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Effectiveness of Digital Pedagogy in Higher Mathematics Education

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Abstract

The use of digital technology in education has revolutionized traditional teaching methods, particularly in mathematics education. This study aims to examine the effectiveness of digital pedagogy in higher mathematics education. The research question focuses on analyzing the effectiveness of digital pedagogy on students' APOS-based learning achievements in higher mathematics. The study used a quantitative research approach, employing a critical Action Research design with pre-and post-test measures to assess the effectiveness of digital pedagogy. The study participants were 126 third-semester students taking a course "Differential Geometry". The used tools are the DP model and mathematics achievement test (MAT). Based on the analysis of data, it is found that DP is effective to enhance APOS-based students learning. So, the study concludes that Digital Pedagogy in higher mathematics education provides insights into how educators can leverage technology to enhance student learning outcomes.

Keywords: Digital Pedagogy, APOS, Mathematics Education, Nepal

1. Introduction

Higher mathematics is abstract by its nature. Many researchers (e.g., Durand-Guerrier, 2016; Lockwood, Ellis, & Lynch, 2016; Zhen, Weber, & Mejia-Ramos, 2016; Dawkins & Roh, 2016) have accepted this fact. It is also proof-oriented mathematics. Proof is an essential component of doing, communicating, and constructing higher mathematics. It is clearly mentioned in the work of Stylianou, Blanton, & Rotou (2015) that the “essence of higher mathematics lies in proofs”. Therefore, teaching and learning in higher mathematics mostly exist with definition-theorem-proof, definition-theorem-proof-based prototype style exist.

Mathematical abstraction can be materialized in a number of ways. Several types of research have demonstrated that mathematical abstraction can be materialized. For example, using examples (Durand-Guerrier, 2016; Lockwood et al., 2016), using graphical images (Zhen et al., 2016), using analogy (Dawkins & Roh, 2016)**,** and using metaphor (Durand-Guerrier, 2016)**,** researchers have tried to materialize mathematical abstraction. Some researchers have tried conceptual and ideational reasoning (Soto-Johnson, Hancock, & Oehrtman, 2016), metalinguistic and mathematical reasoning (Dawkins & Roh, 2016), procedural and conceptual reasoning (Bagley & Rabin, 2016), syntactic and semantic reasoning, cognitive and metacognition reasoning (Mejía-Ramos, Weber, & Fuller, 2015) to materialize proof oriented mathematics meaningfully.

Researcher teaches mathematics courses to master’s students in a teacher education programme at TU and covers many aspects from higher dimensions and abstract concepts. For example, in the DG course, it is hard for him to let students conceptualize that the “tangent line has two-point contact with the curve”. Similar dynamics exist to him to let students understand that profs of the osculating plane and osculating circle have three-point contact with the curve, and the osculating sphere has four-point contact with the curve. So, the researcher feels it is necessary to materialize proof of higher mathematical abstraction.

In this new era of advanced digital technology, it may be a way forward that students need to consolidate their comprehension of mathematical concepts through visual pictures. The researcher sees that mathematical software made interactive and dynamic learning objects are not tested/documented yet. In this aspect, He sees that there is a space to do pedagogical exercises with digital learning objects to materialize mathematical abstraction. Therefore, he felt necessary to investigate the potential of technology to materialize mathematical proofs and abstraction.

This study, therefore, is aimed at utilizing engaged, interactive, and technology-integrated meaningful digital pedagogy to internalize and formalize mathematical concepts in higher education. In this concern, the objective of this study is "to analyze the effectiveness of Digital Pedagogy in students learning higher mathematics".

2. Conceptual and Theoretical Framework

Digital Pedagogy (DP), in general, refers to the integration of digital media and technology into teaching and learning to emphasize the role of technology in facilitating students' learning. Digital pedagogy involves the use of digital tools and virtual learning environments to enhance students' engagement, creativity, and critical thinking skills in their learning process through a dynamic and interactive learning environment that empowers them to take their own control of their learning. With this, in this guideline, Digital Pedagogy (DP) is conceptualized as a planned pedagogical activity integrating ICT and 21st century skills to form a digital learning environment that enhances learners' autonomy and personalized learning. Preparation of the digital environment within DP consists of core pedagogical activities that should be taken systematically. Virtual Learning Environment (VLE)/Digital Learning Environment consists of a Learning Management System (LMS) with seven pedagogical principles.

1) Learning Contents/Curriculum Mapping
2) Setting Learning Outcomes
3) Selecting and organizing Learning Resources
4) Designing Learning Activities/Assignments
5) Ensuring Learning Communication/Interaction/Discussion
6) Learning Feedback/Support, and
7) Learning Assessment/Evaluation.

In this study, the theoretical perspective for DP is based on a constructivist approach of xpedagogy and cpedagogy. The xpedagogy is primarily based on the interaction with content adopting a constructivism learning approach. At the same time, cpedagogy is primarily based on social media to harness interaction with peers adopting a connectivism learning approach. In addition, the theoretical perspective includes the notion "higher the engagement/interaction higher the learning (social constructivism + tools) (Vygotsky, 1978; Siemens, 2005; Downes, 2005), higher the media, higher the learning (Mayer, 2009; Baddeley, 2007; Paivio, 1990) and based on the framework of APSO theory (Arnon, 2014).

APOS is a acronym for Action, Process, Object, and Schema. APOS is a constructivist-based theory of how mathematical concepts can be learned. Dubinsky, around 1983, began to think about it. He started applying Piaget's reflective abstraction into mathematics. At that time, Dubinsky was particularly interested in the use of computer experiences to help students to construct their mathematical concept. During the period 1985–1988, Dubinsky, with various collaborators, developed pedagogical methods using computer programming to induce students to internalize their actions into the process, processes into objects, and apply the relationships to learn new mathematical concepts as schemes, collectively called APOS (Arnon, 2014).

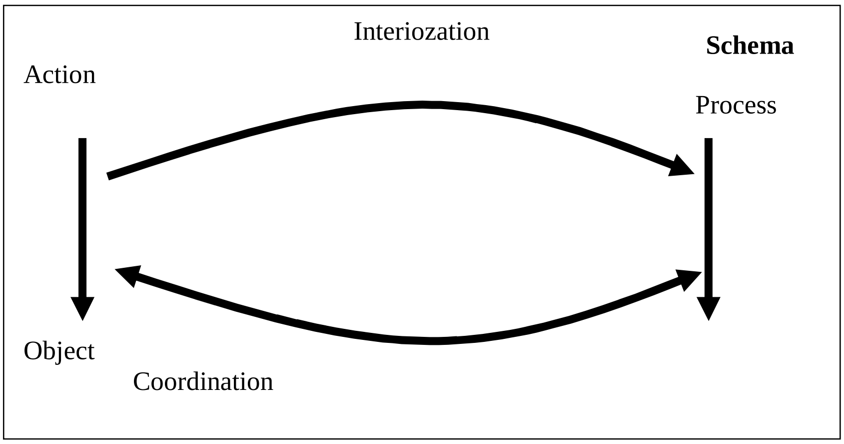

Figure 1: Actions, Processes, Object and Schema through GD

Action is a concept in APOS theory. The concept may be basic or complex, it depends on the context (Arnon et al., 2014). For example, “dividing an interval into specific subintervals of a given size, constructing a rectangle under the curve for each subinterval, calculating the area of each rectangle, and calculating a sum of areas of rectangles” can be an action to learn Riemann integral. Therefore, Action is an external transformation of an object, or manipulation of objects or facts.

Process is a mental structure in APOS theory. The essence of the process is to perform certain operations. The process is usually constructed using one of two mental mechanisms: interiorization or coordination. The main difference between action and process is that, for an action, one must transform (either physically or mentally), but for a process, one must carry out the transformation. For example, “to be able to describe how the Riemann sum is determined decreasing mesh size” is a process to learn the Riemann integral (Arnon et al., 2014). Therefore, Process is internal transformation of an action.

Object is a dynamic structure/connection in APOS theory. This object forms when Action is encapsulated in a Process (Arnon et al., 2014). For example, when one can see area dynamics under a curve and the limit of the Riemann sum, then Object is formed. This Object is formed when Action is applied to Process (Riemann sum) and dynamism is internalized (Arnon et al., 2014). When learners are able to encapsulate the process (Riemann sum) into an Object (Limit of Sum is the area), then the object is formed. Therefore, object forms when students know the process as a totality and identify those transformations acts on it.

Schema is a larger mental structure in APOS theory. It is a new object as connected nodes that forms when Actions are interiorized into Processes, and Process is encapsulated into a mental Object (Arnon et al., 2014). It is complete package connecting how the structures are related and organized into a larger mental structure including Action, Process, and Object. For example, Riemann integral is a schema based on action, process and object as described above (Arnon et al., 2014). The important action for Scheme formation is encapsulation and de-capsulation of Actions applied to a Process and Object with their dynamism. It forms only after genetic decomposition.

Table 1: Examples of APOS

Activities	Explanation	APOS
[1] Dividing an interval into specific subintervals of a given size [2] Constructing a rectangle under the curve for each subinterval. [3] Calculating the area of each rectangle [4] Calculating the sum of areas of rectangles	Action is carried out	A
[1] Can describe how the Riemann sum is determined by decreasing mesh size	Action is automated	P
The area under the curve for a function on a closed interval is the limit of Riemann sums.	Action applied to the Riemann sum (Process)	O
Calculated R-Integral	Student encapsulated the Riemann sum (Process) into an Object	S

Genetic decomposition (GD) in APOS is a model to describe Schema. GD describes specific mental structures in a model required to construct mathematical concepts, in a particular context (Arnon et al., 2014). The model of GD is first outlined, observed, tested in iteration, and validated. For example, in the context of the Riemann integral, if most students “divide an interval into specific subintervals of a given size, construct a rectangle under the curve for each subinterval, calculate the area of each rectangle, and calculate the sum of areas of rectangles” (Arnon et al., 2014). Then it can be considered as an Action in the model. Otherwise, if it is found that students construct the concept differently, than described, then new action needs to be designed until it is validated. The architecture work in a similar manner for Process and Object. And finally, GD will be prepared to describe the model (Arnon et al., 2014).

APOS theory works on spiral hierarchy. It assumes that students can learn any mathematical concept if they are provided with the necessary structures to understand. The main essence of APOS theory is to teach mathematical concepts using a model of the schema (Arnon et al., 2014). Once the model of schema via genetic decomposition is validated, then it can be used as a model for further teaching/learning, and it is supposed to contribute to the construction of the mathematical concept (Arnon et al., 2014). But need to be aware that a genetic decomposition may be different even for the same mathematical concept, in different contexts.

APOS theory is ground to create context-specific model to generalize knowledge construction schema in mathematics in a spiral hierarchy, any content can be learned at any time provided we supply the required backgrounds. It is also tool based, and more importantly, it has also been used to study different complex concepts of mathematics.

3. Methods and Materials

This study is carried out to analyze the effectiveness of DP in higher mathematics education. In this study, Iinquire with pragmatic paradigm because pragmatic paradigm-based research deals with constructed reality. Creswell mentioned that this type of research activity is more problem based and involves appropriate strategies to overcome the existing problem (Creswell, 2014).

This study is based on an experimental sort of educational research method that is “I-we” form Action Research (AR). In general, there are three forms of AR.The researcher avoided the empirical/ technical form of AR (McNiff & Whitehead, 2006) because it tries to maintain the objectivity of the research field to see cause and effect relationship, like ‘if x, then y’, where results are generated by statistical analysis and can be applied and generalized, and replicable in similar conditions. The researcher also avoided the interpretive AR (McNiff & Whitehead, 2006), which tries to observe events in natural settings to describe and explain “what is being/doing there” and to understand “what is happening” and negotiate meanings about objectivity. Beyond both AR, the researcher utilized critical action research to analyze the effectiveness of DP. It is argued that critical AR is a collaborative approach of inquiry to engage “subjects: researcher” and “objects: participants” with an interest in common problems. Therefore, one cycle of AR was carried out in CDED during the academic year 2016 in a course Differential Geometry. Usually, action research is conducted in cycles, but that is taken for implementers'/practitioners' research. Here, the researcher is in academic research to utilize digital pedagogy for the experiment. This experiment was based on the intact group pre-test and post-test design. One possible way to minimize problems related to having no control group is to measure the same dependent variable in one group of participants before (pretest) and after (posttest) a treatment. Using this type of research design, it is possible to measure scores before and again following treatment, then compare the difference between pretest and post-test scores.

The study participants in this research were students of the Master programme in Mathematics Education at CDED in a course Differential Geometry during the 2017 semester. In this semester, 167 students were enrolled in Mathematics Education. However, 126 students participated in both pre-test and post-test surveys. Since a value of *n* as the sample size is greater than 100, it is enough to use inferential statistics, therefore, 126 students were considered as participants of this study.

The mathematics achievement test (MAT) was utilized as data collectiontool. This MAT was prepared to analyze students’ level of thinking in four levels of the APOS dimension. In each level of APOS (action, process, object, and schema), there was one question in each semantic area. The reliability and validity of the APOS framework were established by a pilot test among five students' written test. To ensure the content and construct validity of MAT, the specification grid was validated through APOS. For this, students’ responses were validated with an interview. For reliability, Cronbach coefficient alpha was calculated, which was 0.81, this indicated a 64% reliability factor, therefore accepted.

4. Results

In this study, the DP access score of 126 students was divided into three levels of percentile points: 33, 67, and 100. These levels were coded as low, moderate, and high levels of DP access.

Table 2: Students three cohorts in DP Access

	DP Access Level (N)			DP Access Level (%)			
Gender	Low	Moderate	High	Low	Moderate	High	Chi
Boys	39	38	34	92	90	81	3.17
Girls	3	4	8	8	10	19	

(N=126 of 167)

MAT was analyzed in relation to students' ability according to APOS theory. For this, students' MAT score was categorized according to their APOS ability group. For example, if a student were in Action level but not in the other three, s/he was tagged as an Action level ability student. If a student were in the Action, and Process level but not in other two, s/he was tagged as a Process level ability student. If a student were in Action, Process, and Object level but not in Schema level, s/he was tagged to object level ability student. If a student were in all four levels, s/he was tagged as Schema level ability student. Otherwise, the student was tagged as a varied-level ability student. The table below shows APOS-based student-level statistics.

Table 3 reported that 3 students (3 boys, 0 girls) were in the Action level ability group. This reporting indicated that 3 students were able to get an average or above score in Action level items, but they were not able to score average or above marks in the other three higher-level items according to the genetic decomposition model of APOS theory. Therefore, these 3 students were considered Action level students.

Table 3: APOS Student Level Statistics

Students	APOS Level				
	Action	Process	Object	Scheme	Varied
N	3	9	24	44	46
%	2	7	19	35	37
Boys	3	8	19	38	43
Girls	0	1	5	6	3

(N=126 of 167)

Table 3 reported that 9 students (8 boys, 1 girl) students were in the Process level ability group. This reporting indicated that 9 students were able to get an average or above score in Action level items and Process level items, but they were not able to score average or above marks in the other two higher level items according to the genetic decomposition model of APOS theory. Therefore, these 9 students are considered as Process level students.

Table 3 reported that 24 (19 boys, 5 girls) students were in Object level ability group. This reporting indicated that 24 students were able to get an average or above score in Action level items, Process level items, and Object level items, but they were not able to score average or above marks in Schema level items according to the genetic decomposition model of APOS Theory. Therefore, these 24 students are considered as Object level students.

Table 3 reported that 44 (38 boys, 6 girls) students were in the Schema level ability group. This reporting indicated that 44 students were able to get an average of the above score in all APOS level items according to the genetic decomposition model of APOS Theory. Therefore, these 44 students are considered as Schema level students.

Table 3 reported that 46 (43 boys, 3 girls) students were in the Varied level ability group. These 46 students have scored average or above marks in at least level of APOS Theory, but the level was not consecutive. Therefore, the results also indicated that 46 students were able to get an average of the above score either in Action level items, Process level items, or Object level items or Schema level items, but not in consecutive order on one level after another higher level.

Student achievement was analyzed based on DP access level. The one-way analysis of variance was applied for further analysis. This analysis across DP access level was conducted to compare pre-test results for mean differences across the groups.

Table 4: DP Access level and Student's pre-test score

Source	Sum of Squares	df	Mean Square	F
Between Groups	0.36	2	0.18	0.85
Within Groups	25.3	123	0.02	
Total	25.66	125		

This analysis of variance given in Table 4 showed that there is no significant difference between the pre-test mean score across the DP access level group of students, F (2,122) = 0.85, p >0.05. This pretest score was the students' score in the second semester, which is the end-semester exam score of Math Ed 527; Projective Geometry. The analysis indicated that three groups of students according to DP access level have similar achievement in pre-test scores (Math Ed 527 Projective Geometry).

Table 5: DP access Level and Student's MAT score

Source	Sum of Squares	df	Mean Square	F
Between Groups	165.06	2	82.53	9.88
Within Groups	1027.74	123	8.36	
Total	1192.80	125		

(N=126 of 167)

After, the DP intervention, the analysis of the variance test was examined to test the students MAT score mean differences across the DP access level group of students'. The test given in Table 5 showed that the difference in the mean achievement of students in three DP access level groups was significant, F(2,125) = 9.88, p <0.05. Therefore, it is indicated that DP intervention has made a significant difference in students' MAT scores.

In addition, a chi-square test of independence was performed to examine the relationship between DP access level and students' APOS level mathematics understanding.

Table 6: DP access and Student's APOS Level of Learning

M-VLE Access Level	APOS Level					Chi
	Action	Process	Object	Scheme	Varied	
Low	0	2	9	6	25	31.77***
Moderate	3	2	11	14	12	
High	0	5	4	24	9	

(N=126 of 167)

Table 6 given above shows that the relation between DP access level and students' APOS level mathematics learning is significant, χ^2 (8, N = 126) = 31.78, p<0.05. This result concluded that there is a significant relationship between DP intervention and students' APOS-level mathematics learning.

To ensure the effectiveness of DP intervention, students' end semester final score of Math Ed 537; Differential Geometry was taken as a post-test result.

Table 7: DP access Level and Student's Post-test score

Source	Sum of Squares	df	Mean Square	F
Between Groups	2.14	2	1.07	3.46
Within Groups	37.17	123	0.31	
Total	39.31	125		

(N=126 of 167)

This analysis of variance showed that there is a significance difference between the post-test mean score across DP access level, F (2,125) = 3.46, p<0.05. It indicated that three groups of students according to DP access level have different achievements in the post-test. It shows that DP intervention is effective to enhance student's students' mathematics learning.

5. Discussion and Conclusion

Digital Pedagogy has been increasingly adopted in mathematics education, particularly in the context of the APOS theory of mathematical learning. The APOS theory proposes that students' understanding of mathematical concepts involves four stages: actions, processes, objects, and schemas. Digital pedagogy is found effective in promoting students’ learning at each of these stages.

Research has also shown that digital pedagogy can be effective for APOS mathematics learning. For example, a study by Koichu et al. (2019) found that a digital pedagogy intervention improved students' understanding of calculus concepts in terms of the APOS theory. The intervention included interactive animations, simulations, and problem-solving activities that allowed students to explore and manipulate mathematical concepts. Also, another study by Oliveira and Tall (2017) explored the use of dynamic geometry software in promoting APOS learning in geometry. The software allowed students to manipulate geometric figures and explore geometric properties, which facilitated their development of APOS schemas. The authors found that the use of dynamic geometry software promoted students' understanding of geometric concepts in terms of the APOS theory. Overall, the effectiveness of digital pedagogy in this study also evidenced that APOS-based mathematics learning is well-supported by Digital Pedagogy by providing seven pedagogical principles 1) Learning Contents/Curriculum Mapping 2) Setting Learning Outcomes 3) Selecting and organizing Learning Resources. 4) Designing Learning Activities/Assignments 5) Ensuring Learning Communication/Interaction/Discussion 6) Learning Feedback/Support, and 7) Learning Assessment/Evaluation. This DP provided interactive and engaging learning experiences that promote exploration and manipulation of mathematical concepts, thus digital pedagogy facilitated students' understanding of mathematics in terms of the APOS theory. Therefore, it is concluded that DP intervention is effective to enhance students' mathematics learning.

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