PROCESS CAPABILITY ANALYSIS MADE SIMPLE THROUGH GRAPHICAL APPROACH

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ABSTRACT

The statistical techniques can be used to quantify the variability of the manufacturing process. The process capability analysis helps in quantifying the process variability and assists manufacturing by reducing the variability. Process capability of a manufacturing process can be assessed through the computations of various process capability ratios. Graphical approach can be extended and used to compute capability ratios. A graphical approach of the process capability study is a presentation of process capability parameters in the form of graphs and gives quick information for further computation and analysis.

The process capability can be measured and analyzed through the computations of various process capability indices by analytical methods. This involves more computations and time consuming and not user-friendly. The mean and grand mean computations are replaced by median and grand median. Graphical method of computations of median, grand median and average range is developed. Graphical approach can be used for process capability measurement. The graphical approach of process capability study makes it easier to assess the variability of the manufacturing process with less computation and consumes less time and makes user-friendly analysis. The process capability results using median has less affected by extreme values. The median chart indicates on-line variability. If, the control limits are introduced to median chart it performs like average and range chart with fewer computations and consumes less time. This approach helps to worker in the shop floor to assess the process.

Keywords: Process capability, Process capability analysis, Graphical approach.

INTRODUCTION

The main objectives of process management are to prevent the defects and to minimize the variability in manufacturing process. To reduce the variability it needs to establish the relationship between process variable and product results. Even though product meets the specification limits and the process is in the state of control, the actions may be initiated to quantify and further reducing the process variability to gain marvelous advantages. The statistical techniques can be used to quantify the variability of the manufacturing process. The process capability analysis helps in quantifying the process variability and assists manufacturing by eliminating or greatly reducing the variability.

Process capability analysis: Process capability is inherent variation of the product turned out by the process. Process capability provides the quantified prediction of the process adequacy and refers to the uniformity of the process. It is a measurement with respect to the precision of a manufacturing process.

Process capability analysis is a measurement of quality performance capability of the process with given factors and under statistical control conditions. Estimate of process capability may be in the form of probability distribution having a specified shape, center and spread. In the sense, a process capability analysis may be performed without regard to specifications on the quality characteristic. The process capability can be measured and analyzed through the computations of various process capability indices by analytical methods. Graphical approach can be used for process capability measurement. The graphical approach of process capability study makes it easier to assess the variability of the manufacturing process with less computation and time.

OBJECTIVES

- Predicting the extent of variability that process will exhibit.
- Capability information helps designers to set realistic specifications limits.
- Planning the interrelationship of sequential process.
- Selecting between the competing vendors.
- Reducing the variability of the manufacturing process.

METHOD

Analytical method of measuring process capability: The capability of a manufacturing process is measured through process capability ratios. These ratios are evaluated and tested with respect to the process centering. The process capability ratios are tested with confidence intervals. Hypothesis about the process capability ratio is carried out to demonstrate whether process capability ratio, C_p meets or exceeds the target value

Process Capability Ratios (PCR) and Equations: The assessment of process capability analysis is done through computations of various process capability ratios. It is frequently convenient to have a quantitative way to express process capability. One way to do so is through the process capability ratio. This measures the ability of process to manufacture product that meets specification.

Process capability ratio (PCR) -C_p. $C_p = \frac{(USL - LSL)}{6\sigma}$

....(1)

Where, USL = Upper specification limit, LSL = Lower specification limit and σ = Process standard deviation. The PCR can be interpreted in another way that is Cr index, a process capability index that measures the percentage of tolerance actually used by process.

The quantity is the percentage of specification band that the process uses up. The above equation assumes that process has both upper and lower specification limits. For one-sided specification there is one sided process capability ratio with process mean μ as,

$$C_{pu} = (\underline{USL} - \underline{\mu})$$
 and $C_{pl} = (\underline{\mu} - \underline{LSL})$
3 σ 3 σ

A new process capability ratio that takes process centering into account is calculated with an equation $C_{pk} = Minimum \{ C_{pu}, C_{pl} \}$(2)

Generally, if $C_p = C_{pk}$ the process is centered at mid point of specifications and when $C_{pk} < C_p$ the process is off-center. C_p measures potential capability in the process and C_{pk} measures actual capability. The magnitude of C_{pk} relative to C_p is a direct measurement of how off-center the process is operating.

	Two sided specification	One sided Specification
Existing process	1.33	1.25
New process	1.50	1.45
Critical parameter of	1.50	1.45
existing process.		

Table 1: Recommended Minimum Values of the Process Capability Ratio

About Process Centering: The process capability ratio C_{pk} was initially developed because C_p does not adequately deal with case of a process with mean μ that is not centered between the specification limits. However, C_{pk} alone is still not adequate to measure process centering. For a process, if $C_{pk} = C_p$ implies that process is satisfactorily centered and if $C_{pk} > C_p$ implies that process is off-centered. As C_{pk} depends inversely on σ and becomes large as approaches to zero. This characteristic can make C_{pk} unsuitable, as a measure of centering and large value of C_{pk} does not really tell us anything about location of mean in the interval from USL and LSL. The way to address this difficulty is to use a PCR that is better indicator of centering.

One such ratio is $C_{pm} = (\underline{USL} - \underline{LSL}) \\ 6 \tau$

Where τ is square root of expected squared deviation from target T i.e., $\tau^2 = \sigma^2 + (\mu - T)^2$ and T= ½ (USL+LSL)

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)}^2}$$

$$C_{pm} = \frac{C_p}{\sqrt{1}} + \xi^2 \qquad \text{where,} \quad \xi = (\mu - T)/\sigma$$
.... (3)

Pear et al. (1992) proposed the process capability ratio as below.

$$C_{pm} = \frac{C_{pk}}{\sqrt{1 + ((\mu - T))^{2}} \sigma^{2} \sqrt{1 + \xi^{2}}}$$

This is also called "Third generation" PCR. This PCR has increased sensitivity to departures of process mean μ from designed target T.

Confidence Intervals and Tests on PCR: In industries PCR has focused on computing and interpreting the point of estimate of desired quality. The standard practice is to report confidence intervals for PCR's. If σ is replaced by S in equation for C_p , it produces the usual point estimator C_p . If quality characteristic follows a normal distribution then a 100(1- α)% confidence interval on C_p is obtained from following equation.

Where Chi – square , X^2 (1- $\alpha/2)_{n-1}$, X^2 ($\alpha/2)_{n-1}$ are lower $\alpha/2$ and upper $\alpha/2$, percentage points of X^2 distribution with (n-1) degree of freedom. For more complicated ratios such as C_{pm} and C_{pk} Zhang et al., Bissell(1990), Kushler and Hurlely and Pear.et.al. (1992) have developed approximate confidence intervals. If quality characteristics are normally distributed then, an approximate 100(1- α)% confidence interval on C_{pk} is given below. $C_{pk}[1-Z_{\alpha/2}\sqrt{1/(9nC_{pk}^{-2})} + 1/2(n-1)] \leq C_{pk} \leq C_{pk}[1+Z_{\alpha/2}\sqrt{1/(9nC_{pk}^{-2})} + 1/2(n-1)] \dots (5)$

Testing of Hypothesis: It is necessary to demonstrate that PCR-C_p meets or exceeds particular target value say C_{po.} This problem may be formulated as hypothesis testing problem. Let, H₀ : C_p = C_{po} (Process is not capable). H₁ : Cp > C_{po} (Process is capable). It is required to reject H_o and thereby to demonstrate that process is capable. We can formulate the statistical test in terms of C_p so, as to reject the H_o if C_p exceeds critical value C. Kane (1986) has investigated this test and provides a table of sample sizes and critical values C. Let C_p (High) as process capability that would like to accept with probability (1- α) and C_p (Low) as a process capability that would like to reject with probability (1- β). With the help of the table and the values of C_p (High)/C_p (Low) and C/C_p (Low) ratios hypothesis can be tested.

Sample Size	C _p (High)/ C _p (Low)	C/ C _p (Low)	C _p (High)/Cp(Low)	C/ C _p (Low)
(n)				
10	1.88	1.27	2.26	1.37
20	1.53	1.27	1.73	1.26
30	1.41	1.20	1.55	1.21
40	1.34	1.16	1.46	1.18
50	1.30	1.14	1.40	1.16
60	1.27	1.13	1.36	1.15
70	1.25	1.11	1.33	1.14
80	1.23	1.10	1.30	1.13
90	1.21	1.10	1.28	1.12
100	1.20	1.09	1.26	1.11

Table 2. Sample size and critical value determinations for testing $\alpha = \beta = 0.10$ $\alpha = \beta = 0.05$

Table 3 :Values of $PCR - C_p$ and associated process falls out for a normal distribution process (In defectives ppm) that is statistical control.

	Process falls out (defectives ppm)			
PCR	One side specification	Two side specification		
1.2	159	318		
1.3	48	96		
1.4	14	27		
1.51	4	7		
1.6	1	2		
1.7	0.17	0.34		
1.8	0.03	0.06		
2.0	0.0009	0.0018		

Taguchi's loss function approach: The loss function as reported by Taguchi is based on concept that loss denoted by L is incurred when a product's functional quality characteristic denoted by 'y' deviation from its target or nominal value denoted by 'm', irrespective of quantum of deviation. When there is no deviation, the loss is said to be zero. Taguchi loss function is a way to show how each non-perfect part produced, results in a loss for the company. According to Deming, loss function approach shows that " A minimum loss at nominal value and an ever increasing loss with departure either way from nominal value". The technical definition is " A parabolic representation that estimates the quality loss, expressed monetarily, that results when quality characteristics deviate from target values. The

cost of this deviation increases quadratically as the characteristics moves farther from target value.

Mathematically, the loss function is represented by L = C (y – m)_2 , where C is proportionality constant. Accordingly Ross (1986) suggests the loss function in following form L = C [S² + (X – T)] where, C = Cost constant, S = Process variance, X = Process average and T= Target value. Product quality is independent on both variability and the deviation of quality characteristic from the target value. Since, $C_p = (USL-LSL) / 6\sigma$, where σ is the process deviation, the variance can be written as $S^2 = (USL - LSL) / 2 / (36 C_p ^2)$ using this, loss function is expressed as L=C{[(USL-LSL)²/4][1/(3 C_p)² + K²)]} Where C = Cost constant, USL and LSL are upper and lower specification limits, C_p = process capability ratio and K is scaled distance factor which represents shift in target and is given by K = 2 (X – T) / (USL – LSL). Thus using above equation it is possible to estimate the loss due to particular quality characteristics for different values of C_p, K and tolerance. However, cost constant value is not available in all situations; it is possible to calculate the loss in terms of L/C. By using suitable cost constant the economic interpretation can be done.

Table 4. Case studyACE H -63 CNC Machining center.Part Name :Drive gear. Operation: Boring Specifications: 210.745 mm to 210.795 mm

No	1	2	3	4	5	X	R
1	210.795	210.790	210.790	210.795	210.795	210.793	0.005
2	210.785	210.785	210.775	210.775	210.775	210.779	0.010
3	210.770	210.775	210.770	210.775	210.770	210.772	0.005
4	210.770	210.765	210.770	210.775	210.770	210.770	0.01
5	210.765	210.765	210.765	210.760	210.760	210.763	0.005
6	210.760	210.755	210.785	210.790	210.785	210.775	0.035
7	210.780	210.775	210.775	210.770	210.765	210.773	0.015
8	210.765	210.765	210.765	210.775	210.770	210.768	0.01
9	210.775	210.770	210.770	210.770	210.770	210.771	0.005
10	210.765	210.775	210.775	210.775	210.770	210.772	0.01
11	210.770	210.775	210.770	210.765	210.760	210.768	0.015
12	210.785	210.785	210.780	210.780	210.780	210.782	0.005
13	210.775	210.775	210.775	210.780	210.780	210.777	0.005
14	210.775	210.770	210.765	210.765	210.755	210.766	0.02
15	210.795	210.795	210.785	210.780	210.775	210.786	0.02
16	210.775	210.765	210.760	210.760	210.755	210.763	0.02
17	210.770	210.775	210.770	210.775	210.770	210.772	0.005
18	210.770	210.765	210.765	210.765	210.760	210.765	0.01
19	210.790	210.790	210.790	210.790	210.780	210.788	0.01
20	210.770	210.765	210.765	210.765	210.765	210.766	0.005

Computations and analysis by analytical methos

Process Capability Ratio (*PCR*) $C_{p \ : \ m}$ easured quality characteristics of parts are tabulated. The means of sample means (X = μ) and average range (R) value is computed. With the help of statistical table the value of d₂ for a sample size of five is noted and the process standard deviation is estimated by $\sigma = R/d_2$.

 $\sigma = 0.01125 / 2.326 = 0.00483.$

 $C_p = (USL - LSL) / 6 \sigma = (210.795 - 210.745) / (6 \times 0.00483) = 1.725.$

This value of C_p implies that natural tolerance limits in the process are well inside the specification limits. From the table-3, for PCR = 1.725, it is noticed that the probability of 0.34 ppm defectives are there in manufacturing process.

Percentage of Specification Band Used By the Process: $C_r = (1/C_p) \times 100 = (1/1.725) \times 100 = 57.96\%$

This means that the manufacture process has a wide tolerance band width while it uses 57.96% of specification band.

 $\begin{array}{l} PCR-C_{pk} : \text{The grand mean of the case analysis is } 210.773.\\ C_{pk} = \text{Min} \left\{ \begin{array}{l} C_{pu} = (\underline{210.795} - \underline{210.773}) \\ 3 \ x \ 0.00483 \end{array} \right\}, \\ C_{pl} = (\underline{210.773} - \underline{210.745}) \\ 3 \ x \ 0.00483 \end{array} \right\} \\ C_{pk} = \text{Minimum} \left\{ \begin{array}{l} C_{pu} = 1.518 \\ C_{pl} = 1.932 \end{array} \right\}, \\ C_{pk} = 1.518. \end{array} \\ \text{It should be noted that } C_{p} \text{ measures potential capacity and } C_{pk} \text{ Measures actual capability} \end{array} \right\}$

Estimation of Process Centering

a) Estimation of C_{pm} : $C_{pm} = \frac{C_p}{\sqrt{1+\xi^2}} = \frac{1.725}{\sqrt{1+(0.6211)^2}} = 1.465$ Where, $\xi = (\mu - T)/\sigma$ = (210.773 - 210.77) / 0.00483 = 0.6211 and $T = \frac{1}{2} (\text{USL+LSL}) = \frac{1}{2} (210.795 + 210.745) = 210.770$

b) Estimation of
$$C_{pkm}$$
:
 $C_{pkm} = \frac{C_{pk}}{\sqrt{1+\xi^2}} = \frac{1.518}{\sqrt{1+(0.6211)^2}} = 1.289$

The case analysis reveals that $C_p = 1.725$, $C_{pk} = 1.518$ and $C_{pkm} = 1.289$.Since, C_p , C_{pk} and C_{pm} are not nearer in their magnitude the process under study is not exactly centered.

Confidence Intervals & Tests on PCR

a) Confidence intervals & tests on PCR - C_p $C_p \underbrace{X^2_{(1-\alpha/2), n-1}}_{n-1} \leq C_p \leq \underbrace{C_p X^2_{\alpha/2, n-1}}_{n-1}$ $1.725 \underbrace{74.22}_{99} \leq C_p \leq 1.725 \underbrace{129.56}_{99}$ $1.493 \leq C_p \leq 1.973$

b) Estimation of confidence interval for C_{pk} $C_{pk} [1-Z_{\alpha/2} \sqrt[]{\sqrt{1/(9 n C_{pk}^{-2}) + 1/2}(n-1)}] \le C_{pk} \le C_{pk} [1+Z_{\alpha/2} \sqrt{1/(9 n C_{pk}^{-2}) + 1/2}(n-1)]$ $1.518 [1-1.96 \sqrt{1/(9 x 100 x (1.518)^2) + 1/(2 x 9)}] \le C_{pk} \le C_{$ $1.2966 \leq C_{pk} \leq 1.739$

Based on the confidence intervals and tests on PCR- C_p , reveals that C_p value is $1.493 \leq C_p \leq 1.973$ and estimation of confidence interval for C_{pk} is $1.2966 \leq C_{pk} \leq 1.739$. However, for these values test of hypothesis can be carried out.

Testing and Hypothesis:

Let us $H_o = C_p = 1.725$, $H_1 = C_p < 1.725$ with $\alpha = \beta = 0.10$

 C_p (High) / C_p (Low) = 1.973/1.493= 1.32. From table-2, the ratio 1.32 corresponds to n=50.C = 1.13 x C_p (low) = 1.13 x 1.493 = 1.687. This means that, to demonstrate capability the supplier or process must take a sample of n= 50 parts and the sample PCR- C_p must exceed 1.687.

The case analysis reveals that PCR- C_p is 1.725, which is greater than 1.687 and hence the process under study is capable.

Estimation of Scaled distant factor K. $K = \frac{2 \mid \mu - T}{(USL - LSL)} = \frac{2(210.773 - 210.770)}{(0.050)} = 0.12$

Estimation of Loss Function:

L=C{[(USL -LSL)²/4][(1/3 C_p)²+K²)]}={ [(0.05)²/4][1/(3 x 1.725)²+0.12²]} L/C = 0.00003233

Graphical approach of the process capability studies: The traditional method of process capability computation involves determining the mean values for individual samples containing number of sub-samples and thereby finding the grand mean (means of means), which is the representative of all the sample observations. Though mean is based on the value of every observation in the series, it is unduly affected by the extreme values of observations. It is known that median is also a representative value and is the middle observation of the series arranged in ascending or descending order of magnitude and if number of observations is even, median is average of two observations in the centre of the distribution. Using the property of median, the traditional mean can be replaced by median and grand mean can be replaced by grand median. This makes elimination of the effect of extreme observations. Median is a positional average; it can be computed even if the observations at the extremes are unknown. Median can be also be obtained by the graphical method. This graphical approach of median is used for the computations of process capability. Graphical approach of the process capability study is representation of the data and parameters in the form of graphs and gives quick information with fewer computations. This approach helps to an extent a worker in the shop floor can make the assessment about the process parameters.

Proposed Graphical Model: Traditional method of process capability computation uses grand mean and average range values, which involves computation of individual mean values for each samples and hence computation of grand mean. This method is time consuming, tedious and not user friendly at shop floor levels. The proposed model will make use of a graphical paper having the specifying quality characteristics at vertical axis and sample numbers in horizontal axis. The operator after completing the operation makes the measurement of the quality characteristic and records the same in the form of dots in the appropriate specified box. The mid value of the observation from the top or bottom will be marked with cross sign; say third observation in case of a sample contains five sub-samples. These cross marked values are nothing but medians as the observations are arranged in ascending or descending

order. Now, by joining cross marked observations the median graph is generated. Allot the numbers from the bottom level to top or vice versa starting from number one to the last observation say,1 to 20 incase of twenty samples. Now, consider tenth and eleventh observations and bisect the same to get grand median value. The number of boxes between first and last observations excluding the bottom most if numbers of boxes are considered from top level in each sample will be entered at the bottom portion of the graph as a range. The total number of boxes divided by number of samples and multiplied by scale will give the average range. This average range value can be used to compute the process capability potential and grand median value can be used to compute the capability indices like C_{pk} , C_{pm} and C_{pkm} .

Illustration: The graphical approach of process capability is illustrated through the case study depicted in table 4. The readings are located in the appropriate boxes and cross marks are made for the middle values and joined them to get median chart. Numbers are allocated from one to twenty from the bottom to top level. Tenth and eleventh median values are averaged to get grand median. The number of boxes between top most and bottom most observations are noted in range row for each samples. The average range is computed by total number of boxes divided by number of samples and multiplied by scale. The capability indices are computed with usual procedure.



KATHMANDU UNIVERSITY JOURNAL OF SCIENCE, ENGINEERING AND TECHNOLOGY VOL.I, No.III, JANUARY, 2007.

Computations and analysis of graphical approach.

- 5.1 Grand median from the graphical approach = 210.772.
- 5.2 Average Range = 0.01125
- 5.3 Process Standard Deviation = Average Range / $d_2 = 0.01125/2.326 = 0.00483$
- 5.4 Process Capability Ratio Cp = Tolerance / $6 \sigma = 0.050/(6x0.00483) = 1.725$.

5.5 PCR – $C_{pk} = Min\{(210.795-210.772), (210.772-210.745)\}$

= Min { 1.587, 1.863 }.

Estimation of C_{pm} : $C_{pm} = \frac{C_p}{\sqrt{1+\xi}^2} = \frac{1.725}{\sqrt{1+(0.414)}^2} = 1.593$ Where, $\xi = (\mu - T)/\sigma = (210.772 - 210.770) / 0.00483 = 0.414$ $T = \frac{1}{2} (\text{USL+LSL}) = \frac{1}{2} (210.795 + 210.745) = 210.770$

Estimation of C_{pkm} :

$$C_{pkm} = \frac{C_{pk}}{\sqrt{1+\xi^2}} = \frac{1.587}{\sqrt{1+(0.414)^2}} = 1.466$$

Estimation of Loss Function.

$$\begin{split} & K = \frac{2|\ \mu - \mathbb{T}}{(USL - LSL)} = 2 \ x \ \underline{(210.772 - 210.770)} = 0.16. \\ & L = C\{[(\ USL - LSL \)^2 / 4][(\ 1 / \ 3 \ C_p)^2 + K^2)]\}, \ L / C = \{\ [(0.05)^2 / \ 4] \ [\ 1 / \ (3 \ x \ 1.725)^2 + 0.16^2]\} \\ & L / C = 0.00003933. \end{split}$$

COMPARISON OF RESULTS

Parameter	Analytical Method	Graphical Approach
Process Mean	210.773	210.772
Process Capability –PCR-C _p	1.725	1.725
Process capability ratio - C_{pk}	1.518	1.587
Process capability ratio – C _{pm}	1.465	1.593
Process capability ratio – C _{pkm}	1.289	1.466
Scaled Distant Factor -K	0.12	0.16
Loss function value	0.00003233	0.00003933

Advantages of proposed graphical approach:

- The process capability results do not affected by extreme values.
- The median chart indicates on-line variability.
- If, the control limits are introduced to median chart it performs like average and range chart.
- Involves fewer computations and consumes less time.
- The approach helps to a worker in the shop floor to assess the process.
- Graphical approach is a user-friendly analysis.
- When process is having a reasonably higher tolerance the variation values between analytical and graphical method will have very less effect.

Limitations of the graphical approach

- It helps to an extent of finding average range and grand median but further computations follows usual procedure.
- The approach assumes process is in statistical control.
- The approach is validated with limited case studies and found acceptable.

CONCLUSION

The results of process capability study of the given manufacturing process reveals that, graphical values of parameters approaches very nearer to the magnitude of the analytical values and hence graphical approach could be treated as equivalent to analytical method. Graphical approach can be used to study the variability of manufacturing process. It is one of the tools to convey the results through which it is easy to make inference on the data. The approach helps a worker in the shop floor can make the assessment about the process parameters. Thus it also helps to process management and identifies opportunities for improving quality and operational performance.

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