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# Determination of Variation of Mass with Gravity 

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#### Abstract

The paper demonstrates that the theory of mass and gravity in the existing normally acknowledged version of theory of gravitational red-shift is completely erroneous. What makes it erroneous is that the frequency of particle in a gravitational field is considered to change depending on its location while mass of particle is treated as constant parameter. This paper predicts that a body in a gravitational field of escape velocity $V_{e}$ will be more massive by a factor of $1+\frac{V_{e}{ }^{2}}{C^{2}}$, where C is the speed of light, as compared to mass of a similar body outside the field. It manifests that mass of a body measured at earth surface is different than actual mass because of the influence of gravity. If $m^{\prime}$ and $m$ denote mass of body within and beyond the gravitational field respectively, then the mass ( $m^{\prime}$ ) measured at earth surface as compared to actual mass ( m ) is given by the formula $m^{\prime}=\mathrm{m}$ $\left(1+1.39 \times 10^{-9}\right)$. It reveals that an actual relative mass change at earth $m^{\prime}-\mathrm{m} / \mathrm{m}=1.39 \times 10^{-9}$ is extremely small, with change measure in nanogram. Based on this postulate of variation of mass with gravity, the correct criteria for a particle to be inside black hole turns out to be $m^{\prime}>2 \mathrm{~m}$. Therefore, a gravitational field is a black hole if the mass of a body inside the field will be more than two times of its actual mass. In this highly interesting topic, the particular purpose is to present a succinct and carefully reasoned account of new aspect of gravitational red shift which properly allows to determine the change of mass with gravity.


Keywords: Black hole, Gravitational red-shift, Mass-energy equivalence, Photon energy, Schwarzschild radius.

## 1. INTRODUCTION

In 1900, Max Plank introduced the hypothesis of the quantization of energy: the radiation of frequency $v$ can only exchange energy with matter in discrete units (quanta) of energy [1, 2]
$E=\mathrm{h} v$
Where $\mathrm{h}=6.626 \times 10^{-34} \mathrm{~J}$ s is a universal constant called the Plank constant. Also, Albert Einstein's theory of special relativity [3-6] that expresses the fact that mass and energy are the same physical quantity and can be changed into each other. In the equation,
$\mathrm{E}=m C^{2}$
Where $\mathrm{C}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ is the speed of light. From equation (1) and (2), we have
$m C^{2}=\mathrm{h} \nu$

Here, a photon of frequency $v$ falling in gravitational field gains energy. This gain in energy is manifested as an increase in frequency [7, 8]. As a result, the term containing frequency on the right hand side of equation (3) get change. Since C and h are constant, the term containing photon's mass on the left hand side of equation (3) must therefore be changed to equalize the equation (3), but there is large literature on red-shift [9-12] that neglect the corresponding change in the photon's mass. The behavior of particle nature in gravitational field have not been accounted and it is assumed that the corresponding particle nature (mass of Particle) remains invariant under the effect of gravity. Therefore, these literature have to be modified to give a satisfactory explanation of red-shift for measurement of mass in gravitational field.
de-Broglie [13] suggested that material particle such as electron proton etc. should possess dual
nature like light. According to gravitational redshift (known as Einstein shift in older literature [14, 15], all light in the presence of a gravitational field shifts its frequency and wavelength. Consequently, wave nature of light get change due to change in frequency [16, 17]. It does not describe the particle nature of photon under the influence of gravitational field, although particle and wave nature are interrelated form of photon [18]. Also, the special theory of relativity (STR) provides the clear description of the temporal and spatial phenomenon in inertial frame of reference [19]. In the theory, space, time and mass are relative physical quantities which vary with velocity. On other hand, general theory of relativity (GTR) extends the range of applicability of STR and determines the spatial and temporal behaviour in non-inertial frame of reference [20]. Length and time are treated as variable parameter in both STR and GTR. However mass of particle is treated as constant parameter only in GTR which leads to contradiction between GTR and STR. Therefore, mass of particle must be change in gravitational field as that of gravitational time dilation and space contraction. However, until now we do not have any mathematical solutions appropriate for such problems. Here, we describe corresponding gravitational red-shift that apply for simultaneous measurement of particle and wave nature. If mass is the measurement of particle nature and frequency is measurement of wave nature, then for the new theories, we have simply,
$m^{\prime}=\mathrm{m}\left(1+\frac{V_{e}^{2}}{C^{2}}\right), \quad v^{\prime}=v\left(1+\frac{V_{e}^{2}}{C^{2}}\right)$
where, $(\mathrm{m}, v)$ and $\left(m^{\prime}, v^{\prime}\right)$ denote mass and frequency of particle possessing dual nature outside and inside of gravitational field of escape velocity $V_{e}$. For new theories, unlike Einstein shift, there are obvious point of identification such as criteria for mass, frequency of particle to be in black hole. The purpose of this paper is to present a brief but careful derivation of gravitational red shift for precise and simultaneous measurement of wave and particle nature of matter and photon.
The structure of the remainder of this paper is organized as follows. The relative frequency, wavelength and momentum change of particle in gravitational field is presented into a theoretical form in section2, which describes the mathematical criteria for a particle to be inside black-holes. The theoretical model is based on gravitational red shift. Result and discussions are found in section3, which explains the mechanism behind change of mass at
earth and black-holes due to the influence of gravitational field. In section4, the concluding remarks are presented.

## 2. METHODS

### 2.1 Determination of relative momentum change

Let us consider a particle of mass $m$ moving with a velocity V has a wave associated with it whose wavelength (according to de-Broglie) is given by,
$\lambda=\frac{h}{p}=\frac{h}{m V}$.
If T and $V_{p}$ are the kinetic and potential energy of particle respectively, then total energy E is given by,
$\mathrm{E}=\mathrm{T}+V_{p}$
For free particle $V_{p}=0$
$\mathrm{E}=\mathrm{T}=\frac{m V^{2}}{2}$.
If $v$ is the frequency of the associated matter wave. then from quantum expression
$\mathrm{E}=\mathrm{h} \nu$
From equations (5) and (6), we have
$\frac{m V^{2}}{2}=\mathrm{h} \nu$
$\mathrm{m}=\frac{2 \mathrm{~h} v}{\mathrm{~V}^{2}}$
If particle has velocity equal to velocity of light i.e. $\mathrm{V}=\mathrm{C}$, then mass of particle is
$m=\frac{2 \mathrm{~h} \nu}{\mathrm{C}^{2}}$
Consider initially a particle is outside of gravitational field where it experiences zero potential i.e. $V_{p}=0$. let same particle is put in a gravitational field of a planet of mass M and radius R as shown in figure (1).


Fig. 1: Particle in gravitational field of potential of $V_{p}$.

The potential energy of particle of mass $m$ on planet's surface is

$$
V_{p}=\frac{\mathrm{GMm}}{R}
$$

Substituting mass of particle from equation (7), we get,

$$
V_{p}=\frac{2 G M \mathrm{~h} v}{R C^{2}}
$$

This gain in potential energy is manifested as an increase in frequency of particle from $v$ to $v^{\prime}$. Hence, final energy of the particle in gravitational field is
$E^{\prime}=$ Initial energy + potential energy
$\mathrm{h} \nu^{\prime}=\mathrm{h} v+\frac{2 G M \mathrm{~h} v}{R C^{2}}$
$\mathrm{h} v^{\prime}=\mathrm{h} v\left(1+\frac{2 G M}{R C^{2}}\right)$
or, $\quad \frac{v^{\prime}}{v}=1+\frac{2 G M}{R C^{2}}$
or, $\quad \frac{v^{\prime}}{v}-1=\frac{2 G M}{R C^{2}}$
or, $\quad \frac{v^{\prime}-v}{v}=\frac{2 G M}{R C^{2}}$
The relative frequency change is given by
$\frac{d v}{v}=\frac{v^{\prime}-v}{v}=\frac{v^{\prime}}{v}-1=\frac{2 G M}{R C^{2}}$
From relation between wavelength and frequency,

$$
\begin{aligned}
& \mathrm{C}
\end{aligned}=v \lambda
$$

Differentiating both sides,
or, $\quad \mathrm{d} \lambda=\operatorname{Cd}\left(\frac{1}{v}\right)$
or, $\quad \mathrm{d} \lambda=-\frac{C}{v^{2}} \mathrm{~d} v$
or, $\quad \mathrm{d} \lambda=-\lambda \frac{d v}{v}$
or, $\frac{d \lambda}{\lambda}=-\frac{d v}{v}$
Now, equation (8) becomes,
$\frac{d \lambda}{\lambda}=-\frac{2 G M}{R C^{2}}$
Equation (9) gives relative wavelength change of particle in gravitational field. Also, from equation (4),
$\lambda=\frac{h}{p}$
Differentiating both sides,
or,

$$
\mathrm{d} \lambda=h \mathrm{~d}\left(\frac{1}{p}\right)
$$

or, $\quad \mathrm{d} \lambda=-\frac{h}{p^{2}} \mathrm{~d} p$
or, $\quad \mathrm{d} \lambda=-\lambda \frac{d p}{p}$
or, $\quad \frac{d \lambda}{\lambda}=-\frac{d p}{p}$
Now, equation (9) becomes,
$\frac{d p}{p}=\frac{2 G M}{R C^{2}}$
or, $\frac{d p}{p}=\frac{2 G M}{R^{2} C^{2}} R$
Substituting $g=\frac{G M}{R^{2}}$ we get,
or, $\frac{d p}{p}=\frac{2 g R}{C^{2}}$
or, $\frac{d p}{p}=\frac{V_{e}{ }^{2}}{C^{2}}$
Where, $V_{e}^{2}=2 g R$ be the escape velocity. Equation (10) gives relative momentum change of particle in gravitational field.

### 2.2 Black- Holes

If escape velocity $V_{e} \geq C$ no Particle can leave the gravitational field, since to do so needs greater velocity than velocity of light. A world of this kind cannot radiate and so would be invisible called black-hole in space.
Squaring both sides we get,

$$
\begin{array}{ll} 
& V_{e}^{2} \geq C^{2} \\
\text { or, } & 2 g R \geq C^{2} \\
\text { or, } & \frac{2 G M}{R^{2}} R \geq C^{2} \\
\text { or, } & \frac{2 G M}{R C^{2}} \geq 1
\end{array}
$$

Substituting value of $\frac{2 G M}{R C^{2}}$ from equation (8)
$\frac{d v}{v} \geq 1$
or, $\quad d v \geq v$
It reveal that change in frequency of particle inside black-hole should be greater than its initial frequency. This equation provides required condition for a particle to be inside the black-hole. When relative frequency change is one, the above equation (8) becomes
or, $\quad \frac{d v}{v}=\frac{2 G M}{R C^{2}}$
or, $\quad 1=\frac{2 G M}{R C^{2}}$
or, $\quad R_{S}=\frac{2 G M}{C^{2}}$

This radius of body of mass M is called Schwarz child's radius.
Rewriting above equation
$R_{S}=\frac{2 G M}{C^{2}}$
or, $R_{S}=\frac{2 G M R_{S}{ }^{2}}{R_{S}{ }^{2} C^{2}}$
Substituting $\mathrm{g}=\frac{G M}{R_{S}{ }^{2}}$ we get,
or, $\quad R_{S}=\frac{2 g R_{S}{ }^{2}}{C^{2}}$
or, $\quad R_{s}=\frac{V_{e}{ }^{2} R_{S}}{C^{2}}$
or, $\quad \frac{V_{e}{ }^{2}}{C^{2}}=1$
or, $\quad V_{e}{ }^{2}=C^{2}$

$$
\begin{equation*}
V_{e}=C \tag{11}
\end{equation*}
$$

This is the required condition for the existence of Schwarz child's radius. Thus, the most direct physical interpretation of Schwarz child's radius of the body is that Radius of the body when escape velocity due to gravity is equal to velocity of light.

## 3. RESULTS AND DISCUSSIOS

Consider a photon of frequency $v$, then the energy of photon is $h v$. let m be the equivalent mass of the photonic energy, then from Einstein's mass energy relation
$m c^{2}=\mathrm{h} \nu$
Let this photon is in a gravitational field. Thus there is change in frequency from $v$ to $v^{\prime}$ due to gain of potential energy. Therefore, there must be corresponding change in the equivalent mass of photonic energy from $m$ to $m$, then equation (12) becomes,
$m^{\prime} c^{2}=\mathrm{h} v^{\prime}$
Subtracting equation (12) and (13) we get,
$m^{\prime} c^{2}-m c^{2}=\mathrm{h} v^{\prime}-\mathrm{h} v$
Dividing both sides by $m c^{2}$ we get,

$$
\frac{m^{\prime} c^{2}-m c^{2}}{m c^{2}}=\frac{\mathrm{h} v^{\prime}-\mathrm{h} v}{m c^{2}}
$$

Substituting value of $m c^{2}=\mathrm{h} v$ from equation (12) in right side,
$\frac{m^{\prime} c^{2}-m c^{2}}{m c^{2}}=\frac{\mathrm{h} v^{\prime}-\mathrm{h} v}{\mathrm{~h} v}$
or, $\frac{m^{\prime}-m}{m}=\frac{v^{\prime}-v}{v}$
or, $\frac{m^{\prime}}{m}-1=\frac{v^{\prime}}{v}-1$
Substituting value of $\frac{v^{\prime}}{v}-1$ from equation (8)
$\frac{m^{\prime}}{m}-1=\frac{2 G M}{R C^{2}}$.
Also, from equation (10)
$\frac{d p}{p}=\frac{2 G M}{R C^{2}}$
or, $\quad \frac{p^{\prime}-p}{p}=\frac{2 G M}{R C^{2}}$
or, $\quad \frac{m^{\prime} c-m c}{m}=\frac{2 G M}{R C^{2}}$
or, $\quad \frac{m^{\prime}}{m}-1=\frac{2 G M}{R C^{2}}$
or, $\quad \frac{m^{\prime}}{m}=1+\frac{2 G M}{R C^{2}}$
or, $\quad m^{\prime}=m\left(1+\frac{2 G M}{R C^{2}}\right)$
or, $\quad m^{\prime}=m\left(1+\frac{2 G M R}{R^{2} C^{2}}\right)$
Substituting $\mathrm{g}=\frac{G M}{R^{2}}$ we get,
or, $\quad m^{\prime}=m\left(1+\frac{2 g R}{C^{2}}\right)$
or, $\quad m^{\prime}=m\left(1+\frac{V_{e}^{2}}{C^{2}}\right)$.
Where $V_{e}^{2}=2 \mathrm{gR}$ represents escape velocity of gravitational field. This indicates that the mass of particle in gravitational field depends on escape velocity. Earth's gravitational field also influence the measurement of mass on earth. The escape velocity of earth is equal to $11186 \mathrm{~m} / \mathrm{s}$. putting this value in equation (15), we get

$$
\begin{aligned}
m^{\prime} & =m\left(1+\frac{11186^{2}}{\left(3 \times 10^{8}\right)^{2}}\right) \\
\text { or, } \quad m^{\prime} & =m\left(1+1.39 \times 10^{-9}\right) \\
m & =\frac{m^{\prime}}{\left(1+1.39 \times 10^{-9}\right)}
\end{aligned}
$$

$m^{\prime}$ and m represent mass measured inside and outside of gravitational field respectively. In order to acquire accurate mass, the mass measured on earth need to be divide by the factor $(1+$ $\left.1.39 \times 10^{-9}\right)$. Also, the escape velocity of blackhole must be greater or equal to velocity of light i.e.

$$
\begin{gathered}
V_{e} \geq \mathrm{C} \\
\text { or, } \quad \frac{V_{e}{ }^{2}}{C^{2}} \geq 1
\end{gathered}
$$

From equation (15),

$$
m^{\prime}=m\left(1+\frac{V_{e}^{2}}{C^{2}}\right)
$$

or, $\frac{m^{\prime}}{m}=1+\frac{V_{e}{ }^{2}}{C^{2}}$
or, $\quad \frac{m^{\prime}}{m}-1=\frac{V_{e}{ }^{2}}{C^{2}}$
Using above inequality,
or, $\quad \frac{m^{\prime}}{m}-1 \geq 1$
or, $\quad \frac{m^{\prime}}{m} \geq 2$
or, $\quad m^{\prime} \geq 2 \mathrm{~m}$
This is the condition for a particle to be inside the black hole. This shows that the mass of particle inside the black hole must be equal or greater than two times of its initial mass. Also, for Schwarzschild radius we have from equation (11),

$$
\begin{array}{ll} 
& V_{e}=\mathrm{C} \\
\text { or, } & V_{e}^{2}=C^{2} \\
\text { or, } & \frac{V_{e}^{2}}{C^{2}}=1
\end{array}
$$

Using equation (16)

$$
\begin{array}{ll} 
& \frac{m^{\prime}}{m}-1=1 \\
\text { or, } & \frac{m^{\prime}}{m}=2 \\
\text { or, } & m^{\prime}=2 m
\end{array}
$$

This change in mass of particle in gravitational field occurs when escape velocity of particle is equal to velocity of light. Thus, this mass $m^{\prime}$ can be called as Schwarzschild mass from equation (11).
The de-Broglie wavelength depends upon mass of a particle and influence of the mass due to presence of gravitational field implies the corresponding change in wave nature of particle. A body falling in gravitational field gain energy and the velocity of body at earth surface will be equal to escape velocity of field. From equation (4),

$$
\lambda^{\prime}=\frac{h}{m^{\prime} V_{e}}
$$

$\lambda^{\prime}$ and $m^{\prime}$ 'represent wavelength and mass of falling body or particle at surface of gravitational field. Using equation (15)
$\lambda^{\prime}=\frac{h}{\mathrm{~m}\left(1+\frac{V_{e}{ }^{2}}{C^{2}}\right) V_{e}}$
$\lambda^{\prime}=\frac{\lambda}{1+\frac{V_{e} e^{2}}{C^{2}}}$.

Where $\lambda=\frac{h}{\mathrm{~m} V_{e}}$ represents the wavelength of particle of mass m moving with velocity $V_{e}$. This change in wavelength is in perfect agreement with ordinary gravitational red shift.
Rewriting equation (18) we get,

$$
\begin{array}{ll} 
& \lambda^{\prime}=\frac{\lambda}{1+\frac{V_{e}^{2}}{C^{2}}} \\
\text { or, } & 1+\frac{V_{e}^{2}}{C^{2}}=\frac{\lambda}{\lambda^{\prime}} \\
\text { or, } & \frac{V_{e}{ }^{2}}{C^{2}}=1-\frac{\lambda}{\lambda^{\prime}} \tag{19}
\end{array}
$$

From the relation among velocity $\left(V_{e}\right)$, frequency $\left(v^{\prime}\right)$ and wavelength $\left(\lambda^{\prime}\right)$,

$$
\begin{aligned}
& V_{e}=v^{\prime} \lambda^{\prime} \\
& v^{\prime}=\frac{V_{e}}{\lambda^{\prime}}
\end{aligned}
$$

Using equation (18)

$$
\begin{align*}
v^{\prime} & =\frac{\mathrm{m}\left(1+\frac{V_{e}^{2}}{C^{2}}\right) V_{e}^{2}}{h} \\
v^{\prime} & =v\left(1+\frac{V_{e}^{2}}{C^{2}}\right) \ldots \ldots . \tag{20}
\end{align*}
$$

Where $v=\frac{m V_{e}{ }^{2}}{h}$ represents the frequency of particle of mass $m$ moving with velocity $V_{e}$. This change in frequency is in perfect agreement with ordinary gravitational red shift.
Rewriting equation (20) we get,

$$
v^{\prime}=v\left(1+\frac{V_{e}^{2}}{C^{2}}\right)
$$

or, $\quad 1+\frac{V_{e}{ }^{2}}{C^{2}}=\frac{v^{\prime}}{v}$
or, $\quad \frac{V_{e}^{2}}{C^{2}}=1-\frac{v^{\prime}}{v}$
Equations (18) and (20) give corresponding change in wavelength and frequency due to earth's gravitational field of escape velocity $V_{e}=$ $11186 \mathrm{~m} / \mathrm{s}$ as follows.

$$
\lambda^{\prime}=\frac{\lambda}{1+\frac{V_{e}^{2}}{C^{2}}}
$$

or, $\quad \lambda^{\prime}=\frac{\lambda}{1+\frac{11186^{2}}{\left(3 X 10^{8}\right)^{2}}}$
or, $\quad \lambda^{\prime}=\frac{\lambda}{1+1.39 \times 10^{-9}}$

Similarly, from equation (20),

$$
v^{\prime}=v\left(1+\frac{V_{e}^{2}}{C^{2}}\right)
$$

or, $\quad v^{\prime}=v\left(1+\frac{11186^{2}}{\left(3 \times 10^{8}\right)^{2}}\right)$
or, $\quad v^{\prime}=v\left(1+1.39 \times 10^{-9}\right)$
This equation gives the corresponding change in frequency due to earth's gravitational field. Also, the escape velocity of black-holes must be greater or equal to velocity of light i.e. $V_{e} \geq \mathrm{C}$.
$V_{e}{ }^{2} \geq C^{2}$
$\frac{V_{e}{ }^{2}}{C^{2}} \geq 1$
Using Equation (19) and (22),
$\frac{\lambda}{\lambda^{\prime}}-1 \geq 1$
or, $\quad \frac{\lambda}{\lambda^{\prime}} \geq 2$
or, $\quad \lambda \geq 2 \lambda^{\prime}$
or, $\quad \frac{\lambda}{2} \geq \lambda^{\prime}$.
It is clear that, a gravitational field is a Black hole if the wavelength of a particle inside the field will be less than half of its initial wavelength (outside the field). Also, using equation (21) and (23)

$$
\frac{v^{\prime}}{v}-1 \geq 1
$$

$$
\begin{align*}
& \text { or, } \quad \frac{v^{\prime}}{v} \geq 2 \\
& \text { or, } \quad v^{\prime} \geq 2 v . \tag{24}
\end{align*}
$$

It is clear that, a gravitational field is a black hole if the frequency of a particle inside the field will be more than two times of initial frequency (outside the field) and $\frac{\lambda}{2}=\lambda^{\prime}, v^{\prime}=2 v$ when $V_{e}^{2}=C^{2}$. Thus, this wavelength and frequency can be called as Schwarzschild wavelength and frequency from equation (10).
In order to confirm the variation of mass with gravity, the mass variation formula given by equation (15) is applied on de-Broglie wavelength to determine corresponding wavelength shifts in gravitational field which is given in equation (18). Also, the corresponding frequency shifts due to variation of mass is well accounted in equation (21). This shifts of wavelength and frequency is in perfect agreement with ordinary gravitational red-shift. It manifests that change in frequency takes place if and only if mass variation under the gravity occurs. This variation of mass, wavelength and frequency associated with a particle at earth and black hole have been discussed in this paper and below a few major point have been presented in the table.

Table 1: measurement of mass, wavelength and frequency in the gravitational field.

| S.N. | Escape Velocity | Mass | Wavelength | Frequency |
| :---: | :---: | :---: | :---: | :---: |
| 1 | Gravitational Field ( Escape velocity $=V_{e}$ ) | $m^{\prime}=m\left(1+\frac{V_{e}^{2}}{C^{2}}\right)$ | $\lambda^{\prime}=\frac{\lambda}{1+\frac{V_{e}{ }^{2}}{C^{2}}}$ | $v^{\prime}=v\left(1+\frac{V_{e}^{2}}{C^{2}}\right)$ |
| 2 | Earth (Escape velocity $=11186 \mathrm{~m} / \mathrm{s}$ ) | $\begin{aligned} & \left.m_{\left(1+1.39 \times 10^{-9}\right)}^{\prime}\right) \end{aligned}$ | $\begin{aligned} & \lambda^{\prime}= \\ & \frac{\lambda}{\left(1+1.39 \times 10^{-9}\right)} \end{aligned}$ | $v^{\prime}=v\left(1+1.39 \times 10^{-9}\right)$ |
| 3 | Black Hole $\left(V_{e} \geq C\right)$ | $m^{\prime} \geq 2 \mathrm{~m}$ | $\lambda \geq 2 \lambda^{\prime}$ | $v^{\prime} \geq 2 v$ |
| 4 | Schwarzschild Radius $\left(V_{e}=C\right)$ | $m^{\prime}=2 \mathrm{~m}$ | $\lambda=2 \lambda^{\prime}$ | $v^{\prime}=2 v$ |

## Alternative Method:

From relativistic variation of mass with velocity we have,
$m=\frac{m_{0}}{\sqrt{1-\frac{\mathrm{v}^{2}}{\mathrm{C}^{2}}}}$
$\mathrm{m}=m_{0}\left(1-\frac{\mathrm{V}^{2}}{\mathrm{C}^{2}}\right)^{\frac{-1}{2}}$

As we know,
$(1-x)^{\frac{-1}{2}}=1+\frac{1}{2} x+\frac{3}{8} x^{2}+\frac{5}{16} x^{3}+$. ..........
Putting $x=\frac{\mathrm{V}^{2}}{\mathrm{C}^{2}}$ we get,
$\left(1-\frac{\mathrm{V}^{2}}{\mathrm{C}^{2}}\right)^{\frac{-1}{2}}=1+\frac{1}{2}\left(\frac{\mathrm{v}^{2}}{\mathrm{C}^{2}}\right)+\frac{3}{8}\left(\frac{\mathrm{v}^{2}}{\mathrm{C}^{2}}\right)^{2}+\frac{5}{16}\left(\frac{\mathrm{~V}^{2}}{\mathrm{C}^{2}}\right)^{3}+\ldots \ldots .$.
Neglecting higher terms for $\mathrm{V}<\mathrm{C}$ we get,
$\left(1-\frac{\mathrm{V}^{2}}{\mathrm{C}^{2}}\right)^{\frac{-1}{2}}=1+\frac{1}{2}\left(\frac{\mathrm{~V}^{2}}{\mathrm{C}^{2}}\right)$
From equations (25) and (26),
$\mathrm{m}=m_{0}\left(1+\frac{1}{2}\left(\frac{\mathrm{~V}^{2}}{\mathrm{C}^{2}}\right)\right)$
Let us consider a particle of mass $m$ is at surface of a planet of mass $M$ and radius $R$. In order to escape from gravitational field, the initial kinetic energy of particle must be equal to gravitational binding energy.
Kinetic energy $=$ Gravitational energy
$\frac{m V^{2}}{2}=\frac{\mathrm{GMm}}{R}$
or, $V^{2}=\frac{2 \mathrm{GM}}{R}(28)$
Substituting this value of $V^{2}$ in equation (27),
$\mathrm{m}=m_{0}\left(1+\frac{2 G M}{2 R C^{2}}\right)$
$\mathrm{m}=m_{0}\left(1+\frac{G M}{R C^{2}}\right)$
The de-Broglie wavelength associated with a particle of mass $m$ is given by,
$\lambda=\frac{h}{m V}$
Or, $m V=\frac{h}{\lambda}$
Multiplying both sides by V ,
$m V^{2}=\frac{h V}{\lambda}$
Since, $v=\frac{V}{\lambda}$
$m V^{2}=\mathrm{h} \nu(30)$
In order to escape from gravitational field, the quantum energy $\mathrm{E}=\mathrm{h} v$ must be equal to gravitational potential energy.
Energy of particle = Gravitational energy
$\mathrm{h} v=\frac{\mathrm{GMm}}{R}$
Using equation (30) we get,
$m V_{e}^{2}=\frac{\mathrm{GMm}}{R}$
$V_{e}^{2}=\frac{\mathrm{GM}}{R}(31)$
From equations (29) and (31) we get,
$\mathrm{m}=m_{0}\left(1+\frac{V_{e}{ }^{2}}{C^{2}}\right)$

Above relation suggests at once that mass of particle in gravitational field (m) change by the factor $\left(1+\frac{V_{e}^{2}}{C^{2}}\right)$ as compared to its rest mass $\left(m_{0}\right)$.

## 4. CONCLUSION

The aim of the present paper is to open a discussion towards a new way of formulating the mathematical framework to reveal the variation of mass under the influence of gravitational potential and address to the question of what are the criterion for a particle to be inside the black hole according to variation of mass with gravity. The measurement of gravitational shifts such as change of momentum, frequency have been computed using the proposed variation of mass with gravity. This mathematical formalism suggest the frequency of photon changes with gravity if and only if the corresponding change in mass occurs. Therefore, it has been demonstrated that both frequency and mass must be change to equal Einstein mass energy $\left(m c^{2}\right)$ with photonic energy (hv). On the basis of the proposed mathematical connection, the change of mass and frequency in earth's gravitational field is given by,

$$
m^{\prime}=m\left(1+1.39 \times 10^{-9}\right), v^{\prime}=v\left(1+1.39 \times 10^{-9}\right)
$$

where $\left(m^{\prime}, v^{\prime}\right)$ and $(\mathrm{m}, v)$ denote mass and frequency of particle possessing dual nature inside and outside of gravitational field. If the corresponding variation of mass and frequency is calculated for black-hole, then we may purpose the criterion for particle to be inside black-hole such as,

$$
\mathrm{m}^{\prime} \geq 2 \mathrm{~m}, \quad \mathrm{v}^{\prime} \geq 2 v
$$

Which predicts that the mass $\left(\mathrm{m}^{\prime}\right)$ and frequency $\left(\mathrm{v}^{\prime}\right)$ of particle inside the black hole should be greater than two times of its initial mass $(\mathrm{m})$ and frequency $(v)$ i.e. outside of field.
The primary purpose here has been to provide a deliberate account of change of mass under the influence of gravitational potential and lay down the basic equations of the theories. The determination of gravitational shifts such as change of mass, wavelength and frequency have been thoroughly presented on the table (1). The formulas elucidated here will be of interest in many in other areas of theoretical Physics. Hence, it opens a door for scientist to develop experimental technique for verification of this theory and will play a crucial role to perceive the influence of gravity on mass.

## List of symbols

V Velocity of particle

C Velocity of light
$\mathrm{V}_{\mathrm{e}} \quad$ Escape velocity
$m \quad$ Mass outside the gravitational field
$\mathrm{m}^{\prime} \quad$ Mass inside the gravitational field
$v \quad$ Frequency outside the gravitational field
$\mathrm{v}^{\prime} \quad$ Frequency inside the gravitational field
$\lambda \quad$ Wavelength outside the gravitational field
$\lambda^{\prime} \quad$ Wavelength inside the gravitational field
E Total energy
T Kinetic energy
$\mathrm{V}_{\mathrm{p}} \quad$ Potential energy
h Plank constant
p Linear momentum
M Mass of planet
R Radius of planet
g Acceleration due to gravity
G Gravitational constant

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