Concepts towards cm-geoid for Nepal GPS to replace conventional leveling using airborne gravity

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Abstract

This paper gives the concepts of the principles of graimetric geoid and summarizes the airborne gravity survey and the determination of new national geoid for the height determination by GPS. The paper also deals with the technique of fitting the computed geoid model with leveling benchmarks and therefore improvement of the geoid will primarily be related to improvements in the vertical datum and GPS data.

Introduction:

Geoid is the shape of the earth that is described as the equipotential surface of the earth's gravity field; resultant of rotational and gravitational potential which fits most closely with the mean sea level, ignoring the long periodic effects of dynamic sea surface topography and it extends through the continents. The geoid is commonly used as the elevation datum to which topographic heights are referred. It is then referred to points on coastlines using tide gauges and relative to these points the orthometric heights of the topographic surface is determined using geodetic spirit leveling technique.

The Global Positioning System (GPS) can achieve height differences accurately to a few parts per million of observed base line length when used in differential mode. It is defined in Geocentric Cartesian World Geodetic System 1984, which is in practice identical to the Geodetic Reference System 1980 (GRS 80) ellipsoid (Moritz, 1980a). Now in order to convert GPS derived geometric height differences into the physically relevant orthometric heights, an accurate knowledge of the position of the geoid relative to GRS 80 is required. If the gravimetric geoid is of sufficient precision, then GPS has the potential to be used as a rapid and cost effective alternative for geodetic sprit leveling orthometric heighting.

In addition to the geodetic application, the geoid contains other valuable information of the Earth's geology and geophysics. The geoid is an equipotential surface in the earths gravity field, and thus a complicated surface, the shape of which is the determined by the earth's mass distribution. Geoid heights are produced by sub-surface density excesses, and lows correspondingly by deficiencies. Hence by identifying the short and intermediate wavelength trends and features in the geoid, the shallow geological structure can be deduced.

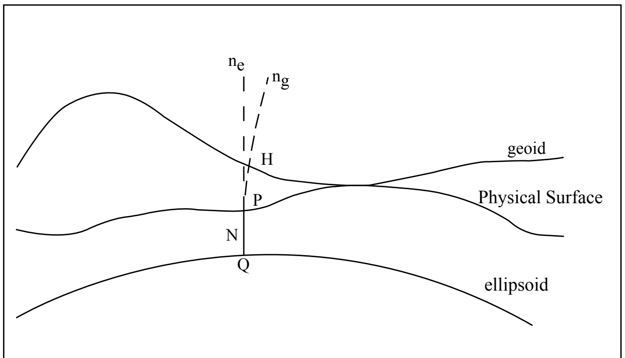
Geoid height with GPS and geodetic leveling :

Here we discuss in relation to quantities such as geoid height, GPS derived geometric height and physical height derived from geodetic leveling. We therefore have a means of determining GPS derived geometric height, also called ellipsoidal height, relative to the GRS 80 reference ellipsoid (h) and the orthometric height (H) relative to the local height datum. The datum equipotential has been defined by mean sea level, which is assumed to be the geoid. This assumption is very important. If it is true then only the network defines the geoid relative to the Earth's physical surface otherwise the datum is not the geoid but another non-parallel equipotential surface. For the case of Nepal, the datum of heights is determined by the Bay of Bengal; this differs from the global average height of the ocean by the local sea-surface topography, which can be up to 1 m or more depending on region.

The difference between the ellipsoidal height and the orthometric height gives the geoid height relative to reference ellipsoid. Therefore the simple equation is

$$N = h^{GPS} - H \qquad (1)$$

All the three quantities have the same unit. Studying in depth we find that the orthometic height is measured along the plumbline, whereas, the ellipsoidal height is measured along the ellipsoidal normal and the two do not coincide. The linear approximation must be extended between the geoid and physical surface of the earth.



Relationship between geoid and ellipsoid quantities

Figure 1

This is validated in the same way as was done between the ellipsoid and geoid. The discrepancy of ~ 2ppm is negligible compared to the uncertainty in either GPS derived or orthometric height can be ignored. Therefore, GPS in conjunction with geodetic leveling can provide geometric estimates of geoid height at discrete points to control the gravimetric geoid. However, it should be remember this control refers to the local sea-level height datum.

Gravimetric determination of geoid :

From the fundamental equation of the physical geodesy the gravimetric geoid height N is determined by Stokes' equation which gives the expression of the geoid height N as an integral of gravity anomalies around the earth (σ)

$$N = \frac{R}{4\pi\gamma}\iint_{\sigma} \Delta g S(\psi) d\sigma \qquad (2)$$

where Δg is the gravity anomaly, R the earth radius, γ normal gravity, and S a complicated function of spherical distance ψ (Heiskanen and Moritz, 1967). In the practical determination of gravimetric geoid, the solution is typically split into three components such as:

$$N = N_{GM} + N_{DEM} + N_{gravity} \qquad (3)$$

Here the first part is the global field, coming from a spherical harmonic expansion of the geopotential. The global spherical harmonic solution is found from the a set of spherical harmonic coefficients C_{nm} and S_{nm} by

$$N_{GM} = \frac{GM}{R\gamma}\sum_{n=0}^{N}\left(\frac{R}{r}\right)^{n}\sum\nolimits_{m=0}^{n}(C'_{nm}\cos m\lambda + S_{nm} snm\lambda)P_{nm}(S\sin\phi) \qquad (4)$$

Where C' indicates that the ellipsoidal part of the coeffecients (primarily due to the earth's flattening) has been removed. The current model in global use is EGM96 and in this model of the geopotential from analysis of satellite data and global mean gravity anomalies are used. e.g. for the current global model EGM96 (Lemoine et. al., 1996).

$$N_{EGM96} = \frac{GM}{R\gamma}\sum_{n=2}^{360}\left(\frac{R}{r}\right)^{n}\sum_{m=0}^{n}(C'_{nm}\cos m\lambda + S_{nm}\sin m\lambda)P_{nm}(\sin\phi) \qquad (5)$$

Here the spherical harmonic coefficients Cnm and Snm for EGM96 complete to degree and order 360 define the long wavelength gravity field (degree 360 corresponds to a resolution of 55km). The online information on EGM96 can be found at www.nga.mil. Correctly a new global model EGM08 is being prepared in cooperation with the International Association of Geodesy; this new spherical harmonic model will have a maximal degree of 2160 (5' resolution), and global data collection activities are ongoing to support this model development, including the planned airborne gravity survey of Nepal, which is primarily supported by NGA.

The second part is the contribution from the local topography. This data is the digital elevation models (DEM's), which provide details of the gravity field variations in the mountainous area. The mass of the mountains can change the geoid by several 10's of cm locally. The handling of digital elevation may be done by analytical prism integration assuming known rock density (Forsberg, 1984). On the global scale, the recently released Shuttle Radar Topography Mission (SRTM) data means that 100 m-resolution DEM's are globally available; therefore geoid determinations in mountainous areas are much improved by these data, especially for the short wavelengts.

The third part is residual contribution from local terrestrial gravity. In the process of determining the gravimetric geoid the computation of gravity anomaly (Δg) which is the difference between observed gravity at the earth's surface reduced to geoid and the normal gravity on the reference ellipsoid. This (Δg) is used in Stokes' formula given by e.g. (2) in the evaluation of the geoid height. If the distribution of the local terrestrial gravity is sparse, then airborne gravity can be used to densify the gravity values in a most efficient way. Airborne gravity has only recently been operational, mainly due to the developmentes in precise kinematic GPS positioning of aircraft.

Finally the data from the spherical harmonic models, local or airborne gravity and DEM's the gravimetric geoid is constructed by remove-restore techniques as the sum

$$N = N_{EGM} + N_{gravity} + N_{DEM}. \quad (6)$$

To be consistent with GPS and local leveling system, a correction between the global and local vertical datums must be made: In principal N_{GPS} = N (gravimetric geoid heights). But we find,

$$N_{GPS} = N + \varepsilon \quad (7)$$

Where "ε" the GPS corrector surface taking into account datum difference and possible error in GPS, spirit leveling and gravimetric geoid "N".

In practice"ε" in determined by fitting the gravimetric geoid at points with coincident GPS and leveling; in these points "ε" can be directly determined by

$$\varepsilon = N_{GPS} - N = h_{GPS} - H_{leveling} - N \quad (8)$$

and "ε" then interpolated to other points by least square collocation.

Test result of GPS derived and leveled height :

The initiative was taken in order to study agreement of heights derived by GPS and leveled heights in Nagarkot area in the course of conducting the detail survey.

$H = h^{GPS} - N$ (H derived from GPS using global geoid height)

H' = orthometric height from geodetic leveling

Leveled height were provided to all the third order GPS points and newly established traverse points. Results are given in table below.

TABLE 1

ST.	PLACE NAME	ELEVATIONS			DIFFERENCES	
		GPS (DERIVED HT.)	LEVEL (MSL HT.) (H)	ELLIPSOID HT. (WGS-84) (h)	GPS-MSL)	(MSL-WGS84) (N)
001	ARMY CAMP T3	2085.488	-	2057.592		
002	ARMY CAMP T2	2085.537	2085.482	2057.643	0.055	-27.839
003	STONE T1	2117.621	-	2089.706		
004	STATION A	2066.951	2067.093	2039.051	-0.142	-28.042
005	PANICHAURE	2099.917	2099.930	2072.014	-0.013	-27.916
006	DEVI DHUNGA	2104.682	-	2076.776		
007	GHIMIRE GAUN	2060.424	2060.460	2032.478	-0.036	-27.982
008	GENERATOR HOUSE	2088.252	2088.279	2060.322	-0.027	-27.957
009	PHULCHWOKI DEVI	2130.784	2130.767	2102.865	0.017	-27.902
010	GUEST HOUSE	2125.107	2125.091	2097.182	0.016	-27.909
011	GUARD HOUSE	2131.064	2131.120	2103.139	-0.056	-27.981
012	LABORATORY	-	2151.711	-		
013	OBSERVATORY HOUSE	-	2154.422	-		
014	OBSERVATORY HOUSE	2154.285	2154.268	2126.344	0.017	-27.924
015	HELIPAD	2158.724	2158.721	2130.780	0.003	-27.941
016	MAHADEV POKHARI	2138.646	2137.630	2110.692	0.016	-27.938
017	TOWER S/W	2145.799	2145.736	2117.837	0.063	-27.899
018	TELICOMUNICATION (T5)	2098.850	2097.813	2070.826	1.037	-26.987
019	TELICOMUNICATION (T4)	2092.655	2092.340	2064.626	0.315	-27.714
-	TOWER 1/157	2165.394	2165.358	-	0.036	
-	DOPPLER POINT	2151.722	2151.787	-	-0.065	
-	TRACKING STATION	2151.780	-	2123.827		

Interesting result is seen. The difference between GPS derived orthometic height using global geoid model and MSL height is observed. The difference is only in the centimeter level in most of the points where as in some points result agreed in the millimeter level. The difference between the MSL height and WGS84 ellipsoidal height, i.e. geoid height, is found to be around 28 meter in this area. The negative sign indicated that geoid in this area is 28 meters below the mathematical surface WGS84 ellipsoid. Thus in a local area GPS is already now useful for determine heights.

Gravimetric geoid information of Nepal:

A geoid NEPAL97 for the whole of Nepal was computed using material (GPS, leveling, gravimetry) collected over many years in both East and West Nepal. The technique used was “GPS-gravimetric”: First a gravimetric geoid was computed using geopotential theory (the Molodensky approach). Then, this geoid was “fitted” to a set of given geoid undulation values obtained from GPS heighting and (classical as well as trigonometric) leveling.

The reference surface obtained this way is a regionally GPS-adapted geoid, i.e. a representation of an equipotential surface near sea level, inside the topographic rocks, which is consistent with the existing Nepal height datum. This means that it can be used together with orthometric heights, as have been traditionally used in Nepal.

The datum of the computed geoid is the *Nagarkot* datum, i.e. in Nagarkot (GPS40) the geoid height is assumed 0.0 m and its height above the reference ellipsoid (GRS80) as determined by GPS is assumed equal to its orthometric height, 2151.78 m. Therefore GPS measurements should be transformed to this datum by adding a constant to all height values *h* before its is attempted to compute orthometric heights from them using NEPAL97 geoid. For the GPS measurements in Eastern Nepal, such a translation had already been made; for the Western Nepal measurements, 22.733 m has to be added (Definition of gravimetric geoid Vol. III)

Determination new gravimetric geoid by airborne gravity measurement:

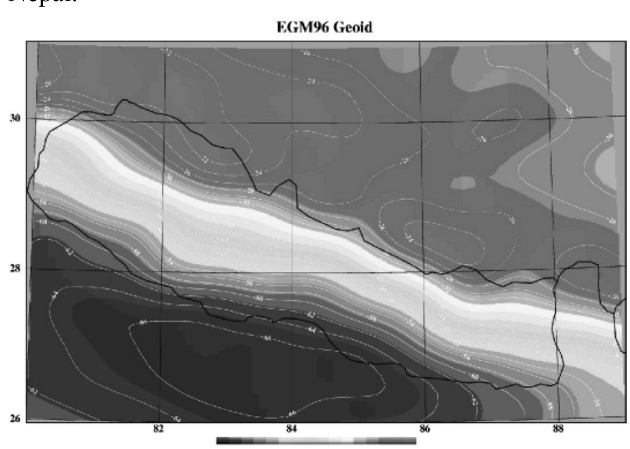
A cooperative venture/assistance programme to allow height determination accurately by modern satellite survey methods in Nepal, and increase scientific research in the field, by Survey Department/Geodetic Survey Branch and The Danish National Space Center is expected in near future. This would foremost involve a complete coverage of Nepal with airborne gravity data, including data in the high mountain regions, where little gravity data exists, and thus makes geoid prone to errors. Cooperation will also involve utilization of new satellite gravity fields, derived e.g. from GRACE.

The new airborne gravity survey is planned for November 2008, and an accuracy of 2 mgal is expected, corresponding to geoid errors at 5-10 cm over most of Nepal.

A very special application of the geoid would be to determine the real height of Sagarmatha/Mount Everest. In recent years we have seen several international press releases of GPS projects on the mountain, none of which have been able to address the geoid issue of this region. If this project is implemented it would settle these issues and likely generate further media attention by providing a much more accurate height for the world’s highest mountains.

Challenges in geoid determination:

The geoid of Nepal is incredibly rough due to the Himalayas, and the complex geology of the region. A rough estimate of the geoid is provided by global models (EGM96), based on satellite information and the sparse available gravity survey data, and shown below. The geoid is only known to an accuracy of a few meter generally in Nepal.



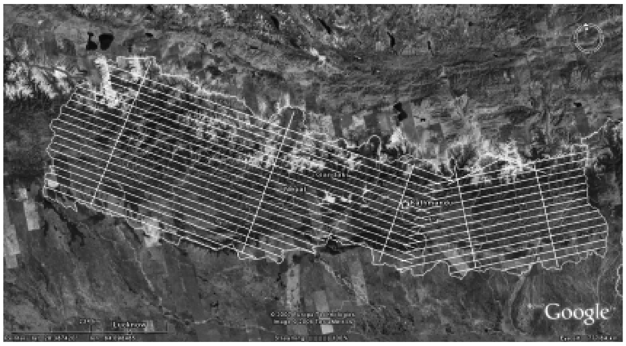
Global geoid of Nepal, 2 m contour interval. Geoid is height of sea-level above GPS reference ellipsoid.

To improve the situation the following data are needed:

- Additional gravity data, to be provided by *airborne gravity measurements.*
- *Digital height data* of the topography (from recent Finnish-Nepalese mapping projects and the recent

US satellite radar topography mission SRTM).

- *GPS observations* on levelling benchmarks of the existing Nepal height network (restricted essentially to the lowland and central valley along roads).



Planned airborne tracks (10 km spacing) of the airborne gravity survey.

Mixing of all these data for the best possible result represent a major scientific challenge, and the Nepal geoid determination might produce global interest for physical geodesy research. It is planned to do the Nepal geoid by the method of least-squares collocation for downward continuation of airborne data and merging with existing surface data; by spherical Fourier transformations (Forsberg and Sideris, 1993); and by using analytical prism integration for the best modeling of the topographic effects on the geoid.

Conclusion and recommendation:

Fundamental geodetic networks are needed for general mapping, environment, development of communication system, irrigation, water supply and hydropower as well as study the crustal dynamics. Measurement of heights was carried out in Nepal by traditional methods of leveling. Because of the topographical irregularities of Nepal, measurement of heights are traditionally a very complicated and time consuming techniques.

Modern geodetic satellite positioning by GPS, survey operations have become much more efficient and accurate. Heights can, however, not be obtained directly from GPS measurements. But in the present context one of the main objective in using GPS with full potential is replacing the geodetic spirit leveling by GPS measurement.

The main practical goal of this project will be a determination of new national geoid of Nepal – consistent with older measurements – accurate at the cm level or better over the main important areas of the country.

Then national height system of Nepal will be much improved if surveys can be carried out using modern satellite techniques. New technique of airborne gravity/ GPS surveys can be used to determine the precise Geoid of Nepal. Precise Geoid is used for defining the height datum of a country and investigations towards the geophysical changes as well as for the strengthening, supplement and expand the existing geodetic control network system of Nepal. In this context Surey Department will take necessary steps regarding this project, and enter the international cooperation.

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