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SOME FIXED POINT RESULTS IN FUZZY METRIC SPACE

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ABSTRACT
The purpose of our paper is to study the fixed point theorems in different types of fuzzy metric space using different types of contractive conditions.

Key words: Fixed point, fuzzy metric space and compatible mappings.

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INTRODUCTION

In 1965, the concept of fuzzy set was introduced by L.A Zade [46]. Then, fuzzy metric spaces have been introduced by O. Kramosil and J. Michalek [21]. A. George and P. Veeramani [8] modified the notion of fuzzy metric spaces with the help of continuous t-norms and also many others have introduced and generalized in different way. Recently, many authors have studied the fixed point theory in the fuzzy metric space and numbers of fixed point theorems have been obtaining in fuzzy metric space by using the contractive condition of self-mappings.

In 1986, G. Jungck [18] introduced the notion of compatible mappings, which are more general than commuting and weakly commuting mappings and also many others have been introduced different types of compatible mappings in metric and fuzzy metric space.

In 2007, Singh and Singh [43] introduced the concept compatible mappings of type (E) in metric space and Manandhar et.al [24] extended into fuzzy metric space. Jha et al. [17] introduced the new notion of compatible mappings of type (K) in metric space and Manandhar et al. [25] extended into fuzzy metric space and established some common fixed point theorems for the pairs of compatible mappings of type (K) with examples.

DEFINITION 1. [46] Let X be any set. A ***fuzzy set*** A of X is a function from domain X and values in [0, 1].

Example 1. Consider U = {a, b, c, d} and A: U $\rightarrow$[0; 1] define as A(a) = 0; A(b) = 0.5; A(c) = 0.2 and A(d) = 1. Then A is a fuzzy set on U. This fuzzy set also can be written as follows A = {(a, 0), (b, 0.5), (c, 0.2), (d, 1)}

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DEFINITION 2. [8] A binary operation $\ast$: [0, 1] × [0, 1] → [0, 1] is said to be a ***continuous t-norm*** if $\ast$ satisfies the following conditions:

(a) $\ast$ is commutative and associative;
(b) $\ast$ is continuous;
(c) $a \ast 1 = a$ for all $a \in [0, 1]$; and
(d) $a \ast b \leq c \ast d$ whenever $a \leq c$ and $b \leq d$, and a, b, c, d ∈ [0, 1].

Example 2. A binary operation *: [0, 1] × [0, 1] → [0, 1] such that a * b = ab for a, b ∈ [0; 1] is a continuous t-norm.

DEFINITION 3. [39] A binary operation ♢: [0, 1] × [0, 1] → [0, 1] is said to be a ***continuous t-conorm*** if $\ast$ satisfies the following conditions:

(a) ♢ is commutative and associative;
(b) ♢ is continuous;
(c) a♢ 0 = a for all a [0, 1];
(d) a ♢ b ≤ c ♢ d whenever $a \leq c$ and $b \leq d$, for each a, b, c, d ∈ [0; 1].

Example 3. A binary operation ♢: [0, 1] × [0, 1] → [0, 1] such that a ♢ b = min(a+ b, 1) is a t -conorms.

DEFINITION 4. [8] A 3-tuple (X,M, $\ast$) is said to be a ***fuzzy metric space*** if X is an arbitrary set, $\ast$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions: for all x, y, z ∈ X and s, t > 0,

(FM1) M(x, y, t) > 0;
(FM 2) M(x, y, t) = 1 if and only if x = y;
(FM 3) M(x, y, t) = M(y, x, t);
(FM 4) $M(x, y, t) \ast M(y, z, s) \leq M(x, z, t + s)$; and
(FM 5) M(x, y, .) : (0,∞) → (0, 1] is continuous.

Then M is called a fuzzy metric on X. The function M(x, y, t) denote the degree of nearness between x and y with respect to t. Also, we consider the following condition in the fuzzy metric spaces (X, M, $\ast$).

(FM6) $\lim_{t\to\infty} M(x, y, t) = 1$, for all $x, y \in X$.

Example 2.1[1] Let (X, d) be a metric space. Denote $a \ast b = ab$ for all a, b ∈ [0, 1] and let M be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows: $M(x, y, t) = \frac{t}{t+d(x,y)}$, for all x, y in X. Then (X, M, $\ast$) is a fuzzy metric space. This fuzzy metric is the standard fuzzy metric space induced by a metric d.

DEFINITION 5. [35] Let (X, M, *) be a fuzzy metric space, and f is a self-mapping of X. Then, ξ is said to be a ***periodic point*** or an eventually fixed point, if there exists a positive integer k such that $f^k(\xi) = \xi$.

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DEFINITION 6. [35] Let (X, M, $*$) be a fuzzy metric space, the mapping f : X →X is said to be a ***fuzzy ε-contractive*** if there exists $0 < \varepsilon < 1$, such that if $1-\varepsilon < M(x, y, t) < 1$, then $M(f(x), f(y), t) > M(x, y, t)$ for all $t > 0$, and $x, y \in X$.

DEFINITION 7. [8] A sequence (x_n) in a fuzzy metric space (X, M, $*$) is a ***Cauchy sequence*** if and only if for each $\varepsilon \in (0, 1)$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1-\varepsilon$ for all $n, m \geq n_0$.

A fuzzy metric space in which every Cauchy sequence is convergent is called a ***complete fuzzy metric space.***

DEFINITION 8. [45] Two mappings A and S of a fuzzy metric space (X, M, *) into itself are ***weakly commuting*** provided there exists some real number R. such that

$$M(ASx, SAx, t) \geq M(Ax; Sx; t) \text{ for each } x \in X \text{ and } t > 0.$$

DEFINITION 9. [45] Two mappings A and S of a fuzzy metric space (X,M, *) into itself are **R-weakly commuting** provided there exists some real number R. such that

$$M(ASx, SAx, t) \geq M(Ax, Sx, t/R) \text{ for each } x \in X \text{ and } t > 0.$$

DEFINITION 10. [2] Let f and g be two self-mappings of a metric space (X, d). Then, f and g said to satisfy the ***property (E. A.)*** if there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t \text{ for some } t \in X.$$

DEFINITION 11. [6] Let (X, M, *) be a fuzzy metric space and $\varepsilon > 0$. A finite sequence $x = x_0, x_1, \ldots, x_n = y$ is called **ε - chainable** from x to y if $M(x_i, x_{i-1}, t) > 1 - \varepsilon$ for all $t > 0$ and i = 1, 2, 3, . . ., n.

DEFINITION 12. [6] A fuzzy metric space (X, M, *) is called ***ε - chainable fuzzy metric space*** if for $x, y \in X$ there exists an ε -chain from x to y.

DEFINITION 13. [27] Two maps A and S of metric space (M, d) are called ***R-weakly commuting*** at a point x if $d(ASx, SAx) \leq R\, d(Ax, Sx)$ for some $R > 0$. Also, A and S are called ***pointwise R-weakly commuting*** on X if given x in X, there exists R > 0 such that

$$d(ASx, SAx) \leq R\, d(Ax, Sx).$$

DEFINITION 14. [28] Two self-mappings A and S of a metric space (X, d) are said to be ***reciprocally continuous*** if $\lim_{n\to\infty} ASx_n = A(t)$ and $\lim_{n\to\infty} SAx_n = S(t)$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = t$ and $\lim_{n\to\infty} Sx_n = t$, for some $t \in X$.

If f and g are both continuous then they are obviously reciprocally continuous but the converse need not be true.

DEFINITION 15. A function $\Psi : [0;1) \rightarrow [0;1)$ is an ***altering distance function*** if $\Psi(t)$ is monotone non-decreasing, continuous and $\Psi(t) = 0$ if and only if $t = 0$.

DEFINITION 16. [21] A sequence $\{x_n\}$ in a fuzzy metric space (X, M, *) is called ***G-Cauchy sequence*** if, $\lim_{n\rightarrow 1}(x_{n+p},\ x_n,\ t) = 1$

for every $t > 0$ and for each $p > 0$. A fuzzy metric space (X, M, *) is complete (respectively G-complete) if, every Cauchy sequence (respectively G-sequence) in X converges in X.

DEFINITION 17. Let (X, M, *) be a fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be ***convergent*** to x in X if for each $\varepsilon > 0$ and each $t > 0$, there exist $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ for all $n \geq n_0$.

DEFINITION 18. [7] Let A and S be self-maps on a fuzzy metric space (X, M, $*$). Then, f and g are said to be ***compatible*** or ***asymptotically commuting*** if, for all $t > 0$,

$\lim_{n\rightarrow\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that,

$\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Sx_n = z$, for some $z \in X$.

DEFINITION 19. [20] The mappings *f* and *g* from a fuzzy metric space (X, M, $*$) into itself are said to be ***weakly compatible*** if they commute at their coincidence point, that is

$Ax = Sx$ implies that $ASx = SAx$.

DEFINITION 20. [13] Let A and S be mappings from a fuzzy metric space (X, M, *) into itself. Then, the mappings A and S are said to be ***semi-compatible*** if, for all $t > 0$,

$\lim_{n\rightarrow\infty} M(ASx_n, Sx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that,

$\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Sx_n = z$, for some $z \in X$.

DEFINITION 21. [33] A fuzzy metric space (X, M, *) is said to be ***sequentially compact*** if every sequence in X has a convergent sub-sequence in it.

DEFINITION 22. [19] The self-mappings A and S of a fuzzy metric space (X, M, *)

are said to be ***compatible of type (A)*** if $\lim_{n\rightarrow\infty} M(ASx_n, SSx_n, t) = 1$ and

$\lim_{n\rightarrow\infty} M(SAx_n, AAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Sx_n = z$ for some z in X and $t > 0$.

DEFINITION 23. [31] The self-mappings A and S of a fuzzy metric space (X, M, *) are said to be ***compatible of type (P)*** if $\lim_{n\rightarrow\infty} M(SSx_n, AAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\rightarrow\infty} Ax_n = \lim_{n\rightarrow\infty} Sx_n = z$ for some z in X and $t > 0$.

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DEFINITION 24. [24] The self-mappings A and S of a fuzzy metric space (X, M, ∗) are said to be ***compatible of type (E)*** iff

$$\lim_{n\to\infty} M(AAx_n, ASx_n, t) = \lim_{n\to\infty} M(AAx_n, Sx, t) = \lim_{n\to\infty} M(ASx_n, Sx, t) = 1 \text{ and}$$

$$\lim_{n\to\infty} M(SSx_n, SAx_n, t) = \lim_{n\to\infty} M(SSx_n, Ax, t) = \lim_{n\to\infty} M(SSx_n, Ax, t)=1.$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X and t > 0.

DEFINITION 25. [26] The self-mappings A and S of a fuzzy metric space (X, M, ∗) are said to be ***compatible of type (K)*** iff

$$\lim_{n\to\infty} M(AAx_n, Sx, t) = 1 \text{ and } \lim_{n\to\infty} M(SSx_n, Ax, t) = 1,$$

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X and t > 0.

The following examples show that the compatible of type (K) in fuzzy metric space is independent with compatible, compatible of type (A), compatible of type (P) and reciprocal continuous.

Example 5. Let X = [0, 2] with the usual metric $d(x, y) = |x - y|$, define $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$, t > 0 and a ∗ b = ab for all a, b ∈ [0, 1] then (X, M, ∗) is a fuzzy metric space. We define self-mappings A and S as

$Ax = 2$, $Sx = 0$ for $x \in [0, 1] - \{1/2\}$, $Ax = 0$, $Sx = 2$ for $x = 1/2$ and $Ax = (2 - x)/2$, $Sx = x/2$ for $x \in (1, 2]$. Then, A and S are not continuous at $x = 1, 1/2$. Consider a sequence $\{x_n\}$ in X such that $x_n = 1 + 1/n$ for all n∈ N.

Then, we have $Ax_n = (2 - x_n)/2 \to 1/2 = x$ and $Sx_n = x_n/2 \to 1/2 = x$. Also, we have

$AAx_n = A((2 - x_n)/2) = 2 \to 2$, $ASx_n = A(x_n/2) = 2 \to 2$, $S(x) = 2$ and $SSx_n = S(x_n/2) = 0 \to 0$, $SAx_n = S((2 - x_n)/2) = 0 \to 0$, $A(x) = 0$.

Therefore, {A, S} is compatible of type (K) but the pair {A, S} is neither compatible nor compatible of type (A) (compatible of type (P), reciprocal continuous).

Example 6. Let X = [0, 2] with the usual metric $d(x, y) = |x - y|$, define $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$, t > 0 and a ∗ b = ab for all a, b ∈ [0, 1] then (X, M, ∗) is a fuzzy metric space. Define self-mappings A and S as $Ax = Sx = 1$ for $x \in [0, 1)$, $Ax = Sx = 4/3$ for $x = 1$ and $Ax = 2 - x$, $Sx = x$ for $x \in (1, 2]$.

Consider a sequence $\{x_n\}$ in X such that $x_n = 1 + 1/n$ for all n∈ N. Then,we have

$Ax_n = (2 - x_n) \to 1 = x$, and $Sx_n = x_n \to 1 = x$. Since, $2 - x_n < 1$ for all n ∈ N,

we have $AAx_n = A(2 - x_n) = 1 \to 1$, $ASx_n = A(x_n) = 2 - x_n \to 1$ and $SSx_n = S(x_n) = x_n \to 1$, $SAx_n = S(2 - x_n) = 1 \to 1$. Also, we have $A(x) = 4/3 = S(t)$ but $AS(x) = AS(1) = A(4/3) = 2/3$, $SA(x) = SA(1) = S(4/3) = 4/3$. However, we have $2/3 = AS(x) \neq SA(x) = 4/3$, at $x = 1$. Therefore, {A, S} is not compatible of type (K) but it is compatible, compatible of type (A) and compatible of type (P).

DEFINITION 26. [41] The 3-tuple (X, M, ∗) is called a ***fuzzy-2 metric*** *space* (shortly FM-space) if X is an arbitrary set, ∗ is a continuous t-norm and M is a fuzzy set in $X^3 \times [0, \infty]$ satisfying the following conditions: for all $x, y, z \in X$ and s, t > 0,

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FM'1. $M(x, y, z, 0) = 0$,
FM'2. $M(x, y, z, t) = 0$ for all $t > 0$ and when at least two of the three points are equal,
FM'3. $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$ and
FM'4. $M(x, y, u, t_1) * M(x, u, z, t_2) * M(u, y, z, t_3) \leq M(x, y, z, t_1 + t_2 + t_3)$,
FM'5. $M(x, y, z, .) : [0,1) \rightarrow [0,1]$ is left continuous.

DEFINITION 27. [41] The 3-tuple $(X, M, *)$ is called a ***fuzzy-3 metric space*** (shortly FM-space) if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set in $X^4 \times [0,\infty]$ satisfying the following conditions: for all $x, y, z \in X$ and $s, t > 0$

FM"1. $M(x, y, z, w, 0) = 0$,
FM"2. $M(x, y, z, w, t) = 1$ for all $t > 0$ (only when simplex <x, y, z, w > degenerate)
FM"3. $M(x, y, z, w, t) = M(x, y, w, z, t) = M(x, w, z, y, t) = M(w, y, z, x, t) = \ldots$
FM"4. $M(x, y, z, u, t_1) * M(x, y, u, w, t_2) * M(x, u, z, w, t_3) * M(u, y, z, w, t_4)$
$\leq M(x, y, z, w, t_1 + t_2 + t_3 + t_4)$, and
FM"5. $M(x, y, z, w, .) : [0, 1) \rightarrow [0, 1]$ is left continuous.

DEFINITION 28. [30] A 5-tuple $(X, M, N, *, \diamond)$ is said to be an ***intuitionistic fuzzy metric space*** if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions;

IFM1. $M(x, y, t) + N(x, y, t) \leq 1$,
IFM2. $M(x, y, 0) = 0$,
IFM3. $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
IFM4. $M(x, y, t) = M(y, x, t)$,
IFM5. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$, for all $x, y, z \in X$, and $s, t > 0$,
IFM6. $M(x, y, \cdot) : [0,\infty) \rightarrow [0, 1]$ is left continuous,
IFM7. $lim_{t\rightarrow\infty} M(x, y, t) = 1$ for all $x, y \in X$,
IFM8. $N(x, y, 0) = 1$,
IFM9. $N(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$,
IFM10. $N(x, y, t) = N(y, x, t)$,
IFM11. $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$, and $s, t > 0$,
IFM12. $N(x, y, \cdot) : [0,\infty) \rightarrow [0, 1]$ is left continuous, and
IFM13. $lim_{t\rightarrow\infty} N(x, y, t) = 0$ for all $x, y \in X$.

1.1 FIXED POINT THEOREMS IN FUZZY METRIC SPACE

In 1983, M. Grabiec expanded well known fixed point theorems of Banach and Edelstein contraction principle in in fuzzy metric space in the sense of Kramosil and Michelek.

THEOREM 1. [9] (***Fuzzy Banach contraction Theorem***)

Let (X;M; _) be a complete fuzzy metric space such that

1. $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$,

2. $M(Tx, Ty, kt) \geq M(x, y, t)$

where $0 < k < 1$. Then T has unique fixed point.

Lemma 1. [9] If $\lim_{t\to\infty} x_n = x$ and $\lim_{t\to\infty} y_n = y$, then

$$M(x; y; t - \varepsilon) \leq \lim_{t\to\infty} M(x_n, y_n, t) \leq M(x; y; t + \varepsilon) \text{ for all } t > 0 \text{ and } 0 < \varepsilon < t/2.$$

THEOREM 2. [9] (***fuzzy Edelstein contraction theorem)*** Let (X, M *) be a compact fuzzy metric space with (x, y, .) continuous for all $x, y \in X$. Let $T : X \to X$ be a mapping satisfying

$$M(Tx, Ty, t) > M(x, y, t) \text{ for all } x \neq y \text{ and } t > 0.$$

Then T has unique fixed point.

In 1983, J. Kramosil and J. Michalek proved the contraction principle in the setting of fuzzy metric spaces. In 1993, P.V. Subramanyam generalized Grabiec's results for pair mappings.

THEOREM 3. [44] Let (X, M, *) be a complete fuzzy metric space and let $f, g:X\to X$ be maps that satisfy the following conditions:

(i) $g(X) \subseteq f(X)$,

(ii) f is continuous, and

(iii) $M(g(x), g(y), \alpha t) \geq M(f(x), f(y), t)$ for all x, y in x and $0 < \alpha < 1$.

Then, f and g have unique common fixed point provided f and g commute.

In 2009, V. Pant obtained the following fixed point theorems for pointwise R-weakly commuting self-mappings in fuzzy metric space.

THEOREM 4. [29] Let f and g be pointwise R-weakly commuting selfmaps of type (A_g) of a fuzzy metric space (X, M, ∗) such that

(i) $fX \subset gX$, and
(ii) $M(fx, fy, t) > \min\{M(gx, gy, t\,h), M(fx, gx, t\,h), M(fy, gy, t\,h), M(fy, gx, t\,h), M(fx, gy, t\,h)\}$, $0 \leq h < 1$, for $t > 0$.

If f and g satisfy the property (E.A.) and the range of either of f or g is a complete subspace of X, then f and g have a unique common fixed point.

THEOREM 5. [29] Let f and g be non-compatible pointwise R-weakly commuting selfmaps of type (A_g) of a fuzzy metric space (X, M, ∗) such that

(i) $fX \subset gX$, and

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(ii) $M(fx,fy,t) > \min\{M(gx, gy, th), M(fx, gx, th), M(fy, gy, th), M(fy, gx, th), M(fx, gy, th)\}$, $0 \le h<1$, for $t > 0$

If the range of f or g is a complete subspace of X, then f and g have a unique common fixed point and the fixed point is the point of discontinuity.

1.2 COMMON FIXED POINT THEOREMS FOR THREE MAPPINGS

In 2002, S. Sharma obtained the following common fixed point results for three self-mappings in fuzzy metric space.

THEOREM 6. [41] Let (X, M, *) be a complete fuzzy metric space with the condition (FM-6) and let S and T be continuous mappings of X, then S and T have a common point in X if there exists continuous mapping A of X into $S(X) \cap T(X)$ which commute with S and T and

$$M(Ax, Ay, qt) \ge \min\{M(Ty, Ay, t), M(Sx, Ax, t), M(Sx, Ty, t)\},$$

for all $x, y \in X$, $t > 0$ and $0 < q < 1$.

Then, S, T and A have a unique common fixed point.

THEOREM 7. [41] Let (X, M, *) be a complete 2- fuzzy metric space and let S and T be continuous mappings in X, then S and T have a common point in X if there exists continuous mapping A of X into $S(X) \cap T(X)$ which commute with S and T and

$$M(Ax, Ay, a, qt) \ge \min\{M(Ty, Ay, a, t), M(Sx, Ax, a, t), M(Sx, Ty, a, t)\},$$

for all $x, y, a \in X$, $t > 0$ and $0 < q < 1$, $\lim_{t\to\infty} M(x, y, z) = 1$ for all x, y, z in X.

Then S, T and A have a unique common fixed point.

THEOREM 8. [41] Let (X, M, *) be a complete 3- fuzzy metric space and let S and T be continuous mappings in X, then S and T have a common point in X if there exists continuous mapping A of X into $S(X) \cap T(X)$ which commute with S and T and

$$M(Ax, Ay, a, b, qt) \ge \min\{M(Ty, Ay, a, b, t), M(Sx, Ax, a, b, t), M(Sx, Ty, a, b, t)\},$$

for all $x, y, a, b \in X$, $t > 0$ and $0 < q < 1$, $\lim_{t\to\infty} M(x, y, z, w, t) = 1$ for all x, y, z, w in X

Then S, T and A have a unique common fixed point.

In 2006, J. H. Jung obtained the following common fixed point theorems for weakly pair of compatible self-mappings in fuzzy metric space.

THEOREM 9. [5] Let (X, M, ∗) be a complete ε-chainable fuzzy metric space and let S be continuous self-mapping of X and T be self-mapping of X. Then, S and T have a common fixed point in X if and only if there exist a continuous self-mapping A of X such that the following conditions are satisfied:

(i) $AX \subset TX \cap SX$,

(ii) the pairs (A, S) and (A, T) are weakly compatible,

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(iii) there exists q ∈ (0, 1) such that for every x, y ∈ X and t > 0,

$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$.

In fact, A, S and T have a unique common fixed point in X.

1.3 COMMON FIXED POINT THEOREMS FOR FOUR MAPPINGS

In 2005, B. Singh and S. Jain obtained the following fixed point theorems in fuzzy metric space using implicit relation under semi-compatibility.

THEOREM 10. [42] Let A, B, S, and T be self-mappings of a complete fuzzy metric space

(X, M, ∗) satisfying that

(i) A(X) ⊆ T(X), B(X) ⊆ S(X);

(ii)The pair (A, S) is semi-compatible and (B, T) is weak compatible, one of A or S is continuous; and

(iii) For some $\phi \in \Phi$, there exists k ∈ (0,1) such that for all x, y ∈ X and t > 0,
$\phi(M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, kt)) \geq 0$,
$\phi(M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, kt), M(By, Ty, t)) \geq 0$.

Then, A, B, S and T have unique common fixed point in X.

In 2005, S. H. Cho and J. H. Jung obtained the following results of common fixed point theorems in ε - chainable fuzzy metric space.

THEOREM 11. [5] Let (X, M, ∗) be a complete ε-chainable fuzzy metric space and let A, B, S and T be self-mappings of X satisfying the following conditions:

(i) AX ⊂ TX and BX ⊂ SX,

(ii) A and S are continuous,

(iii) the pairs (A, S) and (B, T) are weakly compatible, and

(iv) there exists q ∈ (0, 1) such that

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$

for every x, y ∈ X and t > 0. Then, A, B, S and T have a unique common fixed point in X.

In 2007, S. Kutukcu, S. Sharma and H. Tokgoz obtained the following results of fixed point theorem in fuzzy metric spaces for R-weakly commuting pairs of self-maps.

THEOREM 12. [23] Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of complete fuzzy metric space (X, M, $*$) such that

(i) $AX \subset TX$, $BX \subset SX$, and

(ii) $M(Ax, By, t) \geq M(x, y, ht)$, $0 < h < 1$, $x, y \in X$ and $t > 0$.

Suppose that (A, S) and (B, T) are compatible pairs of reciprocally continuous mappings. Then, A, B, S and T have a unique common fixed point.

THEOREM 13. [23] Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of complete fuzzy metric space (X,M, $*$) such that

(i) $AX \subset TX$, $BX \subset SX$, and

(ii) $M(Ax, By, t) \geq M(x, y, ht)$, $0 < h < 1$, $x, y \in X$ and $t > 0$.

Let (A, S) and (B, T) be compatible mappings. If any of the mappings in compatible pairs (A, S) and (B, T) is continuous then A, B, S and T have a unique common fixed point.

THEOREM 14. [23] Let (X, M, $*$) be a complete fuzzy metric space with $a * a \geq a$ for all $a \in [0, 1]$ and the condition (FM.6). Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of X such that

(i) $AX \subset TX$, $BX \subset SX$; and

(ii) there exists $k \in (0, 1)$ such that $M(Ax,By, kt) \geq M(x, y, t)$ for all $x, y \in X$, $\alpha \in (0, 2)$ and $t > 0$. If one of the mappings in compatible pair (A, S) or (B, T) is continuous, then A, B, S and T have a unique common fixed point.

THEOREM 15. [23] Let A, B, S and T be self-mappings on a complete metric space (X, d) satisfying (i) $AX \subset TX$, $BX \subset SX$; (ii) if there exists $k \in (0, 1)$ such that

$$d(Ax, By) \leq k \max\{d(Sx, Ax), d(Ty, By), d(Sx, Ty), [d(Ty, Ax) + d(Sx, By)]/2\}$$
for all $x, y \in X$, then A, B, S and T have a unique common fixed point in X.

In 2008, K. P. R. Rao, G. R. Babu and B. Fisher obtained the following results of common fixed point theorems in fuzzy metric spaces under implicit relations.

THEOREM 16. [34] Let (X, M, $*$) be a complete fuzzy metric space with $t * t \geq t$, for all $t \in [0, 1]$, and let f, g, S and T be self-maps on X such that

(i) $f(X) \subset T(X)$, $g(X) \subset S(X)$,

(ii) S and T are continuous,

(iii) The pairs (f, S) and (g, T) are compatible,

(iv) There exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,

$M(fx, gy, kt) \geq M(Sx, Ty, t) * M(fx, Sx, t) * M(gy, Ty, t) * M(fx, Ty, t)$, and

(v) $\lim_{n\to\infty} M(x, y, t) = 1$, for all $x, y \in X$.

Then, f, g, S and T have a unique common fixed point in X.

THEOREM 17. [34] Let A, B, S and T be self-mappings of a complete L-fuzzy metric space (X, M, T), which has property (C), satisfying:

(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$ and T(X), S(X) are two closed subsets of X;

(ii) the pairs (A, S) and (B, T) are weakly compatible; and

(iii) $M(Ax, By, t) \geq_L M(Sx, Ty, kt)$, for every x, y in X and some $k > 1$.

Then, A, B, S and T have a unique common fixed point in X.

In 2009, M. Abbas, I. Altun and D. Gopal obtained the following results of common fixed point theorems for non-compatible mappings in fuzzy metric spaces.

THEOREM 18. [2] Let (X, M, ∗) be a fuzzy metric space. Let A, B, S and T be maps from X into itself with $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ and there exists a constant $k \in (0, \frac{1}{2})$ such that

$M(Ax, By, kt) \geq \phi\,(M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, \alpha t), M(By, Sx, (2-\alpha)t))$,

for all $x, y \in X$, $\alpha \in (0, 2)$, $t > 0$ and $\phi \in \Psi$. Then A, B, S and T have a unique common fixed point in X provided the pair (A, S) or (B, T) satisfies (E. A.) property, one of A(X), T(X), B(X), S(X) is a closed subset of X and the pairs (B, T) and (A, S) are weakly compatible.

THEOREM 19. [2] Let (X, M , *) be a fuzzy metric space. Let A, B, S and T be maps from X into itself such that

$M(Ax, By, kt) \geq \phi\,(M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Sx, \alpha t), M(By, Sx, (2-\alpha)t)))$

for all $x, y \in X$, $k \in (0, \frac{1}{2})$, $\alpha \in (0, 2)$, $t > 0$ and $\phi \in \Psi$.

Then A, B, S and T have a unique common fixed point in X provided the pair (A, S) and (B, T) satisfy common (E.A.) property, T(X) and S(X) are closed subsets of X and the pairs (B, T) and (A, S) are weakly compatible.

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In 2009, R.K. Saini and M. Kumar proved the following fixed point theorem in fuzzy metric space using implicit relation.

THEOREM 20. [37] Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of complete fuzzy metric space (X, M, *) such that

(i) $AX \subset TX$, $BX \subset SX$,

(ii) $M(Ax, By, t) \geq M(x, y, ht)$; $0 < h < 1$, $x, y \in X$ and $t > 0$.

Suppose that (A, S) and (B, T) is compatible pairs of reciprocally continuous mappings. Then, A, B, S and T have a unique common fixed point.

THEOREM 21. [37] Let (A, S) and (B, T) be pointwise R-weakly commuting pairs of self-mappings of complete fuzzy metric space (X, M, *) such that

(i) $AX \subset TX$, $BX \subset SX$,

(ii) $M(Ax, By, t) \geq M(x, y, ht)$; $0 < h < 1$, $x, y \in X$ and $t > 0$.

Let (A, S) and (B, T) be compatible mappings. If any of the mappings in compatible pairs (A, S) and (B, T) is continuous, then A, B, S and T have a unique common fixed point.

THEOREM 22. [37] Let (X, M, *) be a complete fuzzy metric space with $t * t \geq t$, for all $t \in [0, 1]$ and the condition (FM 6). Let (A, S) and (B, T) be point wise R–weakly commuting pairs of self-maps on X satisfying

(i) $AX \subset TX$ and $BX \subset SX$;

(ii) (A, S) and (B, T) are compatible pairs and one of the mapping in each pair is

continuous;

(iii) there exists $k \in (0, 1)$ such that

$F(M(A^2x, B^2y, kt), M(S^2x, A^2x, t), M(T^2y, S^2x, t), M(T^2y, B^2y, kt), M(A^2x, T^2y, t)) \geq 0$,

for all $x, y \in X$ and $t > 0$, where $F \in \mathbf{F}^*$, then A, B, S and T have a unique common fixed point in X.

In 2010, A. S. Ranadive and A. P. Chouhan obtained the following results on absorbing mappings and _xed point theorem in fuzzy metric spaces.

THEOREM 23. [32] Let (X, M, *) be a complete ε-chainable fuzzy metric space and let A, B, S and T be self-mappings of X satisfying the following conditions:

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(i) AX ⊂ TX and BX ⊂ SX;

(ii) A and S are continuous;

(iii)the pairs (A, S) and (B, T) are weakly compatible; and there exists q ∈ (0, 1) such that $M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$, for every $x, y \in X$ and $t > 0$.

Then A, B, S and T have a unique common fixed point in X.

In 2010, C. T. Aage and J. N. Salunke obtained the following results of fixed point theorems in fuzzy metric spaces.

THEOREM 24. [1] Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be occasionally weakly compatible. If there exists q∈(0, 1) such that

$$M(Ax, By, qt) \geq \min\{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)\}$$

for all $x, y \in X$ and for all $t > 0$, then there exists a unique fixed point w in X such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.

THEOREM 25. [1] Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be occasionally weakly compatible. If there exists q∈(0, 1) such that

$$M(Ax, By, qt) \geq \phi(\min\{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t)\})$$

for all $x, y \in X$ and $\phi : [0, 1] \to [0, 1]$ such that $\phi(t) > t$ for all $0 < t < 1$,
then there exists a unique common fixed point of A, B, S and T.

THEOREM 26. [1] Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be occasionally weakly compatible. If If there exists q∈(0, 1) such that

$$M(Ax, By, qt) \geq \phi(M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t))$$

for all $x, y \in X$ and $\phi : [0, 1] \to [0, 1]$ such that $\phi(t, 1, 1, t, t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A, B, S and T.

THEOREM 27. [1] Let (X, M, ∗) be a complete fuzzy metric space and let A, B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} are occasionally weakly compatible. If there exists a point q∈ (0, 1) such that for all $x, y \in X$ and $t > 0$,

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$. Then there exists a unique common fixed point of A, B, S and T.

THEOREM 28. [1] Let (X, M, ∗) be a complete fuzzy metric space. Then continuous self-mappings S and T of X have a common fixed point in X if and only if there exists a self-mapping A of X such that the following conditions are satisfied

(i) $AX \subset TX \cup SX$

(ii) the pairs {A, S} and {A, T} are weakly compatible, and

(iii) there exists a point $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$M(Ax, Ay, qt) \geq M(Sx, Ty, t)\ M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$. In fact A, S and T have a unique common fixed point.

THEOREM 29. [1] Let (X, M, ∗) be a complete fuzzy metric space and let A and S be self-mappings of X. Let the A and B are occasionally weakly compatible. If there exists a point $q \in (0, 1)$ such that for all $x, y \in X$ and

$$t > 0,\ M(Sx, Sy, qt) \geq \alpha\, M(Ax, Ay, t) + \beta \min\{M(Ax, Ay, t), M(Sx, Ax, t), M(Sy, Ay, t)\},$$

for all $x, y \in X$, where $\alpha, \beta > 0$, $\alpha + \beta > 1$. Then, A and S have a unique common fixed point.

In 2011, S. Kumar and B. Fisher obtained the following results of a common fixed point theorem in fuzzy metric space using property (E.A.).

THEOREM 30. [22] Let (X, M , *) be a complete fuzzy metric space and let A, B , S and T be self –mappings of X. Let the pairs {A, S} and {B, T} be occasionally weakly compatible. If there exists $q \in (0,1)$ such that

i) $M(Ax, By, qt) \geq \phi(\min\{M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t),$

ii) $M(By, Sx, t)\})$, for all $x, y \in X$ and $\phi : [0,1] \rightarrow [0, 1]$ such that $\phi(t) > t$

for all $0 < t < 1$, then there exists a unique common fixed point of A, B, S and T.

THEOREM 31. [22] Let (X, M, *) be a complete fuzzy metric space and let A, B , S and T be self –mappings of X. Let the pairs {A, S} and {B, T} be occasionally weakly compatible. If there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq \phi(M(Sx, Ty, t), M(Sx, Ax, t), M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t))$$

for all $x, y \in X$ and $\phi : [0,1]^5 \rightarrow [0, 1]$ such that $\phi(t, 1, 1, t, t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A , B , S and T.

THEOREM 32. [22] Let (X, M, *) be a complete fuzzy metric space and let A, B , S and T be self–mappings of X. Let the pairs {A, S} and {B, T} be occasionally weakly compatible. If there exists $q \in (0, 1)$ for all $x, y \in X$ and $t > 0$

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t).$$

Then there exists a unique common fixed point of A, B, S and T.

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In 2013, K. Jha obtained the following results of common fixed point theorem for four mapping in complete fuzzy metric space.

THEOREM 33. [14] Let (X, M, *) be a complete fuzzy metric space with additional condition (vi) and with a * a ≥ a for all a ∈ [0, 1]. Let A, B, S and T be mappings from X into itself such that

(i) AX ⊂ TX, BX ⊂ SX, and
(ii) M(Ax, By, t) ≥ r(N(x, y, t)),

where r : [0, 1] → [0, 1] is a continuous function such that r(t) > t for some 0 < t < 1 and for all x, y ∈ X and t > 0. If (A, S) or (B, T) is semi-compatible pair of reciprocally continuous maps with respectively (B, T) or (A, S) as weakly compatible maps, then A, B, S and T have a unique common fixed point in X.

In 2014, K. B. Manandhar and K. Jha introduced the compatible mappings of type (K) and established the following results of common fixed point theorem for four mapping in complete fuzzy metric space using compatible mappings of type (E).

THEOREM 34. [24] Let (X, M, ∗) be a complete fuzzy metric space with a ∗ a ≥ a for all a ∈ [0, 1] and with the condition (fm6). Let one of the mapping of self-mappings (A, S) and (B, T) of X be continuous such that

(a) X ⊂ TX, BX ⊂ SX;
(b) There exists k ∈ (0, 1) such that M(Ax, By, kt) ≥ N(x, y, t)

for all x, y ∈ X, α ∈ (0, 2) and t > 0.
If (A, S) and (B, T) compatible of type of (E) then A, B, S and T have a unique common fixed point.

1.4 COMMON FIXED POINT THEOREMS FOR SEQUENCE OF MAPPINGS

In 2005, A. Razani proved the following results of a contraction theorem in fuzzy metric spaces.

THEOREM 35. [35] Let (X, M,*) be a fuzzy metric space, and A is a fuzzy contractive mapping of X into itself such that there exists a point x ∈ X whose sequence of iterates ($A^n(x)$) contains a convergent subsequence ($A^{n_i}(x)$), then $\xi = \lim_{i\to\infty} A^{n_i}(x) \in X$ is a unique fixed point.

THEOREM 36. [35] Let (X, M, ∗) be a fuzzy metric space, where the continuous t-norm ∗ is defined as a∗b = min{a, b} for a, b ∈[0, 1]. Suppose f is a fuzzy ε-contractive self-mapping of X such that, there exists a point x ∈ X whose sequence of iterates ($f^n(x)$) contains a convergent subsequence ($f^{n_i}(x)$), then $\xi = \lim_{i\to\infty} f^{n_i}(x)$ is a periodic point.

In 2010, K. Jha obtained the following results of a common fixed point theorem for sequence of mapping in fuzzy metric space.

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THEOREM 37. [12] Let (X, M, ∗) be a fuzzy metric space with additional condition (FM6.) and with $a * a \geq a$ for all $a \in [0, 1]$. Let $\{A_i\}$, I = 1, 2, 3, 4,… , S and T be self-mappings of a fuzzy metric space from (X, M, ∗) such that

(i) $A_1X \subset TX$, $A_i X \subseteq SX$ for i >1,

(ii) there exists $r \in (0, 1)$ such that $M(A_1x, A_ix, r\,t) \geq M_{1i}(x, y, t)$ for all $x, y \in X$, $\alpha \in (0, 2)$ and $t>0$.

If one of $A_i X$, SX or TX is complete subspace of X and if the pairs (A_1, S) and (A_k, T), for some k > 1 are weakly compatible then all the mapping $\{A_i\}$, S and T have a unique common point.

In 2012, K. Jha obtained the following results of generalized common fixed point theorem for sequence of mapping in fuzzy metric space.

THEOREM 38. [13] Let (X, M, *) be a fuzzy metric space. Let $\{A_i\}$, i = 1, 2, 3, ..., S and T be mappings of a fuzzy metric space from X into itself such that

(i) $A_1X \subset TX$, $A_iX \subset SX$, for i > 1, and

(ii) for a function $\Psi : [0, 1) \to [0,1)$ with (r) > 0 for r > 0, $\Psi(0) = 0$ and an altering distance function ϕ such that for i > 1, the relation $\phi\left(\frac{1}{M(A_1x, A_iy, t)} - 1\right) \leq \phi\left(\frac{1}{M_{1i}(x,y,t)} - 1\right) - \Psi\left(\frac{1}{M_{1i}(x,y,t)} - 1\right)$ holds for every $x, y \in X$ and each $t > 0$.

If one of A_iX, SX and TX is a G-complete subspace of X, if the pair (A_1, S) and (A_i, T), for i > 1, are weakly compatible, then all the mappings A_i, S and T have a unique common fixed point in X.

1.5 COMMON FIXED POINT THEOREMS USIMG COMPATIBLE MAPPINGS OF TYPE (K)

In 2014, K. B. Manandhar and K. Jha introduced the compatible mappings of type (K) and established the following results of common fixed point theorems in complete fuzzy metric space using compatible mappings of type (K).

THEOREM 39. [25] Let (X, M, ∗) be a complete fuzzy metric space and A, B, S and T be a self-mappings of X satisfying the following conditions:

(i) $A(X) \subset T(X)$, $B(X) \subset S(X)$,

(ii) $M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Bx, Ty, t) * M(Ax, Ty, t)$,
for all $x, y \in X$, $k \in (0,1)$ and $t > 0$, and

(iii) S and T are continuous.

If (A, S) and (B, T) compatible of type of (K), then A, B, S and T have a unique common fixed point.

THEOREM 40. [26] If A, B, S and T are self-mapping on a complete Intuitionistic fuzzy metric space (X, M, N, ∗ , ◊), satisfying the conditions:

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(i) A $(X) \subseteq T(X)$, $B(X) \subseteq S(X)$.

(ii) There exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,

$$M(Ax, By, kt) \geq \min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t), M(Ax, By, t), M(Sx, By, t)\}, \text{ and}$$

$$N(Ax, By, kt) \leq \max\{N(Sx, Ty, t), N(Ax, Sx, t), N(By, Ty, t), N(Ax, Ty, t), N(Ax, By, t), N(Sx, By, t)\}.$$

(iii) B and T weakly compatible mappings.

If the pair of mappings (A, S) is compatible of type (K) and one of the mapping is continuous then A, B, S and T have a unique common fixed point in X.

In 2014, M. Hakwadiya, R.K. Gujetiya and D.K. Mali, established the following result of common fixed point theorems in complete fuzzy metric space using compatible mappings of type (K).

THEOREM 41. [11] Let (X, M, *) be a complete Fuzzy 2-metric space and A, B, P, Q, S and T be a self-mapping of X satisfying the following condition:

i. $P(X) \subset BT(X)$ and $Q(X) \subset SA(X)$
ii. SA and BT are continuous.
iii. (P, SA) and(Q, BT) compatible of type of (K).
iv. $[1 + aM(SAx, Px, akt)] * M(Px,Qy, a, kt) \geq$
$a[M(x,SAx, a, kt) * M(BTy, Qy, a, kt) * M(BTy,Px, a, kt)] + M(BTy,SAx, a, t) *M(Px,SAx, a, \alpha t) * M(Qy, BTy, a, (2 - \alpha)t) *M(Qy, SAx, a, \alpha t) * M(Px,BTy, a, (2 - \alpha)t)$

For all $x, y, a \in X$, $\alpha \in (0, 2)$, $a \geq 0$ and $t > 0$

If (P,SA) and (BT, Q) are commute, Then A, B, P, Q, S and T have a unique common fixed point.

In 2015, K. B. Manandhar and K. Jha established the following result of common fixed point theorems in complete intuitionistic fuzzy metric space using compatible mappings of type (K).

THEOREM 42. Let A, B, S and T be self-maps of a complete intuitionistic fuzzy metric spaces $(X, M, N, *, \Diamond)$ with continuous t-norm * and continuous t-conorm $\Diamond$ defined by $a * a \geq a$ and $a \Diamond a \leq a$ for all $a \in [0; 1]$ satisfying the following condition:

i. $A(X) \subset T(X)$ and $B(X) \subset S(X)$,
ii. the pairs (A, S) and (B, T) are compatible mappings of type (K),
iii. If A, S and one of the mapping of pair (B, T) is continuous.
iv. there exist $k \in (0; 1)$ such that
$M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t)*M(By, Ty, t) * M(Ax, Ty, t)$ and
$N(Ax, By, t) \leq N(Sx, Ty, t) \Diamond N(Ax, Sx, t)\_N(By, Ty, t) \Diamond N(Ax, Ty, t)$,
for every x, y in X and $t > 0$.

Then A, B, S and T have a unique common fixed point in X.

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CONCLUSION

The study of common fixed point theorems of mappings satisfying certain contractive type conditions in fuzzy metric space has been a very active field of research. Many established common fixed point theorems in metric space and fuzzy metric space may generalize and extend by using newly defined compatible mapping of type (K).

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