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FIXED POINT THEOREMS ON EXPANSION TYPE MAPS IN INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

Ever since the introduction of fuzzy sets by Zadeh [1] , the fuzzyness invaded almost all the branches of crisp mathematics. Deng [3] kaleva and Seikalla [2] and Kramosil and Michalek [5]Have introduced the concept of fuzzy metric space, George and Veeramani [4] modified the concept of fuzzy metric space introduced by kramosil and michalek [5]. In thia paper effort has been made to obtain some results on fixed points of expansion type mapping in Intuitionistic fuzzy metric space

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [1] . Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek [5], George and Veeramani [4] modified the notion of fuzzy metric space with the help of continuous t-norms.

As a generalization of fuzzy sets, Atanassov [21] introduced and studied the concept of intuitionistic fuzzy sets. Using the idea of intuitionistic fuzzy sets Park [16] defined the notion of intuitionistic fuzzy metric space with the help of continuous t norm and continuous t conorm as a generalization of fuzzy metric space , George and Veeramani [4] had showed that every metric induces an intuitionistic fuzzy metric and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete. Choudhary [22] introduced mutually contractive sequence of self maps and proved a fixed point theorem . Kramosil and michalek [5] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach [10] . Turkoglu et al [20] gave the generalization of Jungek,s common fixed point theorem [24] to intuitionistic fuzzy metric space.

PRELIMINARIES

Definition 2.1 [7] A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if

([0,1], $*$) is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever

$a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples of t-norms are $a * b = ab$ and $a * b = \min \{a, b\}$

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Definition 2.2 [7] A binary operation $\lozenge : [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-conorm if $\lozenge$ is satisfying the following condition

2.2.1 $\lozenge$ is commutative and associate

2.2.2 $\lozenge$ is continuous

2.2.3 $a \lozenge 0 = a$ for all $a \in [0,1]$

2.2.4 $a \lozenge b \leq c \lozenge d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 2.3 [4] the 3-tuple (X, M, $*$) is called a fuzzy metric space (FM-space) if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$.

2.3.1 $M(x, y, 0) > 0$

2.3.2 $M(x, y, t) = 1, \forall\, t > 0 \; iff \; x = y$

2.3.3 $M(x, y, t) = M(y, x, t),$

2.3.4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

2.3.5 $M(x, y, \ .) : [0, \infty] \rightarrow [0,1]$ is continuous.

Remark 2.1 since $*$ is continuous, it follows from (2.3.4) that the limit of a sequence in FM-space is uniquely determined

Definition 2.4 [16] A five –tuple (X, M, N, $*, \lozenge$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set $*$ is a continuous t – norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

2.4.1 $M(x, y, t) + N(x, y, t) \leq 1$

2.4.2 $M(x, y, t) > 0$

2.4.3 $M(x, y, t) = M(y, x, t)$

2.4.4 $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

2.4.5 $M(x, y, .): (0, \infty) \to (0,1)$ is continuous

2.4.6 $N(x, y, t) > 0$

2.4.7 $N(x, y, t) = N(y, x, t)$

2.4.8 $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$

2.4.9 $N(x, y, .): (0, \infty) \to (0,1]$ is continuous

Then (M, N) is called an intuitionistic fuzzy metric on X , the function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively

Remark 2.2 Every fuzzy metric space (X, M, $*$) is an intuitionistic fuzzy metric space form$X, M, 1 - M,*,\lozenge$) such that t-norm $*$ and t-conorm $\lozenge$ are associated ie $x \lozenge y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in [0,1]$ but the converse is not true

Example 2.1 (induced intuitionistic fuzzy metric space [16])

Let (X,d) be a metric space denote $a * b = ab$ and $a \lozenge b = \min\{1, a + b\}$ for all $a, b \in [0,1]$ and let M, and N be fuzzy sets on $X^2 \times (0,1)$ defined as follows

$$M(x, y, t) = \frac{t}{t + d(x,y)}$$

$$N(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$$ then

$(X, M, N,*\lozenge)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric

***R*emark2.3** note that the above example holds even with the t-norm $a * b = \min\{a, b\}$ and the t-conorm

$a \lozenge b = \max\{a, b\}$

***E*xample 2.2** Let $X = N$ define $a * b = \max\{0, a + b - 1\}$ and $a \lozenge b = a + b - ab$ for all $a, b \in [0,1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ as follows

$M(x, y, t) = \{ \frac{\frac{x}{y} if\ x \le y}{\frac{y}{x} if\ y \le x}$

$N(x, y, t) = \{ \frac{\frac{y-x}{y} if\ x \le y}{\frac{x-y}{x} if\ y \le x}$

$f$or all $x, y \in X$ and $t > 0$ then $(X, M, N, *, \lozenge)$ is an intuitionistic fuzzy metric space

Remark 2.4 Note that in above example, t-norm $*$ and t- conorm $\lozenge$ are not associated, and there exist no metric d on X satisfying $M(x, y, t) = \frac{t}{t+d(x,y)}$, $N(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$

Where $M(x, y, t)$ and $N(x, y, t)$ are defined in above example, also note that the above functions (M,N) is not an intuitionistic metric with the t-norm and t-conorm defined as $a * b = \min\{a, b\}$ $a \lozenge b = \max\{a, b\}$

Definition 2.5 [16] let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space then

(a)A sequence $\{x_n\}$ in X is said to be convergent x in X if for each $\varepsilon > 0$ and each $t > 0$ there exists $n_0 \in N$ such that $M(x_n, x, t) > 1 - \varepsilon$ and $N(x_n, x, t) < \varepsilon$ for all $n \ge n_0$

(b) An intuitionistic fuzzy metric space in which every caucy sequence is convergent is said to be complete

Lemma 2.1 in intuitionistic fuzzy metric space X , $M(x, y, .)$ is non decreasing and $N(x, y, .)$ is non increasing for all $x, y \in X$

Lemma 2.2 let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space if there exist $k \in (0,1)$ such that $M(x, y, kt) \ge M(x, y, t)$ and $N(x, y, kt) \le N(x, y, t)$ for all $x, y \in X$ then $x = y$

Proof: since $M(x, y, kt) \ge M(x, y, t)$ and

$N(x, y, kt) \le N(x, y, t)$ then we have

$M(x, y, t) \ge M(x, y, \frac{t}{k})$ and

$N(x, y, t) \le N(x, y, \frac{t}{k})$

By reapeted application of above inequality as we have

$M(x, y, t) \ge M\left(x, y, \frac{t}{k}\right) \ge M\left(x, y, \frac{t}{k^2}\right) \ge - - - - M\left(x, y, \frac{t}{k^n}\right) \ge - - - - - - - - -$

And

$N(x, y, t) \le N\left(x, y, \frac{t}{k}\right) \le N\left(x, y, \frac{t}{k^2}\right) \le - - - - - - - - - - N\left(x, y, \frac{t}{k^n}\right) \le - - - -$

For $n \in N$ which tends to 1 and 0 as $n \to \infty$ respectively thus

$M(x, y, t) = 1$ and $N(x, y, t) = 0$ for all $t > 0$ and we get $x = y$

Lemma 2.3 let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space if there exists a number $k \in (0,1)$ such that

$M(y_{n+2}, y_{n+1}, qt) \geq M(y_{n+1}, y_n, t)$ (1)

$N(y_{n+2}, y_{n+1}, qt) \leq N(y_{n+1}, y_n, t)$ for all $t > 0$

And $n = 1,2 ----$ then $\{y_n\}$ is a Cauchy sequence in X

Proof: for $t > 0$ and $k \in (0,1)$ we have

$M(y_2, y_3, kt) \geq M(y_1, y_2, t) \geq M(y_0, y_1, \frac{t}{k})$ or

$M(y_2, y_3, t) \geq M(y_0, y_1, \frac{t}{k^2})$ and

$N(y_2, y_3, kt) \leq N(y_1, y_2, t) \leq N(y_0, y_1, \frac{t}{k})$ or

$N(y_2, y_3, t) \leq N(y_0, y_1, \frac{t}{k^2})$

by simple induction with the condition (1) we have for all $t > 0$ and $n = 0,1,2, ----$

$M(y_{n+1}, y_{n+2}, t) \geq M(y_1, y_2, \frac{t}{k^n})$

$N(y_{n+1}, y_{n+2}, t) \leq N(y_1, y_2, \frac{t}{k^n})$ (2)

$t$hus by (2) and (2.4.5) and (2.4.8) for any positive integer p and real number $t > 0$, we have

$M(y_n, y_{n+p}, t) \geq M\left(y_n, y_{n+1}, \frac{t}{p}\right) * \dots \dots p\ times \dots .. * M(y_{n+p-1}, y_{n+p}, \frac{t}{p})$

$N(y_n, y_{n+p}, t) \leq N\left(y_n, y_{n+1}, \frac{t}{p}\right) \Diamond \dots \dots p\ times \dots \dots \Diamond\ N(y_{n+p-1}, y_{n+p}, \frac{t}{p})$

$w$hich$\rightarrow 1$ and $\rightarrow 0$ as $n \rightarrow \infty$ respectively thus $lim_{n\rightarrow\infty} M(y_n, y_{n+p}, t) = 1$ and $lim_{n\rightarrow\infty} N(y_n, y_{n+p}, t) = 0$

Which implies that $\{y_n\}$ is a Cauchy sequence in X this completes the proof

RESULTS

Theorem 3.1 let $(X, M, N, *, \Diamond)$ be a complete IFM-space and f be a self map of X, onto itself there exist a constant $k > 1$ such that

$M(f_x, f_y, kt) \leq M(x, y, t)$ and

$N(f_x, f_y, kt) \geq N(x, y, t)$ (1)

$f$or all $x, y \in X$ and $t > 0$. Then f has a unique fixed point in X

Proof: let $x_0 \in X$ as f is onto, there is an element $x_1 \in f^{-1}x_0$. In the same way $x_n \in f^{-1}x_{n-1}$. For all $n = 2,3,4 \dots \dots \dots ..$ thus we get a sequence $\{x_n\}$, if $x_m = x_{m-1}$ for some m then x_m is a fixed point of f now suppose

$x_n \neq x_{n-1}$ for all $n = 1,2 \dots \dots \dots$ then it follows from (1) that

$M(x_n, x_{n+1}, kt) = M(f_{x_{n+1}}, f_{x_{n+2}}, kt) \leq M(x_{n+1}, x_{n+2}, t)$ and
$N(x_n, x_{n+1}, kt) = N(f_{x_{n+1}}, f_{x_{n+2}}, kt) \geq N(x_{n+1}, x_{n+2}, t)$ for all $t > 0$ and for all $N = 0,1,2$ ..... therefore by **lemma 2.3** $\{x_n\}$ is a Cauchy sequence in X since X is complete $\{x_n\}$ has limit $u \in X$ as f is onto there is an element $v \in X$ such that $v \in f^{-1}u$. Now
$M(x_n, u, kt) = M(f_{x_{n+1}}, f_v, kt) \leq M(x_{n+1}, v, t)$ and
$N(x_n, u, kt) = N(f_{x_{n+1}}, f_v, kt) \geq N(x_{n+1}, v, t)$
which as $n \to \infty$ gives $M(u, v, t) = 1$ and $N(u, v, t) = 0$ for all $t > 0$, therefore by (IFM 2.4.2) it follows that $u = v$ yielding thereby $fu = u$ and so u is the fixed point of f . let u and v be the two fixed points of f ie $fu = u$ and $fv = v$ then (1) yields
$M(u, v, kt) = M(fu, fv, t) \leq M(u, v, t)$ and
$N(u, v, kt) = N(fu, fv, t) \geq N(u, v, t)$ for all $t > 0$ hence in view of lemma 2.2 we obtain $u = v$ which shows the uniqueness of u as a fixed point of f this completes the proof

theorem 3.2 let $(X, M, N, * \lozenge)$ be a complete IFM- space with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ for all $t \in [0,1]$ and f be mapping from X onto itself there exist a number $k > 1$ such that

$M(f_x, f_y, kt) \leq M(x, y, t) * M(x, f_x, t) * M(y, fy, t)$ (1) and
$N(f_x, f_y, kt) \geq N(x, y, t) \lozenge N(x, f_x, t) \lozenge N(y, fy, t)$
for all $x, y \in X$ and $t > 0$ then f has a unique fixed point in X

Proof : a sequence $\{x_n\}$ is developed similarly as in theorem 3.1, if $x_{m-1} = x_m$ for some m, f has a fixed point x_m , suppose $x_{n-1} \neq x_n$ for every positive integer n then from (1)

$M(x_n, x_{n+1}, kt) = M(f_{x_{n+1}}, f_{x_{n+2}}, kt)$
$\leq M(x_{n+1}, x_{n+2}, t) * M(x_{n+1}, f_{x_{n+1}}, t) * M(x_{n+2}, f_{x_{n+2}}, t)$
$= M(x_{n+1}, x_{n+2}, t) * M(x_{n+1}, x_n, t) * M(x_{n+2}, x_{n+1}, t)$ and
$N(x_n, x_{n+1}, kt) = N(f_{x_{n+1}}, f_{x_{n+2}}, kt)$
$\geq N(x_{n+1}, x_{n+2}, t) \lozenge N(x_{n+1}, f_{x_{n+1}}, t) \lozenge N(x_{n+2}, f_{x_{n+2}}, t)$
$= N(x_{n+1}, x_{n+2}, t) \lozenge N(x_{n+1}, x_n, t) \lozenge N(x_{n+2}, x_{n+1}, t)$
yielding thereby
$M(x_n, x_{n+1}, kt) \leq M(x_n, x_{n+1}, t) * M(x_{n+2}, x_{n+1}, t)$ and
$N(x_n, x_{n+1}, kt) \geq N(x_n, x_{n+1}, t) \lozenge N(x_{n+2}, x_{n+1}, t)$ (2)
now suppose
$M(x_{n+1}, x_{n+2}, t) < M(x_n, x_{n+1}, t)$ and
$N(x_{n+1}, x_{n+2}, t) > N(x_n, x_{n+1}, t)$

For all $t > 0$ then in view of lemma 2.3 $\{x_n\}$ is a Cauchy sequence in X which is complete therefore there exists some $u \in X$ such that $x_n \to u$ since f is onto there is an element $v \in f^{-1}u$ now

$M(x_n, u, kt) = M(f_{x_{n+1}}, f_v, kt)$

$\leq M(x_{n+1}, v, t) * M(x_{n+1}, x_n, t) * M(v, u, t)$ and

$N(x_n, u, kt) = N(f_{x_{n+1}}, f_v, kt)$

$\geq N(x_{n+1}, v, t) \lozenge N(x_{n+1}, x_n, t) \lozenge N(v, u, t)$

Which as letting $n \to \infty$ gives $M(u, v, t) = 1$ for all $t > 0$

Therefore by (IFM 2.4.2) it is noting that $u = v$ and so $fu = u$ ie u is a fixed point of f the uniqueness of u can be shown easily from (1) hence the theorem proved

Theorem 3.3 let $(X, M, N, *, \lozenge)$ be a complete IFM- space with $t * t \geq t$ and $(1 - t) \lozenge (1 - t) \leq (1 - t)$ for all $t \in [0,1]$ and f,g be two self maps of X onto itself if there exist a number $k > 1$ such that

$M(f_x, g_y, kt) \leq M(x, y, t) * M(x, f_x, t) * M(y, g_y, t)$ and

$N(f_x, g_y, kt) \geq N(x, y, t) \lozenge N(x, f_x, t) \lozenge N(y, g_y, t)$ (1)

for all $x \in X$ and $t > 0$ then f has a unique common fixed point in X

proof: choose an element $x_0 \in X$ as f is onto there is an element $x_1 \in f^{-1}x_0$ since g is onto there exist an element x_2 satisfying $x_2 \in g^{-1}x_1$ thus in general a sequence $\{x_n\}$ is defined as $x_{2n+1} \in f^{-1}x_{2n}, x_{2n+2} \in g^{-1}x_{2n+1}$, for all $n = 0,1,2, \ldots \ldots \ldots \ldots$ now we have two cases as follows

case (1) when $x_m \neq x_{m+1}$ for all $m = 0,1,2 \ldots \ldots$ in this case it follows from (1) that

$M(x_{2n}, x_{2n+1}, kt) = M(f_{x_{2n+1}}, g_{x_{2n+2}}, kt)$

$\leq M(x_{2n+1}, x_{2n+2}, t) * M(x_{2n+1}, x_{2n}, t) * M(x_{2n+1}, x_{2n+2}, t)$

$\leq M(x_{2n+1}, x_{2n+2}, t) * M(x_{2n}, x_{2n+1}, t)$ and

$N(x_{2n+1}, x_{2n+2}, t) = N(f_{x_{2n+1}}, g_{x_{2n+2}}, kt)$ (2)

$\geq N(x_{2n+1}, x_{2n+2}, t) \lozenge N(x_{2n+1}, x_{2n}, t) \lozenge N(x_{2n+1}, x_{2n+2}, t)$

$\geq N(x_{2n+1}, x_{2n+2}, t) \lozenge N(x_{2n}, x_{2n+1}, t)$

Suppose

$M(x_{2n+1}, x_{2n+2} kt) < M(x_{2n}, x_{2n+1}, t)$ and

$N(x_{2n+1}, x_{2n+2} kt) > N(x_{2n}, x_{2n+1}, t)$

then from (2) we obtain

$M(x_{2n}, x_{2n+1}, kt) \leq M(x_{2n}, x_{2n+1}, t)$ and

$N(x_{2n}, x_{2n+1}, kt) \geq N(x_{2n}, x_{2n+1}, t)$

Which in view of lemma(2.2) implies $x_{2n} = x_{2n+1}$ which is a contradiction therefore

Let $M(x_{2n+1}, x_{2n+2}, t) \geq M(x_{2n}, x_{2n+1}, t)$ and
$N(x_{2n+1}, x_{2n+2}, t) \geq N(x_{2n}, x_{2n+1}, t)$
Then (2) yields
$M(x_{2n}, x_{2n+1}, kt) \leq M(x_{2n+1}, x_{2n+2}, t)$ and
$N(x_{2n}, x_{2n+1}, kt) \geq N(x_{2n+1}, x_{2n+2}, t)$
$f$or all $t > 0$ similarly it can be shown that
$M(x_{2n+1}, x_{2n+2} kt) \leq M(x_{2n+2}, x_{2n+1}, t)$ and
$N(x_{2n+1}, x_{2n+2} kt) \geq N(x_{2n+2}, x_{2n+1}, t)$
$f$or all $t > 0$, thus in general we obtain
$M(x_n, x_{n+1}, kt) \leq M(x_{n+1}, x_{n+2}, t)$ and
$N(x_n, x_{n+1}, kt) \geq N(x_{n+1}, x_{n+2}, t)$
$f$or all $t > 0$ and $n = 0,1,2 \ldots \ldots$ hence in view of lemma (2.3) $\{x_n\}$ is a Cauchy sequence in X which is complete therefore $\{x_n\}$ has a limit point in x since $\{x_{2n}\}$ and $\{x_{2n+1}\}$ are subsequence of $\{x_n\}$,$x_{2n} \to u$ and $x_{2n+1} \to u$ as $n \to \infty$ as f and g are onto there exist $v, w \in X$ satisfying $v \in f^{-1}u$ and $w \in f^{-1}u$ now
$M(x_{2n}, u, kt) = M(f_{x_{2n+2}}, gw, kt)$
$\leq M(x_{2n+1}, w, t) * M(x_{2n+1}, x_{2n}, t) * M(w, gw, t)$
And
$N(x_{2n}, u, kt) = N(f_{x_{2n+1}}, gw, kt)$
$\geq N(x_{2n+1}, w, t) \lozenge N(x_{2n+1}, x_{2n}, t) \lozenge N(w, gw, t)$
Which as $n \to \infty$ gives $M(u, w, t) = 1$, $N(u, w, t) = 0$ for all $t > 0$ then by (IFM 2.4.2) it follows that $u = w$ in the similar pattern taking $x = v$ and $y = x_{2n+2}$ in (1) and therefore proceeding as above we obtain $u = v$ therefore $u = v = w$ which immediately implies $fu = gu = u$ and so u is a common fixed point of f and g now let u and v be two common fixed point of f and g i.e $fu = gu$ and $fv = gv = v$ then
$M(u, v, kt) = M(fu, fv, kt)$
$\leq M(u, v, t) * M(u, fu, t) * M(v, gv, t)$
$= M(u, v, t) * 1 * 1 = M(u, v, t)$ and
$N(u, v, kt) = N(fu, fv, kt)$
$\geq N(u, v, t) \lozenge N(u, fu, t) \lozenge N(u, fu, t)$
$= N(u, v, t) \lozenge 0 \lozenge 0 = N(u, v, t)$
For all $t > 0$ further by application of lemma (2.2) we obtain $u = v$
Case II : when $x_{m-1} = x_m$ for some m here m may be even or odd, positive integer without loss of generality suppose m is an integer say $m = 2p$ then $x_{2p-1} = x_{2p}$ i.e

$g_{x_{2p}} = f_{x_{2p-1}}$ which implies $x_{2p} = x_{2p+1}$ (as we have $fx \neq fy$ if $x \neq y$) therefore we have $x_{2p-1} = x_{2p} = x_{2p+1} = \cdots$ which shows that $\{x_n\}$ is convergent sequence and so Cauchy sequence in X the rest of the proof is similar to as in case (I) and this complete the proof

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