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Bayesian causal relation effect in quantiles regression models

Adewole, Ayoade I.*[a], Bodunwa,Oluwatoyin K.[b], and Adebola, Femi B.[b]

[a]Department of Mathematics, Tai Solarin University of Education Ijagun Ogun State Nigeria.
[b]Department of Statistics, Federal University of Technology Akure, Ondo State Nigeria.

Abstract

Analysis of causal relationships held an important part of the theoretical and empirical contribution in quantitative economic theory. This research explored the performance of Bayesian quantile regression with Granger causality showing that Bayesian inference can be undertaken in the context of quantiles regression. Causality Bayesian inferences in the context of quantile regression were achieved by applying the framework of the generalized linear model using asymmetric Laplace distribution for the error term. The developed scheme allows assessing the impact of the explanatory variables on all quantiles range of the conditional distribution of GDP growth. In Practical usage of macroeconomics variables, the scheme can be used to estimate parameters with causality effect which is synonyms to time series data. This research contributed to the versatile application of quantile regression in the contest of statistical research, the study estimated the regression quantiles parameter estimate applying Bayesian procedures. Furthermore, compared to frequentist estimates, Bayesian estimates established the superiority of the Bayesian regression method to the frequentist approach.

Keywords: Bayesian Inference; Causality Bayesian quantile regression models; Causal relations and Quantiles regression

1. Introduction

The central location, the scale, the skewness, and other higher-order properties not central location alone characterize a distribution, thus mean models are inherently ill-equipped to characterize the relationship between a response distribution and predictor variable. Developed by [15], quantile regression complements and improves the classical mean regression models, in this situation of homogeneity assumption violated, quantile regression quantifies the heterogeneous effect of covariates through conditional quantiles of the outcome variable and provides a comprehensive scan of the whole distribution of the outcome. Since the path-breaking of work of [15], quantile regression models have been increasingly used in many applied areas in economics due to their flexibility to allow researchers to investigate the relationship between economic variables not only at the center but also over the entire conditional distribution of the dependent variable. Several proposals in the literature applied quantile regression techniques, among many others [4,7,14]. Regression analysis seeks to find the relationship between one or more independent variables and a dependent variable, certain widely used methods of regression such as ordinary least squares have favorable properties if their underlying assumptions are true, but can give wrong inference and misleading decisions if those assumptions are not true; thus, OLS is said not too robust to violations of its assumptions. The Bayesian models allow for the inclusion of prior distributions of the estimated parameters leading to an altogether different set of considerations than that of the classical approach. In recent years, many of the perceived difficulties of implementing the Bayesian paradigm can be tackled through the application of Markov Chain Monte Carlo simulation methods. Bayesian methods do not need to be tested for their sampling properties [10] instead they are concerned with the facts that the correct likelihood and prior are being employed so that Markov Chain Monte Carlo (MCMC) methods converge to the implied posterior distribution.

Accounting for uncertainty is paramount to Bayesian analysis, as to the computations associated with most common tasks e.g estimations, predictions, evaluation of hypothesis are typically integrations. In some situations, it is possible to perform such integration exactly either by taking advantage of conjugate structure in the prior- likelihood or by using dynamic programming when the dependencies between random variables are appropriately simple. The Bayesian framework of regression quantiles implemented via the Markov Chain Monte Carlo method provides a convenient way of incorporating uncertainty into predictive inferences [22].

The Bayesian paradigm is fundamentally about integration: integration computes posterior estimates and measures of uncertainty, eliminates nuisance variables or missing data, and averages models to compute predictions or perform the model comparison. Bayesian inference depends on prior and likelihood functions. Based on empirical justification, it was observed from research that Asymmetric Laplace distribution for response is robust to underlying likelihoods. Asymmetric Laplace distribution has good performance on data generated error distributions [18,24] among others) and theoretic justification.

Furthermore, skewed Laplace distribution possesses an attractive attribute, it can be represented as a scale mixture of normal distribution, [17,21]. The mixtures representations allow the quantile regression model to be expressed as a normal regression model. This property appears in [18], [23], and [3] in conducting Bayesian quantile regression via Gibbs sampler. The goal of many sciences is to understand the mechanism by which variables come to take on the values they have (i.e. to find a generative model) and to predict what the values of those variables would be if the naturally occurring mechanisms were subject to outside manipulations. The definition of Granger non-causality is defined in terms of the conditional distribution, testing non causality in conditional mean

*Corresponding author. Email: hayorhade2005@gmail.com

based on the linear model is very common in literature [11]. Moreover, non-causality is defined in variance [9,11] and non-causality in other moments. Several new emerging methods are in the literature for testing causality that deals with different data generating processes [6,9,11].

Granger causality is based on precepts that cause preceded effect and the causal series that had information about the effect that was not contained in any other series according to conditional distributions [13]. Causal relations between variables are typically examined by testing Granger non-causality. [9] examined causal effects in quantiles accounting for heteroscedasticity using Metropolis's sampling Algorithm. This research adopted the conceptual and empirical framework of Granger causality and Bayesian quantile regression in estimations of regression quantiles across the entire distribution. The testing procedure is based on a multivariate extension of the classical Granger causality in mean, the test is extended to test non-causality in different quantile and identify the quantile range which causality is relevant using the Bayesian paradigm. This research will fill the vacuum in the literature by examining the Granger causality effect on Bayesian quantile regression models using Gibb's sampling algorithm.

2. Methodology

$$y_t = X_t^T \beta_\tau + \epsilon_t \tag{1}$$

t = 1, ..., n where y_t is the response variable and x_t, a $kx1$ vector of covariates for the t^{th} observation. ϵ_t is the error term whose distribution is restricted to have τ^{th} quantile equal to zero, that is

$$\int_{-\alpha}^{0} F_\tau(\epsilon_t) d\epsilon_t = \tau \tag{2}$$

let $Q_\tau(x_t)$ denote t^{th} $(0 < \tau < 1)$ quantile regression function of y_t is given x_t. The relationship is

$$Q_\tau(x_t) = x_t^{'}\beta \tag{3}$$

where β_τ is a vector of unknown parameters of interest. Quantile regression estimation β_τ proceeds by minimizing

$$\hat{\beta}_\tau = argmin_{\beta e R^k} \sum_{t=1}^{n} \rho_\tau(y_t - x_t^{'}\beta_\tau) \tag{4}$$

where the loss function ρ is simplified as

$$\rho_\tau(u) = [\tau - I(u < 0)]u \tag{5}$$

and the model's residuals are formulated as an indicator function with

$$Iu = \begin{bmatrix} 1 & for u < 0 \\ 0 & otherwise \end{bmatrix} \tag{6}$$

The quantile $\hat{\beta}_\tau$ is the τ^{th} quantile.

The loss function is not differentiable; solutions to the minimization cannot be derived explicitly. The linear programming method in R was designed to obtain quantile regression estimates for $\hat{\beta}_\tau$. this linear programming problem was solved using the simplex iteration procedure of Koenker and Dorey (1993), the minimum was obtained at the vertices of the feasible region.

3. Causality in quantiles

Given that distribution is completely determined by its quantiles, Granger non-causality can be expressed in terms of conditional quantiles. If $Q_{y_i}(\tau|F)$ denotes the τ^{th} quantile of $F_{y_i}(.|F)$

$$Q_{y_i}(\tau|y,x)_{t-1} = Q_{y_i}(\tau|y_{t-1}) \tag{7}$$

Granger causality is usually considered in the context of linear regression models and since the true distribution and parameters values are unknown a statistical test of

$$Q_{\tau yt}(y_t|y,x)_{t-1} = Q_{yt}(y_t|y_{t-1}) \tag{8}$$

for all $\forall \tau \epsilon (0,1)$ was performed using an observed sample of data on a specified model. Then the conditional quantile at a probability level τ is

$$Q_{y_t}(\tau) = y_{t-1}^{\prime}\alpha(\tau) + x_{t-1}^{\prime}\beta(\tau) \tag{9}$$

x is said not to granger cause y in all quantile if equation (8) holds. Bayesian causality testing was performed by erecting a standard credible interval on the relevant parameters 2.5th and 97.5th qauntiles of each MCMC sample of parameter iterates to form 95% credible intervals. If 0 is not contained in this interval the parameter is considered significantly different to 0 that is testing

$$H_0 : \beta(\tau) = 0 vs H_1 : \beta(\tau) \neq 0$$

Causality quantile regression model was considered as follows;

$$y_t = Y_{t-1}^{\prime}\alpha(\tau) + X_{it-1}^{\prime}\beta\tau + \varepsilon_t \tag{10}$$

where

$$Y_{t-1}^{\prime} = (1, y_{t-1}, ..., y_{t-p})^{\prime}, \tag{11}$$

$$X_{it-1}^{\prime} = (x_{it-1}, x_{i1,t-q}, ..., x_{it-1}, ..., x_{ir,t-q})^{\prime}, \tag{12}$$

$$\alpha(\tau) = (\alpha_0(\tau), \alpha_1(\tau), ..., \alpha_p(\tau))^{\prime}, \tag{13}$$

$$\beta(\tau) = (\beta_{1,1}(\tau), \beta_{1,q_1}(\tau), ..., \beta_{r,q_r}(\tau))\prime, \tag{14}$$

where

$$Y\prime_{t-1}, X_{it-1}^{\prime}$$

are the regressors; $\alpha(\tau), \beta(\tau)$ are coefficient of causality parameters and the ε_t follows the process;

$$\varepsilon_t = \varepsilon_{t-1} + U_t \tag{15}$$

where

$$U \sim iiid(N0, \sigma^2). \tag{16}$$

the model inference theoretically requires the initial values $(y_0...y_t - p)$ and $(x_1...x_t - p)$, noting that error terms U_t are independent normal, the assumption that errors are independent overall individual and periods implies that the transformed model simply reduces to the standard linear regression framework, the density can be expressed as

$$f(y_t^* | X_{t-1}^{\prime *}, \alpha, \beta,) = \frac{1}{(2\pi\sigma^2)^{(t-p)/2}} \exp\left[\frac{-(y_t^* - X_{t-1}^*)^T (y_t^* - X_{t-1}^{\prime *})}{2\sigma^2}\right] \tag{17}$$

3.1. Causuality Bayesian quantile regression

Bayesian estimation of parameters in the model in (10) begins by erecting a likelihood that follows the asymmetric Laplace distribution. Due to the complexity of the asymmetric Laplace distribution and thus difficulty to maximize its likelihood function, the asymmetric Laplace distribution was represented as a location of scale mixture of normal distribution where the mixing distribution follows an exponential distribution, for effective and easy draw.

$$y_t^* = Y_{t-1}^{\prime *}\alpha(\tau) + X_{it-1}^{\prime *}\beta(\tau) + \theta z_i + \delta\sqrt{z_i u_i} \tag{18}$$

A scale parameter was introduced in the model in $equation(18)$ to account for the spread of the distribution across the entire quantiles, this is given as:

$$y_t^* = Y_{t-1}'^{*}\alpha(\tau) + X_{it-1}'^{*}\beta(\tau) + \sigma\theta z_i + \delta\sqrt{z_i u_i} \quad (19)$$

Reparametrizing $equation(19)$ for easy sampling gives;

$$y_t^* = Y_{t-1}'^{*}\alpha(\tau) + X_{it-1}'^{*}\beta(\tau) + \theta v_i + \delta\sqrt{\sigma v_i u_i} \quad (20)$$

This leads to the likelihood function

$$f\left(y_t^* \left| Y_{t-1}'^{*}\alpha, X_{t-1}'^{*}\beta, v, \sigma, , \tau\right.\right)$$
$$\exp\left[\frac{-\sum_{t=1}^{n}\left(y_t^* - Y_{t-1}'^{*}\alpha(\tau) - X_{t-1}'^{*}\beta(\tau) - uv_i^2\right)}{2\delta^2\sigma v_i}\right]\prod\frac{1}{\sigma v_i} \quad (21)$$

Given the above, to proceed in the Bayesian analysis since the Bayesian inference depends on prior and likelihood function, conjugate prior for $\alpha, \beta, \sigma$and v was chosen separately.

$$Prior of \alpha, \beta \sim N(\beta_p, B_p) \quad (22)$$

Where β_p and $B_p$are the prior mean and the covariance respectively,
all the posterior moment of$\alpha$and β exist, the posterior of α,βstill follows normal distribution i. e.,

$$\alpha, \beta\, |y, v, \sigma \sim N(\beta_p, B_p) \quad (23)$$

with

$$B_p = \left(\frac{\sum_{t=1}^{n} X_{t-1}X^T}{\delta^2\sigma v_i}\right) + B_0^{-1}) \quad (24)$$

and

$$\beta_p = \beta_p\left(\frac{\sum_{i=1}^{n}(y_i - uv_i)}{\delta^2 v_i}\right) + B_p^{-1}\beta_o \quad (25)$$

For the prior on σ, inverse gamma distribution I G(a,b), inv Gamma (shape = $n_{0,}$ scale = $s_{0,}$ was chosen with density

$$f(x\left|n_0, s_0\right.) = \frac{{s_0}^{n_0}}{\Gamma(n_0)}x^{-n_0-1}\exp\left(-\frac{s_0}{x}\right) \quad (26)$$

with parameters $a = \frac{n_0}{2}$ and $b = \frac{s_o}{2}$ the posterior distribution for σ follows an inverse Gamma distribution

$$\sigma\, |y, \alpha, \beta, v \sim IG(\frac{n^*}{2}\frac{s^*}{2}) \quad (27)$$

with $n^* = n_o + 3n$ and

$$s^* = s_o + 2\sum_{i=1}^{n}\mu_i v_i + \sum_{i=1}^{n}\left(\frac{(Y_t - X_{t-1}'\beta - \mu v_i)^2}{\sigma^2 v_i}\right) \quad (28)$$

The prior of ν_i follows a generalized inverse Gaussian distribution

$$v_i\, |y_t, \alpha, \beta, \sigma \sim GIG(\frac{1}{2}, \alpha_i, \gamma_i) \quad (29)$$

where the probability density function of $GIG(v, \alpha, \gamma)$is given by

$$f(x\left|v, \alpha, \gamma\right.) = \frac{(\gamma\,|\alpha)^v}{2kv(\alpha\lambda)}x^{v-1}\exp\left(-\frac{1}{2}(\alpha^2 x^{-1} + \gamma^2 x)\right) \quad (30)$$

$x > 0, -\infty < v < \infty, \alpha, \gamma \geq 0$, and $kv(\alpha\gamma)$ is a modified Bessel function of the third kind.

However, the posterior distribution for ν_i still follows a generalized inverse Gaussian distribution

$$v_i\, |y_t, \alpha, \beta, \sigma \sim GIG(\frac{1}{2}, \alpha_i, \gamma_i) \quad (31)$$

with

$$\alpha_i^2 = \frac{(Y_t - x_{t-1}'\beta)}{\delta^2\sigma} \quad (32)$$

and

$$\gamma_i^2 = \frac{2}{\sigma} + \frac{\mu^2}{\delta^2\sigma} \quad (33)$$

The fully conditional posterior distribution of $\alpha, \beta, \sigma, \nu$ is not of tractable form, so therefore we employed Gibb's sampling method to estimate the posterior.

The Gibb's samplers sampled from

$$\sigma \mid \alpha, \beta, \nu, y$$
$$v \mid \alpha, \beta, \sigma, y$$
$$\alpha, \beta \mid \sigma, \nu, y$$

Which converges to the joint conditional posterior distribution combining the likelihood function of the data, $L(y_t\,|\alpha, \beta, \sigma, \nu)$ given by

$$p(\alpha, \beta, \sigma, \nu\,|y) \propto L(y_t\,|\alpha, \beta, \sigma, \nu)\ .p(\sigma\,|\alpha, \beta, \nu, y_t).$$
$$p(v\,|\alpha, \beta, \sigma, y_t).p(\alpha, \beta\,|\sigma, \nu, y_t) \quad (34)$$

Combining the likelihood density in (21) with the prior specification for $\alpha, \beta, \sigma$and v the mixing function, in equations (22), (26) and, (29). The joint posterior distribution of (α, β, σ and v) becomes $\pi(\alpha, \beta, \sigma, \nu, |y_t, Y_{t-1}'^{*}, X_{t-1}'^{*}, \tau) \propto L\left(y_t Y_{t-1}'^{*}\alpha, X_{t-1}'^{*}\beta, \sigma, v, \tau\right) \times$ joint priors of (α, β, σ and v)

This yields the following full conditional posteriors

$$\begin{bmatrix} \alpha, \beta\,|y_t, \sigma, v \sim N(\beta_p, B_p) \\ \sigma\,|y_t, \alpha, \beta, v \sim IG\left(\frac{n^*}{2}, \frac{s^*}{2}\right) \\ v\,|y_t, \alpha, \beta, \sigma \sim GIG\left(\frac{1}{2}, \alpha_i\gamma_i\right) \end{bmatrix} \quad (35)$$

Based on the conditional posterior densities of α, β, σ and v which are not analytically tractable in equation (41), we turn to the MCMC computation method using Gibb's sampling to draw samples from the posterior. Gibb's sampler is an iterative Monte Carlo scheme designed to extract conditional posterior distribution from intractable joint distribution.

3.2. Empirical applications

To illustrate the estimation of Causality regression quantiles empirically and using Bayesian paradigms, the data set from Nigeria Economy was considered which comprised of the GDP growth as the dependent variable while the independent variable under study is the export rate, import rate, inflation and, past GDP at time $t-1$ ranging from the period of 1985 - 2020. The model used was:

$$y_t = \alpha_o + \alpha_1 y_{t-1} + \beta_1 x_{1t-1} + \beta_2 x_{2t-1} + \beta_3 x_{3t-1}\beta_4 x_{4t-1} + \epsilon t \quad (36)$$

where y_t = GDP growth; x_{1t-1} = import rate at time t-1; x_{2t-1} = export rate at time t-1;x_{3t-1} = inflation rate at time t-1 and x_{4t-1} = exchange rate at time t-1
Bayesian inferences depend on prior and likelihood function, the likelihood function that was based on asymmetric Laplace distribution in equation (21) was employed, both intercept and coefficient of the predictor were sampled from the posterior distribution defined in equation (35) using Gibb Sampling iterations. Causality Bayesian quantile regression was analyzed empirically by examining the causal relationship among GDP growth in Nigeria and other macro-economic variables, it was checked whether

the GDP growth is driven by past GDP itself and or other macroeconomic variables, the causal Bayesian estimation method was analyzed by examining the significance of the estimated parameters. All Bayesian estimates inferences were based on 120,000 Gibbs Sampling iterates with 20,000 burn-in replications discarded and 100,000 replications retained. The comparison was made between the effectiveness of Causality Bayesian Quantile Regression and Causality Quantile Regression estimates. Mean square error was used as a criterion of validation to measure the relative effectiveness of quantile regression in exploring the data at τ^{th} quantile.

4. Results and discussion

Table 1 presented the estimates of selected quantiles covering the lower, middle, and upper quantiles of Causality Bayesian quantile regression (CBQR); Causality Quantile Regression (CQR**),** and the empirical analysis on how the past GDP and other explanatory variables cause the GDP using Bayesian and frequentist approach. The Causality Bayesian Quantile Regression (CBQR) coefficient parameters in the model stated in equation 20 were obtained from the estimation of conditional posteriors derived in equation 35 using the Gibb's Sampling iteration procedures itemized from equation 18 to equation 34 while the quantile regression method of estimation highlighted in equation 1 to equation 16 was used to estimate the coefficient of causality parameters (CQR) in equation 14 applying the model stated in equation 9 using the lower quantiles (0.05, 0.10, 0.15); middle quantiles (0.50, 0.55, 0.60) and upper quantiles (0.85, 0.90, 0.95) respectively.
Posterior estimates with their standard deviation in parentheses are reported with 95% credible intervals formed by 2.5th and 97.5th samples quantiles of the MCMC iterates which are considered significant does not include zero and are shown bold below in Table 1. From results based on quantile effect using Causality Bayesian quantile regression (CBQR) models and Causality Quantile Regression (CQR**)** models in Table 1, the following summaries were drawn out:

- The posterior means are all contained in the credible intervals.
- The posterior means of CBQR are all contained in the 95% credible intervals.
- The omitting lagged dependent variable does not create inconsistency in the estimates of the parameter involved
- The estimated parameters vary widely with τsince the location is quantile-dependent.
- The statistically and positive effect was more reflected at the extreme tails of the distribution.
- In Bayesian analysis,the smaller data set can be analyzed without losing the power of precisions
- Bayesian quantiles regression estimates produce a smaller standard error
- The Bayesian estimate is similar to those based on quantile regression indicating the approach is practical and parameter uncertainty has been established

In the empirical analysis, the adaptive Bayesian Markov chain Monte Carlo scheme using Gibb's sampler was designed to illustrate the examination of possible causal, interactions between Nigeria's GDP and other macro-economic variables, over a range of any specific quantile. The Granger causality was used to expatiate the causal effects of the macroeconomic variables understudy on Nigeria's GDP and also to reflect whether the past GDP growth granger causes the present economic growth in Nigeria. Bayesian method of estimation was used to estimate the causality parameters in which inferences were drawn using the credible intervals. The results proved the evidence of dependencies among the selected macroeconomic variables under the study years. The GDP growth is the response variable while the import rate, export rate, inflation rate, the exchange rate are the explanatory variables. The impact of the exchange rate is statistically significant at the lower tail and the upper part of the conditional distribution of the GDP growth. The statistical effect of the exchange rate in the tail ends granger cause the economic effect on the growth of the conditional distribution of the GDP in the data set as it was discovered by [19]. The exchange rate plays an important role in determining the position of a country in terms of international trade. Moreover, the negative relationship between the conditional distribution of GDP growth and the export rate is related to the declining growth noticed in the economic growth in Nigeria justifying the research work of [1]. The results also show a positive relationship between the inflation rate and the economic growth in Nigeria across the quantiles of time specification under study. This result justifies the work of [20] that revealed that inflation is one of the major macroeconomic variables that undermine the growth of Nigeria's economy across the quantiles. Moreover, the magnitude of the effect of the import rate tends to granger cause the GDP growth than another macroeconomic variable in the lower tail of the distribution while the past GDP does not have a significant causality effect on the present GDP growth.
(CBQR), (CQR) represent Causality Bayesian quantile regression model and Causality Quantile regression model respectively.
From Table 2 above, CBQR procedures report smaller MSE in all the quantiles, the differences in terms of the MSE are less evident in the lower tail while they become more noticeable in the heavy tail in CBQR than CQR
Comparing the frequentist approach with the Bayesian approach using table 4.23, it was revealed that the Bayesian approach produced minimal MSE which implies that the Bayesian approach in estimating causality regression quantiles outperformed the frequentist approach in terms of MSE.

5. Conclusion

This study expatiated the estimation of regression quantiles under the Bayesian approach. The research work measured quantile causal relations using the likelihood-based approach, estimating regression quantiles, and explored the predictive ability of a model on a data set.

Quantile regression provides location, scale, and slope shift information on the conditional distribution of the response variable. The research provided a practical framework for allowing Bayesian parameter estimation to be implemented on more complex quantile regression models in a relatively straightforward approach. Bayesian inference to quantile regression models regards unknown parameters as random variables and the parameter uncertainty is taken into account without relying on asymptotic approximations Causality Bayesian quantile regression model provides estimates that are more efficient and less biased. Granger causality is based on precedence and predictability.

Empirically, this paper investigates the relationship and effect of some key macroeconomic variables on Nigeria's real GDP growth. The research illuminates the discussion of the causality relationship between inflation rate, exchange rate, import, export rate, and GDP growth in Nigeria. The empirical analysis used quantile-based locations procedures which allow the research to investigate more general notions of location beyond the center of a distribution. The results revealed that there is a more asymmetric relationship between the GDP growth and the macroeconomic variables which may not be shown using causality mean regression analysis. The economic implication of this is that the macroeconomic vari-

Table 1: The posterior estimates of the parameter from the Bayesian approach and Frequentist estimates.

τ	0.05				0.10				0.15			
	CBQR	Credible Intervals		CQR	CBQR	Credible Intervals		CQR	CBQR	Credible Intervals		CQR
		2.5^{th}	97.5^{th}			2.5^{th}	97.5^{th}			2.5^{th}	97.5^{th}	
$\widehat{\alpha}_0$	0.2640 (0.038)	0.0312	0.9528	0.4518 (0.15)	0.2595 (0.033)	0.0317	1.786	0.2536 (0.22)	0.2659 (0.074)	0.2181	0.9265	0.4755 (0.5)
$\widehat{\alpha}_1$	0.7651 (0.046)	0.2143	2.2307	0.6930 (0.12)	0.7518 (0.025)	0.3862	1.287	0.8862 (0.42)	0.7680 (0.062)	0.2302	1.1289	0.7789 (0.296)
$\widehat{\beta}_1$	1.0128 (0.031)	1.0056	2.3816	1.0248 (0.19)	1.0016 (0.049)	1.0012	1.63	1.1343 (0.24)	1.0438 (0.051)	0.0008	3.1592	1.0471 (0.48)
$\widehat{\beta}_2$	-0.0446 (0.069)	-0.0029	0.1638	-0.0487 (0.21)	-0.0486 (0.017)	-0.0296	1.5527	-0.0742 (0.37)	-0.0586 (0.061)	-0.0029	0.0714	-0.0457 (0.29)
$\widehat{\beta}_3$	1.0041 (0.017)	0	2.7426	1.1108 (0.37)	1.0029 (0.068)	0.4916	3.1094	1.2774 (0.11)	1.0059 (0.049)	1.0002	1.1978	1.0719 (0.14)
$\widehat{\beta}_4$	1.1714 (0.027)	1.0021	1.2054	1.1916 (0.15)	1.1036 (0.074)	1.0291	2.8125	1.2809 (0.6)	1.1817 (0.04)	1.0623	2.0653	1.2487 (0.16)

τ	0.50				0.55				0.60			
	CBQR	Credible Intervals		CQR	CBQR	Credible Intervals		CQR	CBQR	Credible Intervals		CQR
		2.5^{th}	97.5^{th}			2.5^{th}	97.5^{th}			2.5^{th}	97.5^{th}	
$\widehat{\alpha}_0$	0.3969 (0.018)	-0.205	1.8853	0.4951 (2.70)	0.2659 (0.074)	0.1975	1.0897	0.4935 (0.97)	0.4637 (0.102)	0.0117	1.1863	0.5498 (0.73)
$\widehat{\alpha}_1$	0.9016 (0.05)	0.0312	2.3684	0.8736 (0.18)	0.7680 (0.062)	0.2296	2.0984	0.8951 (0.02)	0.9613 (0.06)	0.3729	3.0096	0.9839 (0.26)
$\widehat{\beta}_1$	1.2468 (0.032)	0.4142	2.5932	1.1274 (0.60)	1.0438 (0.051)	0.0018	3.2269	1.2930 (0.26)	1.1725 (0.09)	0.2047	2.5771	1.3437 (0.51)
$\widehat{\beta}_2$	-0.079 (0.045)	-0.0027	0.1954	-0.0658 (0.99)	-0.0586 (0.061)	-0.0046	0.0005	-0.0748 (0.30)	-0.0822 (0.024)	-0.0257	0.0833	-0.0859 (0.64)
$\widehat{\beta}_3$	1.1994 (0.008)	0.1165	1.9973	1.2311 (0.12)	1.0059 (0.049)	0.0007	2.7729	1.1672 (0.10)	1.3071 (0.019)	0.0555	4.1862	1.3485 (0.17)
$\widehat{\beta}_4$	1.1937 (0.024)	0.2781	2.8355	1.2536 (0.77)	1.1817 (0.160)	0.0019	3.1832	1.2903 (0.32)	1.3386 (0.052)	0.3094	2.6793	1.3794 (0.57)

τ	0.85				0.90				0.95			
	CBQR	Credible Intervals		CQR	CBQR	Credible Intervals		CQR	CBQR	Credible Intervals		CQR
		2.5^{th}	97.5^{th}			2.5^{th}	97.5^{th}			2.5^{th}	97.5^{th}	
$\widehat{\alpha}_0$	0.4857 (0.05)	-0.0703	1.7361	0.4948 (0.12)	0.2716 (0.082)	0.037	0.8836	0.4952 (0.44)	0.4803 (0.09)	0.1963	0.5527	0.5855 (0.61)
$\widehat{\alpha}_1$	1.0534 (0.018)	0.0284	1.9703	0.8897 (0.06)	0.7725 (0.050)	0.3392	1.9803	0.8969 (0.27)	0.9774 (0.586)	0.164	3.1776	1.003 (0.27)
$\widehat{\beta}_1$	1.3298 (0.027)	0.5103	3.0561	1.1285 (0.428)	1.0449 (0.063)	1.0024	2.9467	1.2948 (0.31)	1.1854 (0.035)	0.437	2.9946	1.3628 (0.05)
$\widehat{\beta}_2$	-0.0618 (0.06)	-0.0028	0.0592	-0.0686 (0.92)	-0.0596 (0.205)	-0.0503	0.3752	-0.0748 (0.02)	-0.0345 (0.062)	-0.0069	0.0097	-0.0576 (0.37)
$\widehat{\beta}_3$	1.2848 (0.082)	0.1078	2.6422	-0.0182 (0.20)	1.0096 (0.018)	0.1168	2.8746	1.2978 (0.10)	1.3625 (0.017)	0.2774	3.4476	1.3571 (0.62)
$\widehat{\beta}_4$	1.3751 (0.007)	1.0013	3.153	1.2405 (0.17)	1.1849 (0.032)	1.0016	2.1748	1.2918 (0.74)	1.3856 (0.048)	1.0925	3.883	1.3853 (0.53)

Table 2: Empirical data MSE of quantile causality parameter estimates.

τ	CBQR	CQR
0.05	0.0064	0.00258
0.1	0.0008	0.00165
0.15	0.0049	0.0022
0.2	0.0015	0.00315
0.25	0.0053	0.072
0.35	0.006	0.0376
0.4	0.031	0.0295
0.45	0.0016	0.0826
0.5	0.002	0.0945
0.55	0.0032	0.0413
0.6	0.0061	0.019
0.65	0.0096	0.0364
0.7	0.0008	0.0266
0.75	0.0047	0.081
0.8	0.0006	0.0297
0.85	0	0.061
0.9	0.0002	0.0294
0.95	0.0005	0.0233

ables studied are elastic indicating that their variations will yield proportionate change in the level of GDP in Nigeria. From the summary of the empirical results, it was revealed that Bayesian estimate results provide exact estimation which duly accounts for parameter uncertainty which is all consistent with the theoretical results as it was established by [8]. The mean of the posterior is similar to those based on quantile regression, indicating the approach is practical and parameter uncertainty has been established.

Government should maintain stable macroeconomic policies by strengthening appropriate economic policies that will reduce the problem associated with the high inflation rate and exchange rate.

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