, Grid Controls, Integration

ABSTRACT

Heterogeneous measurements from various surveying methods need to be integrated to accomplish any survey project. The use of Global Navigation Satellite System (GNSS) and Total Station (TS) to establish a control framework along with utilizing existing national grid controls, for any survey project has been the standard practice. However, the successful integration of GNSS, TS, and existing grid controls could be challenging at times. Both survey professionals, and surveying stakeholders would benefit from the successful augmentation of heterogeneous measurements from GNSS, TS, and existing grid controls in order to provide a control framework for their survey project. In the past, the distance discrepancy/mismatch of GNSS-derived distance which is obtained from indirect measurements from GNSS survey and TS distance which is ground-based direct measurements, has created confusion. Herein, we have analyzed in detail, the combined effect of elevation of topography and map projection's distortion on the distance; demonstrate the magnitude of combined effect by numerical examples; tested this formulation with real-world GNSS and TS measurements. This way we proposed a solution to solve distance discrepancy/mismatch between various survey methods. The magnitude of the combined effect would be substantial with higher elevations and longer distances and could cross the threshold of specified/required accuracy. The effect would be more pronounced in mountainous regions suggesting combined effect should be properly taken into account. Taking combined effect into consideration brings the compatibility and comparability of GNSS, TS, and existing grid controls together. Thus, allowing both survey professionals and surveying stakeholders to utilize the mix of GNSS, TS, and existing grid controls to achieve required precision and accuracy in an economical, timely, and easy manner.

1 INTRODUCTION

The field of surveying and mapping gathers heterogeneous survey measurements from different surveying methods. The integration of such heterogeneous survey measurements should result required/specified precision and accuracy of the survey project in an economic manner in order to facilitate successful completion of survey project.

Control survey is the fundamental first step of every survey project. Nowadays, it has been standard practice that: major controls are established by the Global Navigation Satellite System (GNSS) survey method (Seeber, 2003) and further extension and densification of controls is done by Total Station (TS) survey methods. Similarly, the control points/network of any survey project starts from existing national grid controls as the reference points. In later case, the method of controls survey could be both GNSS-based and TS-based. In this study, we focused on this mix/integration of GNSS measurements, TS measurements, and existing grid controls and proposed a solution to make these measurements compatible and comparable with each other.

In Nepal, survey projects such as engineering surveys, cadastral surveys have been using a mix of GNSS and TS in order to establish the control framework. The GNSS-derived coordinates are from indirect GNSS measurements while the TS provides the ground-based direct measurements (Schofield & Breach, 2001). The computational way of GNSS measurements processing to get final coordinates differs from that of TS measurements. Due to this difference in nature, we have experienced the distance discrepancy between these two measurements. Similarly, the distance discrepancy occurred when taking existing grid controls and comparing them with ground-based TS measurements.

In this study, we analyzed the combined effect of elevation of topography (Torge & Müller, 2012) and map projection distortion (Krakiswky, 1973) on distance measurement together and proposed a solution on how measurements from GNSS, TS, and existing grid controls can be made comparable with each other. As we solve the distance discrepancy by considering the combined effect, survey professionals can easily and confidently use the GNSS method for major control framework and TS for further extension and densification. Similarly, the survey professionals can easily and confidently take existing grid controls from the Survey Department (SD) as reference controls for their survey project and extend and densify further controls using both GNSS and TS as per their will.

The use of GNSS to establish a control framework; the use of existing grid controls; and integrate these with TS measurements would motivate any surveying stakeholders to accomplish the control survey phase achieving specified/required precision in an economical, timely and easy manner.

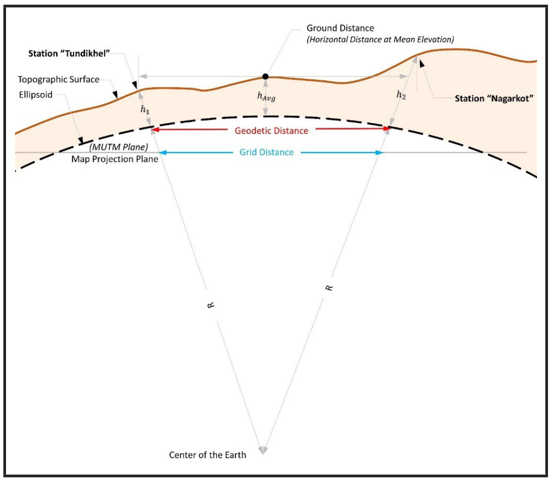
The objective of this study is to provide a theoretical explanation behind making GNSS measurements, TS measurements, and existing grid controls (Ghilani, 2010; Ghilani & Wolf, 2012; Schofield & Breach, 2001) comparable and compatible with each other, specifically dealing with solving distance discrepancy we have been facing in the control establishment phase of engineering and cadastral survey projects. The detailed analysis of the combined effect into distance measurements has been done by providing demonstrative numerical examples and tested in GNSS and TS measurements from real survey projects. We discussed how this would solve the distance mismatch problems of real application scenarios. We argue this combined effect would be more pronounced in mountainous regions and also the way to deal with. In addition, this study makes a point that further well-designed research is necessary in order to have a comprehensive understanding of the problem.

This section presents the background of heterogeneous measurements from different surveying methods and technology (e.g. integration of GNSS and TS measurement for any survey project) and associated implications. Method section presents the theoretical aspect of distance reduction and map projection and elaborates on how change in elevation and change in scale factor can be taken into account in order to solve the problem that this paper is talking about. In the results section, numerical examples are presented to demonstrate the effect of elevation variation and distortion individually and their combined effect in various scenarios. Discussion section describes the potential problems and solution; necessity of further well-designed research in order to have well-rounded understanding of the problem that this paper is talking about. Finally, we conclude our paper.

2 METHOD

2.1 Geodetic reduction

Surveyors perform observations such as distance and angle measurement using TS, height difference measurement, etc. on the surface of topography. The immediate distance measurement from TS subject to geodetic reduction (Torge & Müller, 2012; Vanicek, 1986) and map projection (Krakiswky, 1973; Snyder, 1982) before using it for surveying and mapping activities. The ground distance is the distance measured at the surface of topography (see Figure 1). This ground distance is reduced to the surface of the reference ellipsoid by the geodetic reduction process (Torge & Müller, 2012; Vanicek, 1986). The reduced distance is called geodetic distance and also called ellipsoid distance. In this study, we only deal with geodetic distance reduction. Figure 1 below depicts the concept of geodetic distance reduction.



Source: Sickle, 2015

Figure 1: Distance between any two points at different surfaces. The ground distance is on the mean-elevation of topography; the geodetic (ellipsoid) distance is on the curved surface of the reference ellipsoid; and the grid distance is on the 2D rectangular grid plane.

Suppose we take two stations; "Tundikhel", and "Nagarkot" on the topography of Kathmandu Valley. The former is located in the valley and the latter is on the hilltop. The TS measurement provides horizontal distance at mean-elevation between them. The following relationship between ground distance on topography and corresponding geodetic distance on the surface of the reference ellipsoid exists (Torge & Müller, 2012).

$$s_0 = \sqrt{\frac{hd^2}{(1 + h_1/R)(1 + h_2/R)}} \qquad \text{(i)}$$

Where, hd is the horizontal distance at mean-elevation, h_1 and h_2 are the elevation of two stations, is the mean-radius of the Earth, and s_0 is the geodetic distance.

2.2 Conformal map projection

We are accustomed to the map. We can easily compute distance, azimuths, assess the size of features, etc. for our navigation and other objectives. Performing the same activities on the surface of the ellipsoid in terms of geodetic coordinates, distance, and azimuth would be a difficult task. The map is convenient. Therefore, the point on the curved surface of the reference ellipsoid is projected to the grid plane.

Grid plane is a flat 2D rectangular plane. The linear or polygon feature on reference ellipsoid is projected to grid plane with some distortion. Imagine, taking out a peel of an orange and try to make the peel flat. The peel becomes irregular, misshaped, with uneven bumps, wrinkles, etc. The peel now lacks uniformity, regularity, and smoothness as in its original form.

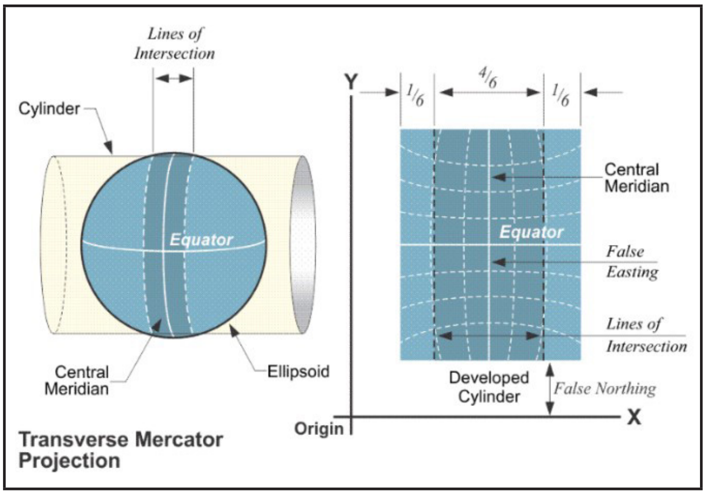
The angle between any two lines in the curved surface of the ellipsoid doesn't change between same lines in a flat grid plane through conformal map projection (Krakiswky, 1973; Shrestha, 2011; Snyder, 1982). Conformal map projection is the class of map projection that preserves the angular distance between lines. As a result, the conformal projection keeps the shape of any feature on the surface of the ellipsoid same in grid (map) plane. However, the length gets changed/distorted (see Figure 1). The navigational purpose map, the topographic map adopts this kind of projection.

The Transverse Mercator is a conformal cylindrical projection (Krakiswky, 1973; Shrestha, 2011; Snyder, 1982). The ellipsoid is supposed to be wrapped around by the developable surface. Here, the developable surface is a transverse secant cylinder, where the cylinder touches the ellipsoid along two meridians. These two meridians are called standard lines and the scale factor is unity along these lines.

The amount of distortion in length can be explained and quantified by scale factor (Krakiswky, 1973; Snyder, 1982). The scale factor is the ratio of map distance to geodetic (ellipsoid) distance (see Figure 1). The unit distance in the curved ellipsoid corresponds to the planar map distance by the amount of scale factor. Hence, the ellipsoid distance is scaled up/down by the scale factor to the map distance.

2.3 Modified Universal Transverse Mercator (MUTM) projection

In Nepal, we have adopted the MUTM projection. Each MUTM grid is made to cover a 3° span of longitude. As Nepal's total longitudinal span is about 9°, it requires 3 separate MUTM grids (Geodetic Survey Division, 1990; S. M. Shrestha, 2017).



Source: Sickle, 2015

Figure 2: Description of transverse secant cylinder as developable surface and the MUTM grid

Table 1 shows the projection parameters of each MUTM grid zone adopted in Nepal.

Table 1: Nepal adopts 3 different MUTM grids: MUTM81, MUTM84, and MUTM87. Different longitude is chosen as the central meridian for each MUTM grid.

Parameters	**MUTM81 Grid**	**MUTM84 Grid**	**MUTM87 Grid**
Central meridian	81° E longitude	84° E longitude	87° E longitude
Origin of Longitude	81° E longitude	84° E longitude	87° E longitude
Origin of Latitude	0° N latitude	0° N latitude	0° N latitude
Scale Factor (along central meridian)	0.9999	0.9999	0.9999
False Easting	500000 m	500000 m	500000 m
False Northing	0 m	0 m	0 m
Reference Ellipsoid	Everest1830	Everest1830	Everest1830

Source: FINNMAP, 1993; Nepal & FINNMAP, 1997b, 1997a

The longitudinal edge of each MUTM grid lies 1°30′00″ away on either side of the central meridian. The secant lines, where the scale factor is unity and distortion is zero, lie 55′00″ away on either side of the central meridian (Geodetic Survey Division, 1990; Shrestha, 2017).

For a 1 km distance in an ellipsoid, it would be 10 cm less along the central meridian, 0 cm less along secant lines, and 18 cm excess along extreme bound lines, in a grid plane while applying MUTM projection.

2.4 Scale Factor Variation

The scale factor (k) varies at each point. The equation to compute the point scale factor can be expressed as (LINZ, 2008; Redfearn, 1948).

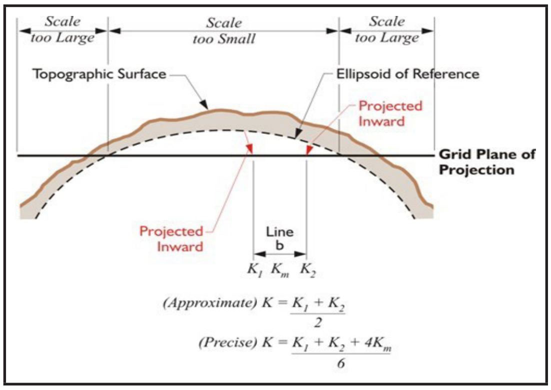
$$
\begin{aligned}
k &= k_0(1 + Term1 + \mathrm{Term2} + \mathrm{Term3})\\
\mathrm{Term1} &= \frac{\omega^2}{2}\psi\cos^2\phi\\
\mathrm{Term2} &= \frac{\omega^4}{24}\cos^4\phi\,[4\psi^3(1-6t^2) + \psi^2(1+24t^2) - 4\psi t^2]\\
\mathrm{Term3} &= \frac{\omega^6}{720}\cos^6\phi\,(61 - 148t^2 + 16t^4)
\end{aligned}
\qquad \text{(ii)}
$$

where, $k=k_0$ is scale factor at central meridian, $\omega=\lambda-\lambda_0$, is central meridian, and ϕ is latitude of point of interest.

The line scale factor (K) is the ratio of the planar grid (map) distance to the corresponding ellipsoidal distance between two points (see Figure 3). The rigorous formula to compute the line scale factor can be found in the same document (LINZ, 2008). Here we use an approximate formula (Sickle, 2015).

$$K = \frac{k_1 + k_2 + 4k_m}{6} \qquad \text{(iii)}$$

where, k_1 and k_2 are the scale factor at both stations and $k_m = (k_1 + k_2)/2$ is the mean-scale factor.

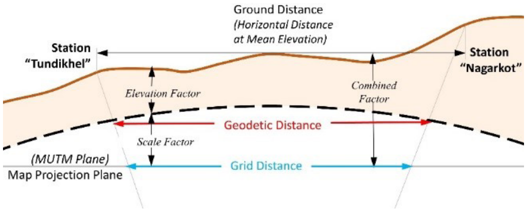


Source: Sickle, 2015

Figure 3: Point scale factor and line scale factor

2.5 Elevation factor, scale factor, and combined factor

The ground distance is reduced to the corresponding geodetic (ellipsoid) distance by elevation factor (see Figure 4).



Source: Sickle, 2015

Figure 4: Description of how ground measured distance is converted to geodetic distance by elevation factor; how the geodetic distance is converted to grid (map) distance by scale factor; and when both elevation factor and scale factor effect together by combined factor.

Suppose the considered stations are at elevations of h_1 and h_2. We compute the mean-elevation (h_m) as in equation (iv). The horizontal distance at mean-elevation (h_m) is. To compute ellipsoid distance, we use for both stations' elevation in equation (i) above and rearrange. The resulting expression is equation (v) (Sickle, 2015; Torge & Müller, 2012)

On the right-hand side of equation (v), the second term is called elevation factor as shown in equation (vi).

$$h_m = \frac{h_1 + h_2}{2} \qquad \text{(iv)}$$

$$s_0 = hd * \frac{R}{R + h_m} \qquad \text{(v)}$$

$$Elevation\ Factor = \frac{R}{R + h_m} \qquad \text{(vi)}$$

Next step is to project ellipsoid distance into a grid (map) distance by scale factor. For the distance between two stations in our case, the line scale factor K is computed by taking point scale factor k_1 and k_2 at both stations as given in above equation (iii) (Sickle, 2015).

The combined scale factor is given by the following expression.

$$\begin{aligned} &Combined\ Factor \\ &= Elevation\ Factor * K \end{aligned} \qquad \text{(vii)}$$

The combined factor directly converts ground distance to grid (map) distance and the inverse combined factor converts back grid distance to ground distance.

3 RESULTS

3.1 Ground distance to geodetic distance and elevation factor

We have described the concept of geodetic reduction and elevation factor in method section. In the subsequent discussion, a numerical demonstration illustrating how ground measured distance is reduced to ellipsoid distance by elevation factor at various elevations of the topography, is provided. Figure 5 below shows the changes in elevation factor as the elevation changes. As the elevation of topography increases from zero elevation at sea level to higher elevation, the elevation factor goes decreasing. As a result, the distance measured at higher elevation reduced more compared to that measured at lower elevation. Figure 5 is for the unit distance. If we take the distance e.g. 100m, then the elevation effect on distance reduction is shown in Table 2.

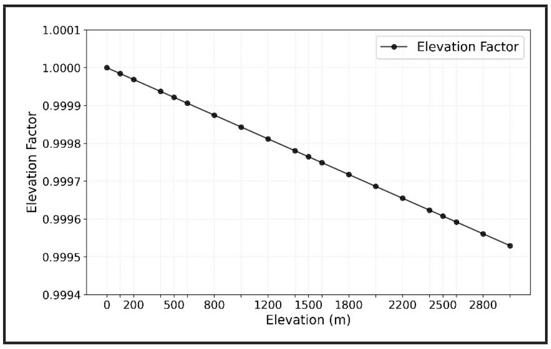


Figure 5: The effect of elevation in distance reduction.

Table 2: Ground distance is reduced to corresponding ellipsoid distance by elevation factor at various elevations of the topography. Different elevation factor applies to different elevations. Higher the elevation, higher the magnitude of the elevation factor.

Ground Distance (m)	Mean-Elevation (m)	Elevation Factor	Ellipsoid Distance (m)
100.00	0.00	1	100
100.00	100.00	0.9999843	99.998
100.00	200.00	0.99996861	99.997
100.00	400.00	0.99993722	99.994
100.00	500.00	0.99992153	99.992
100.00	600.00	0.99990583	99.991
100.00	800.00	0.99987445	99.987
100.00	1000.00	0.99984306	99.984
100.00	1200.00	0.99981168	99.981
100.00	1400.00	0.9997803	99.978
100.00	1500.00	0.99976461	99.976
100.00	1600.00	0.99974893	99.975
100.00	1800.00	0.99971755	99.972
100.00	2000.00	0.99968618	99.969
100.00	2200.00	0.9996548	99.965
100.00	2400.00	0.99962343	99.962
100.00	2500.00	0.99960775	99.961
100.00	2600.00	0.99959207	99.959
100.00	2800.00	0.9995607	99.956
100.00	3000.00	0.99952934	99.953

We take a distance of 100 m in topography at various elevations of range 0 m to 3000 m. We computed the elevation factor at each elevation according to equation (vi). Then, the 100 m ground distance is converted to the equivalent ellipsoid distance. We showed the result in the above Table 2. As the elevation increases, the higher the magnitude of elevation factor, thus the distance is reduced by a greater amount. At 0 m elevation, the distance remains same; at 1000 m elevation, the distance is reduced by 1.6 cm; at 2000m elevation, the distance is reduced by 3.1cm and at 3000m elevation the distance is reduced by 4.7cm.

This suggests that, in mountainous regions, the amount of reduction is big and should be careful when using grid coordinates with ground measurement. In addition to elevation effect, the scale factor amplifies the effect.

3.2 Ellipsoid distance to grid / map distance and scale factor

Conformal map projection converts the ellipsoid distance into grid/map plane distance by the scale factor. Here, we have shown the effect of scale factor that results various grid/ map distance for equal distance in ellipsoid by numerical example. Figure 6 below shows the scale factor effect to distance at different longitudes between central meridian and the edge of the MUTM zone. The scale factor is unity at 55′ away from central meridian, less than unity before 55′ and greater than unity after 55′.

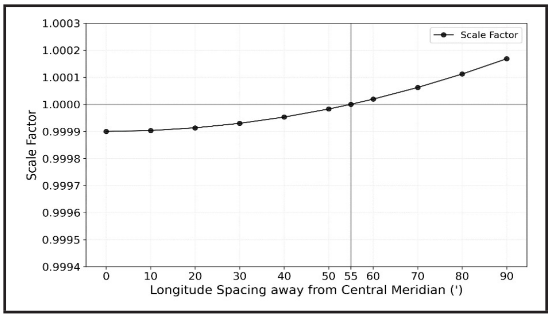


Figure 6: Scale factor variation away from central meridian.

Table 3: Scale factor variation in longitudinal direction at selected designed points and the effect while projecting ellipsoid distance into grid plane.

Ellipsoid Distance (m)	Longitude Spacing (')	Latitude (DD)	Longitude (DD)	Scale Factor	Grid Distance (m)
100.00	0	28.00	84.00000000	0.9999	99.99
100.00	10	28.00	84.16666667	0.99990332	99.99
100.00	20	28.00	84.33333333	0.99991326	99.991
100.00	30	28.00	84.50000000	0.99992984	99.993
100.00	40	28.00	84.66666667	0.99995304	99.995
100.00	50	28.00	84.83333333	0.99998288	99.998
100.00	55	28.00	84.91666667	1.00000029	100
100.00	60	28.00	85.00000000	1.00001936	100.002
100.00	70	28.00	85.16666667	1.00006246	100.006
100.00	80	28.00	85.33333333	1.0001122	100.011
100.00	90	28.00	85.50000000	1.00016857	100.017

We chose 84° E longitude as the central meridian which corresponds to the MUTM84 grid. We designed the point at every 10 ′

longitude interval resulting 11 points. These points have varying longitude away from the central meridian and reflect the scale factor variation of transverse Mercator projection (see Table 3). The secant lines is along 55′ away on either side of the central meridian. The scale factor is set to 0.9999 at the central meridian and increases outward, is unity at 55 ′ away and again increases outward up to the edge of the grid.

We choose a 100 m distance at every point and see the changes by scale factor. For a 100 m distance on the ellipsoid surface, the amount of distortion is -1.0 cm at the central meridian; 0 cm at 55′ away from the central meridian; and +1.7 cm at the extreme edge. This applies to either side of the central meridian. The scale factor effect will be bigger for larger distances such as 500 m, and 1000 m, resulting bigger distortion amount.

3.3 Ground to Ellipsoid to Grid and Combined Factor

The ground measured distance at a certain elevation is reduced to ellipsoid by elevation factor, and that ellipsoid distance is converted to grid /map distance by scale factor. The elevation factor varies as elevation varies and the scale factor varies as longitude varies in Transverse Mercator projection. When both the elevation factor and scale factor are combined, we call it a combined factor (see Equation vii). Figure 7 shows the combined effect.

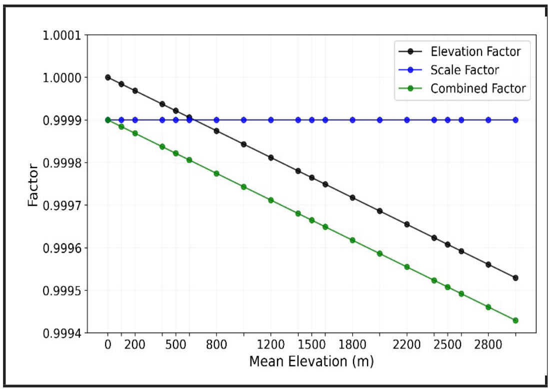


Figure 7: When the elevation factor (black line) combined with scale factor (blue line) results the combined factor (green line). This figure shows the combined effect for unit distance.

Table 4: The combined factor considers both elevation variation and longitude variation effect while plotting ground measured distance into grid plane or vice versa. The combined effect is larger compared to the individual elevation factor effect or scale factor effect.

Ground Distance (m)	Mean-Elevation (m)	Elevation Factor	Ellipsoid Distance (m)	Point Location (Lat, Lon)	Scale Factor	Combined Factor	Grid Distance (m)
100.00	0	1	100	28.0,84.0	0.9999	0.9999	99.99
100.00	100	0.9999843	99.998	28.0,84.0	0.9999	0.99988431	99.988
100.00	200	0.99996861	99.997	28.0,84.0	0.9999	0.99986861	99.987
100.00	400	0.99993722	99.994	28.0,84.0	0.9999	0.99983723	99.984
100.00	500	0.99992153	99.992	28.0,84.0	0.9999	0.99982153	99.982
100.00	600	0.99990583	99.991	28.0,84.0	0.9999	0.99980584	99.981
100.00	800	0.99987445	99.987	28.0,84.0	0.9999	0.99977446	99.977
100.00	1000	0.99984306	99.984	28.0,84.0	0.9999	0.99974308	99.974
100.00	1200	0.99981168	99.981	28.0,84.0	0.9999	0.9997117	99.971
100.00	1400	0.9997803	99.978	28.0,84.0	0.9999	0.99968032	99.968
100.00	1500	0.99976461	99.976	28.0,84.0	0.9999	0.99966464	99.966
100.00	1600	0.99974893	99.975	28.0,84.0	0.9999	0.99964895	99.965
100.00	1800	0.99971755	99.972	28.0,84.0	0.9999	0.99961758	99.962
100.00	2000	0.99968618	99.969	28.0,84.0	0.9999	0.99958621	99.959
100.00	2200	0.9996548	99.965	28.0,84.0	0.9999	0.99955484	99.955
100.00	2400	0.99962343	99.962	28.0,84.0	0.9999	0.99952347	99.952
100.00	2500	0.99960775	99.961	28.0,84.0	0.9999	0.99950779	99.951
100.00	2600	0.99959207	99.959	28.0,84.0	0.9999	0.99949211	99.949
100.00	2800	0.9995607	99.956	28.0,84.0	0.9999	0.99946075	99.946
100.00	3000	0.99952934	99.953	28.0,84.0	0.9999	0.99942939	99.943

In Table 4, we demonstrate the effect of the combined factor by a numerical example. We used 100 m ground measured distance at various elevations, used central meridian with a 0.9999 scale factor and computed the grid /map distance. The changes amount to +1.0 cm at 0 m elevation, +1.8 cm at 500 m elevation, and 2.6 cm at 1000 m elevation, 4.9 cm at 2000 m elevation and 5.7 cm at 3000 m elevation. The combined effect is larger. For topography having a higher altitude than 1000 m, the combined effect is greater than 2 cm in magnitude. The combined effect is 5cm at 2000 m elevation, which is a significant effect. The similar effect can be seen at the edge of each MUTM zone (see Table 5 below).

Table 5: The combined factor and its effect at the edge of the MUTM zone (e.g. at 85.5 ° E longitude).

Ground Distance(m)	Mean-Elevation	Elevation Factor	Ellipsoid Distance (m)	Point Location (Lat, Lon)	Scale Factor	Combined Factor	Grid Distance (m)
100.00	0	1	100	28.0,85.5	1.00016857	1.00016857	100.017
100.00	100	0.9999843	99.998	28.0,85.5	1.00016857	1.00015288	100.015
100.00	200	0.99996861	99.997	28.0,85.5	1.00016857	1.00013718	100.014
100.00	400	0.99993722	99.994	28.0,85.5	1.00016857	1.00010578	100.011
100.00	500	0.99992153	99.992	28.0,85.5	1.00016857	1.00009009	100.009
100.00	600	0.99990583	99.991	28.0,85.5	1.00016857	1.00007439	100.007
100.00	800	0.99987445	99.987	28.0,85.5	1.00016857	1.000043	100.004
100.00	1000	0.99984306	99.984	28.0,85.5	1.00016857	1.00001161	100.001
100.00	1200	0.99981168	99.981	28.0,85.5	1.00016857	0.99998023	99.998
100.00	1400	0.9997803	99.978	28.0,85.5	1.00016857	0.99994884	99.995
100.00	1500	0.99976461	99.976	28.0,85.5	1.00016857	0.99993315	99.993
100.00	1600	0.99974893	99.975	28.0,85.5	1.00016857	0.99991746	99.992
100.00	1800	0.99971755	99.972	28.0,85.5	1.00016857	0.99988608	99.989
100.00	2000	0.99968618	99.969	28.0,85.5	1.00016857	0.9998547	99.985
100.00	2200	0.9996548	99.965	28.0,85.5	1.00016857	0.99982332	99.982
100.00	2400	0.99962343	99.962	28.0,85.5	1.00016857	0.99979195	99.979
100.00	2500	0.99960775	99.961	28.0,85.5	1.00016857	0.99977626	99.978
100.00	2600	0.99959207	99.959	28.0,85.5	1.00016857	0.99976057	99.976
100.00	2800	0.9995607	99.956	28.0,85.5	1.00016857	0.9997292	99.973
100.00	3000	0.99952934	99.953	28.0,85.5	1.00016857	0.99969783	99.97

In mountainous regions (e.g. elevation greater than 1000 m and up to 2500 m), the play of elevation factor combined with scale factor results significant magnitude effect between ground measurement and grid measurement. This suggests to be careful and to well considering this combined effect while mixing both ground measurements from TS with grid/map measurements from existing grid coordinates and existing grid control points.

3.4 Application to integrated measurements from GNSS and TS

We test the effect to both elevation variation and scale factor with real measurement from GNSS observation and TS measured ground distance. We performed GNSS observation over 6 stations, forming 3 sides. We measured the ground distance of 3 sides using TS. The GNSS coordinates based on the global reference ellipsoid (Moritz, 1980) are transformed to local reference ellipsoid i.e. Nepal Datum ellipsoid (KC & Acharya, 2023; UK, 1985) by 7P transformation model (Adhikary, 2002; Manandhar & Bhattarai, 2002; K. G. Shrestha, 2011) and projected to MUTM84 grid. The inverse process: MUTM84 grid distance are scaled back to ellipsoid distance, and ellipsoid distance is scaled back to ground distance. The result is shown in Table 6 below.

Table 6: Inverse operation results the ground distance from MUTM84 grid coordinates and compared with actual TS measured ground distance. Both ground distances seem comparable.

Side	Grid Distance (m)	Mean-Elevation (m)	Elevation Factor	K	Combined Factor	Ground Distance (m)	Measured Ground Distance (m)	Difference Measured & Computed (m)
100-1002	157.219	161.096	0.99997471	0.99990038	0.9998751	157.239	157.237	0.002
1003-1004	300.311	164.941	0.99997411	0.99990097	0.99987509	300.349	300.361	-0.012
1005-1006	80.583	162.332	0.99997452	0.99990056	0.99987509	80.593	80.589	0.004

The ellipsoidal height is used to compute the elevation factor. Instead of the point scale factor, (k) the line scale factor (K) is used (see equation 3). We compared ground distance obtained from inverse operation with actual TS measured ground distance. The deviation is very small attributed to random errors, showing comparable results. Thus demonstrating the effect of elevation variation and scale factor variation is significant and needs to be considered accordingly when mixing TS and GNSS measurements in survey projects.

4 DISCUSSION

Cadastral survey is being carried out in the various parts of the nation. It has been standard practice that the required control network is established by the GNSS survey. Then, the traverse survey using TS is performed for further control densification by taking GNSS-established control points as the reference points. Here, the TS measurements are ground measurements while the GNSS-established control points have MUTM grid based coordinates. In order to make these heterogeneous measurements compatible with each other, the combined effect of both elevation and scale factor variation should be taken into account.

Recently, we faced the problem of mismatch/discrepancy between ground based TS measurements and grid coordinates (e.g. discrepancy in TS orientation) that occurred during the cadastral survey. This analysis of combined effects what might have been the case.

The combined effect would be more pronounced in the mountainous part. The high elevation nature of the mountainous part means the higher the effect of the elevation factor. Similarly, in mountainous regions, the length of the traverse leg or side or baseline tends to be longer. The longer the side, the elevation effect and scale factor effect would result in larger magnitude combined effect. For a traverse leg of 500 m in length at an elevation of 2000 m, the combined effect would be ~ 22 cm. Similarly, for a traverse leg of 1000 m in length and at elevation of 2000 m, the combined effect would be ~ 41 cm. these are substantial amount. This means that, while doing surveying works such as control point establishment by GNSS and later densification by TS as in our standard practice of cadastral survey, in hilly and mountainous parts, this combined effect must be taken into account.

The problem of mismatch/discrepancy between GNSS derived grid coordinates and ground based TS measurements have been a problem in engineering surveying works also. It has been found that the GNSS-derived measurements don't match with TS measurements between any two tower locations of transmission line projects. A similar has been found in reference control of hydropower projects. Here, taking the combined effect into account would bridge the gap between heterogeneous measurements.

Whenever the integration of GNSS and TS measurements needs to be utilized for any kind of surveying projects, this combined effect of both elevation and scale factor should be taken into account.

In the Terai lowland, the elevation factor would be very close to unity and would have very minimal effect, smaller than the tolerance specified/required by the survey project. Only taking the scale factor into account would suffice. This kind of practice has been in practice in the traverse survey of the India-Nepal border survey where only after the grid coordinates are scaled up by scale factor and compared with ground based TS measurements.

SD has established a horizontal control network of first, second, third, and fourth order categories (KC & Acharya, 2022). The fourth order control points are 200 m - 2000 m spacing distant (Geodetic Survey Division, 1990) and are MUTM grid coordinates. When we take TS measurements between any pairs of these fourth order controls, these TS measurements first should be corrected for combined effect, then only these measurements can be compared with MUTM grid coordinates. Otherwise, the discrepancy between the same pairs of fourth order controls would throw you off.

A survey project may have MUTM grid coordinates from past triangulation surveys, may have GNSS derived coordinates from GNSS control survey, and may have to do TS measurements further. This is exactly where the combined effect of elevation and scale factor becomes crucial and well taken care of.

A further well designed research study needs to be carried out in order to answer various questions such as 1) what would be the effective coverage region of the combined factor; 2) will the single combined factor work for the entire survey project area or do we need multiple factors, if so how to properly deal with that situation; 3) what if the same project area has very low and very high elevation region; 4) what if one only uses the GNSS survey during entire project period, would he/she need to take care of combined effect also.

5 CONCLUSION

The field of surveying and mapping science needs to deal with the mixture of heterogeneous measurements from different and various surveying methods. Integrating heterogeneous measurements that meet the precision and accuracy of survey projects could be challenging at times. Specifically, in this study, we focused on the integration of TS and GNSS measurements, particularly to solve the distance problem/discrepancy.

We analyzed the combined effect of elevation of topography and map projection distortion on distance; demonstrated the magnitude of the effect by numerical examples. We showed that by taking combined effect into consideration, ground based TS measurements can be seamlessly augmented/mixed with GNSS measurements. In fact, several surveying projects such as engineering surveys, cadastral and topographical mapping, etc rely on both TS and GNSS measurements for control establishment. We demonstrated that the combined effect would be larger in mountainous regions. However, considering the combined effect would eliminate the discrepancy between TS and GNSS measurements.

Our past experiences showed that, the distance discrepancy between GNSS-derived grid coordinates and TS measurements or existing grid controls and TS measurements has been a severe issue in the control survey part of engineering surveys and cadastral surveys in Nepal. This study has researched the problem with a theoretical aspect, further clarified by numerical examples, and evident by actual GNSS and TS measurements.

This study showed that by considering the combined effect of elevation and distortion, one can easily mix GNSS-derived coordinates, existing grid controls, and TS measurements smoothly in any survey project. By considering the combined effect, the indirect measurements/

coordinates from the GNSS survey are made comparable with direct measurements from TS. This caused increased confidence and motivation of surveying professionals to leverage the GNSS technology in addition to their existing technologies. Next, the integrity of existing grid controls provided by SD can be tested and can be used in any survey project. This would lead to the reduction of the cost of additional control establishment. Next, extending the control survey from existing national geodetic grid control or tie-in of control survey to the existing national geodetic control within the specified tolerance. This way, surveying projects can easily tie to the national grid.

We recommend further well-designed research in order to fully understand problem and devise a solution. The combined effect and mix of various surveying methods should be studied in categories: low-land, hilly, and mountainous regions. Research should be carried out to determine and answer: the effective region of combined factor, multiple combined factors for the same project; optimal combined factor and the guidelines to achieve uniformity.

This study would increase the understanding and clarity when dealing with heterogeneous measurements from GNSS, TS, and existing national grid controls; when making coordinates from indirect GNSS measurements compatible with direct TS measurements. Similarly, the surveying and mapping stakeholders would confidently use mix of existing grid controls, GNSS survey, and TS survey, tied to national grid controls. All these eventually would lead to successful completion of survey projects.

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Author's Information

Name	:	Shanker KC
Academic Qualification	:	BE in Geomatics
Organization	:	Geodetic Survey Division, Survey Department
Current Designation	:	Survey Officer
Work Experience	:	6 years
Published paper/article	:	5