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Radiation Shielding Properties of Oxides $(Al_2O_3, PbO, and Fe_2O_3)$ based on Klein-Nishina Cross-section

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ABSTRACT

In this study, we investigate the Klein-Nishina differential cross-section equation for total crosssection and extend it to calculate the total molecular cross-section for compounds Al_2O_3 , PbO, and Fe₂O₃. Our findings reveal that the total molecular cross-section of these compounds is significantly larger, with values 5 times greater than the total atomic cross-section. Furthermore, we determine that the molecular cross-section of Al_2O_3 , PbO, and Fe₂O₃ is 73, 132, and 118 times greater than the total electronic cross-section, respectively, while the atomic cross-section of these compounds is 15, 66, and 24 times greater. At low energy levels ranging from 1-5 MeV, the entire molecular, atomic, and electronic cross-section dominates due to Compton scattering. However, as the photon energy increases, Compton scattering becomes negligible, and a slight contribution from pair production scattering is observed. We also establish the adequate atomic numbers for Al_2O_3 , PbO, and Fe₂O₃, which are determined to be 15, 66, and 24, respectively. These results highlight the significance of mass attenuation, cross-section purposes. The findings from this study provide valuable insights into the properties and behavior of these compounds, enabling informed decisions in radiation shielding applications.

Keywords: Radiation shielding material, Cross-section, Compton scattering, Compound, Mass attenuation.

INTRODUCTION

The Mass attenuation coefficients (MAC) of silver(Ag)/copper(Cu)/zinc(Zn)allov with Ag 14.80%, Cu 57.61%, and Zn 27.59% weight fraction determined in the energy range 220 to 662 keV with gamma rays has good results agree with the theoretical values with error less than 1%. However, MAC decreased with increasing gamma ray's energies because photons interacted with Ag/Cu/Zn alloy [1]. Gamma attenuation behavior on commercial stainless steels and boron steels was observed by [2] using XCOM computer code and found that Theoretical and experimental MAC are closely related. Shield material selections based on the energy of photon and Lead is a highly shielding material with a suitable density of 11.35g/cm³, high atomic number, and inexpensive. Furthermore, MAC, MEAC, and kerma relative to air and kerma values differed between Fe–Ce and Fe-Ni alloys due to photoelectric cross-sections that vary with an atomic number [3].

Several composite materials assigned are d for X-ray and gamma photon interactions as shielding materials. The shielding material thickness for the authors satisfies $2 \le ln \left(\frac{I_0}{I}\right) \le 4$ with transmission $0.5 \ge T \ge 0.25$.MAC for materials composed of various elements, mixture rule, is necessary and expressed by Morabad and Kerur in 2010 as

The probability of a photon interacting with the material per unit path length is called LAC. Photon attenuation coefficients depend on the photon energy and the material density, which is important for radiation shielding [4]. Here w_i , $(\mu_m)_i$ denoted asweight fraction and MAC of ith element and material composed of multi-elements is expressed as,

Here A_i , n_i ; represent the atomic weight of the ith element and the number of formula units. Now, the total molecular cross-section (σ_t^m) it can be written as

$$(\sigma_t^m) = \frac{1}{N_A} \sum_i \left(\frac{\mu}{\rho}\right)_i n_i A_i \dots (3)$$

Also, the total atomic cross-section (σ_t^a) for the element is expressed [5] as

Also, the total electronic cross-section (σ_t^e) for the element is expressed as

Here f_i, Z_i called a fractional abundance of the element ith and atomic number atoms, the Effective Atomic number of the compounds represented [6] as $Z_{eff} = \frac{\sigma_t^a}{\sigma_t^e}$

Theoretical Formulation

The emergent flux Φ of a beam of monochromatic photons with an incident energy E_0 , and a photon flux Φ_0 interacting with a sample of a homogeneous material of thickness x, can be described by the Beer-Lambert equation: $(\Phi = \Phi_0 e^{-\mu x})$. For energy values over 10 keV, when the molecular binding energy is low, it is reasonable to assume that $\mu (cm^{-1})$ is directly proportional to the physical density as $(\Phi = \Phi_0 e^{-(\frac{\mu}{\rho})\rho x})$. Here, $\frac{\mu}{\rho} (cm^2 g^{-1})$ is the mass attenuation coefficient. The linear attenuation coefficient obtained is as,

where N_A is the Avogadro's number $(6.02 \times 10^{23} atom/mol)$, Z is the atomic number, and A is the atomic material mass. In terms of electron density, the linear attenuation coefficient expressed is as $(\mu = \sigma_t^e(E)\delta_e)$. Here σ_t^e It denoted a total cross-section.

Klein-Nishina Cross Section for Compton Scattering

Compton scattering is the primary scattering process observed for 511Kev photons. In this process, the incident gamma-ray interacts with an atomic electron, leading to atomic ionization. The angle of scattering, denoted as θ , is determined by the Klein-Nishina differential cross-section equation.

$$\left(\frac{d\sigma}{d\Omega}\right)_{a} = \frac{Zr_{\theta}^{2}}{2} \left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^{2} \left(\left(1+\cos^{2}\theta\right) + \frac{\alpha^{2}(1-\cos\theta)^{2}}{\left[1+\alpha(1-\cos\theta)\right]}\right) \quad \dots \tag{7}$$

This equation is a differential atomic cross-sectional area equation for K-N. Also, the total K-N crosssection per atom is written as,

$$\sigma_t^a = 2\pi \int_0^\pi \left(\frac{d\sigma}{d\Omega}\right)_a \sin\theta d\theta \dots$$
(8)

Here θ is scattering angle overall photons. Now from (7) and (8), we get

$$\sigma_t^a = 2\pi \int_0^\pi \frac{Zr_e^2}{2} \left(\frac{1}{1+\alpha(1-\cos\theta)}\right)^2 \left((1+\cos^2\theta) + \frac{\alpha^2(1-\cos\theta)^2}{[1+\alpha(1-\cos\theta)]}\right) \sin\theta d\theta \dots (9)$$

On solving the total KN cross-section per atom is obtained as,

$$\sigma_{t}^{a} = Z2\pi r_{0}^{2} \left\{ \frac{1+\alpha}{\alpha^{2}} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+2\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^{2}} \right\}$$

Since Klein-Nishina atomic cross-sections were obtained by multiplying electronic cross-sections with charge number Z of each element that is $\sigma_a = Z \cdot \sigma_e$, therefore from equation (9), the electronic cross-sectional area for KN is

Where, $r_0 = 2.818 \times 10^{-13} m$ is the classical electron radius, Z is the nuclear charge of the target molecule, and $\alpha = \frac{E}{m_e c^2} = \frac{hf}{0.511 MeV}$ [7]. On putting the value of $\sigma_e \ln \frac{\mu}{\rho} = \frac{\sigma_e Z N_A}{A}$ we get,

Therefore, this equation gives mass attenuation coefficient in terms of KN parameters and known as Compton mass attenuation coefficient is provided by using, $\frac{\mu}{\rho} = N_A Z \frac{\sigma_e}{A}$. Where N_A is the Avogadro's number (6.02 × 10²³ atom/mol), Z is the atomic number, and A is the material atomic mass [8].

Shielding Properties of compound Al_2O_3

Now from equation (11) for Al_2O_3 , we have mass attenuation coefficient is obtained as,

The total molecular cross-section of Al_2O_3 from equation (3) and (12) is obtained as

The total atomic cross-section of Al_2O_3 from equation (4) and (12) is obtained as

The electronic cross-section area of Al_2O_3 from (5) and (12) is obtained as

Now, adequate atomic numbers for Al_2O_3 are obtained as $(Z_{eff})_{Al_2O_3} = \frac{(\sigma_t^a)_{Al_2O_3}}{(\sigma_t^e)_{Al_2O_3}} \approx 15$.

Shielding Properties of compound PbO

Now from equation (11) for *Pb*, *O* we have mass attenuation coefficient is obtained as,

$$\left(\frac{\mu}{\rho}\right)_{PbO} = 0.38N_A \ 2\pi r_0^2 \left\{\frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha}\right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2}\right\}.$$
(16)

The total molecular cross-section of PbO from equations (3) and (16) obtained as

$$a(\sigma_t^m)_{Pb0} = 4473.36 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}.$$
(17)

The total atomic cross-section of PbO from equations (4) and (16) obtained as

$$(\sigma_t^a)_{Pb0} = 2236.68 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\} \text{ f.....(18)}$$

The electronic cross-section area of PbO from (5) and (16) calculated as

$$(\sigma_t^e)_{Pb0} = 33.99 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}.$$
(19)

Now, adequate atomic number for *PbO* is accommodating as, $(Z_{eff})_{PbO} = \frac{(\sigma_t^a)_{PbO}}{(\sigma_t^e)_{PbO}} \approx 66.$

Shielding Properties of compound Fe₂O₃

Now from equation (11) for Fe_2O_3 we have mass attenuation coefficient is obtained as,

$$\left(\frac{\mu}{\rho}\right)_{Fe_2O_3} = 0.50N_A \ 2\pi r_0^2 \left\{\frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha}\right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2}\right\}.$$
(20)

The total molecular cross-section of Fe_2O_3 from equation (3) and (20) is obtained as

The total atomic cross-section of Fe_2O_3 from equation (4) and (20) is obtained as

The total electronic cross-section area of Fe_2O_3 from (5) and (20) is obtained as

$$(\sigma_t^e)_{Fe_2O_3} = 33.99 \times 10^{-26} \left\{ \frac{1+\alpha}{\alpha^2} \left[\frac{2(1+\alpha)}{1+2\alpha} - \frac{\ln(1+2\alpha)}{\alpha} \right] + \frac{\ln(1+\alpha)}{2\alpha} - \frac{1+3\alpha}{(1+2\alpha)^2} \right\}$$

Now, adequate atomic numbers for Fe_2O_3 are obtained as $(Z_{eff})_{Fe_2O_3} = \frac{(\sigma_t^a)_{Fe_2O_3}}{(\sigma_t^e)_{Fe_2O_3}} \approx 24.$

RESULT AND DISCUSSION

The adequate atomic number of compounds Al_2O_3 , *PbO* and Fe_2O_3 was obtained as 15, 66, and 24, respectively. The cross-section formula calculation depends upon the Klein-Nishina. Table 1 shows the comparison of the entire molecular, atomic, and electronic cross-sections. The comparison shows

that the total molecular cross-section is greater than the whole atomic and electronic cross-section; the nuclear cross-section is also more significant than the electronic cross-section. To calculate the crosssection area,1MeV to 400MeV energy of incidence photon for compound Al_2O_3 , *PbO*, and Fe_2O_3 .

Comparison of total molecular and atomic Cross section ratio	Comparison of total atomic and electronic Cross section ratio	Comparison of total molecular and electronic cross-section ratio
$(\sigma_t^m)_{Al_2O_3}\approx 5\times (\sigma_t^a)_{Al_2O_3}$	$(\sigma_t^m)_{Al_2O_3}\approx 73\times (\sigma_t^e)_{Al_2O_3}$	$(\sigma^a_t)_{Al_2O_3}\approx 15\times (\sigma^e_t)_{Al_2O_3}$
$(\sigma_t^m)_{PbO} \approx 2 \times (\sigma_t^a)_{PbO}$	$(\sigma_t^m)_{PbO} \approx 132 \times (\sigma_t^e)_{PbO}$	$(\sigma_t^a)_{PbO} \approx 66 \times (\sigma_t^e)_{PbO}$
$\overline{(\sigma_t^m)_{Fe_2O_3}} \approx 5 \times (\sigma_t^a)_{Fe_2O_3}$	$(\sigma_t^m)_{Fe_2O_3} \approx 118 \times (\sigma_t^e)_{Fe_2O_3}$	$(\sigma_t^a)_{Fe_2O_3} \approx 24 \times (\sigma_t^e)_{Fe_2O_3}$

Table 1: Comparison of Cross Section of Compound

Since we are considering cross-section, which depends upon the scattering, the energy of radiation decreases after interaction with the compound, and hence reducing radiation by combination protects us from radiation. The higher cross-section during diffusion with a mix shows better for shielding radiation.

Total Mass Attenuation Coefficient of compound Al_2O_3 , *PbO*, and Fe_2O_3

AC (Attenuation Coefficient) quantifies the extent

of radiation penetration through a material. A higher penetration of radiation corresponds to a lower MAC (Mass Attenuation Coefficient) value, whereas a higher MAC value indicates a greater reduction in radiation penetration. Therefore, when selecting a shield material for radiation, the MAC value can be used as a determining factor. Figure 1 illustrates that the MAC values of Al₂O₃ and Fe₂O₃ compounds are nearly equal, whereas PbO exhibits a different MAC value compared to the other

materials considered. In this study, Fe_2O_3 exhibits a higher MAC value than the other compounds, indicating that it provides superior radiation shielding compared to the other materials under consideration.



Fi. 1: MAC of oxide compound with photon energy.

Total Cross-section Coefficient of compound Al_2O_3 , PbO, and Fe_2O_3

The Klein-Nishina cross-section equation is utilized in Figure 2 to visualize the entire molecular and atomic interaction. The calculations reveal that compound PbO exhibits a lower molecular cross-section compared to others at the same incident photon energy. The total cross-section, represented as $log(\sigma)$, demonstrates that the molecular cross-sections of PbO and Fe₂O₃ are nearly equal. However, a significant shift is observed in the total atomic cross-section, particularly for the Fe₂O₃ compound.

Since the cross-section is influenced by the energy of the incident photons, the power of the photons decreases as they pass through PbO, resulting in a higher cross-section for PbO. This reduction in energy offers radiation protection when PbO is used as a shielding material. Notably, the total electronic cross-section for all compounds is equivalent and exhibits dependence on the incident photon energy.



Fig. 2: Cross-section of oxide compound with incidence energy

The cross-section is higher at low energy (MeV) for all compounds and this range of energy causes Compton scattering. This scattering caused the change in energy of photons after interaction with radiation shielding. Therefore, cross-section is high while with increase the power of incidence photon pair production take place. However, during the pair production, only a few interactions take place; hence the cross-section is low.

CONCLUSION

The calculation reveals that the entire molecular cross-section of each compound is more significant than the atomic and electronic cross-sections. As the incidence energy of photons increases, the cross-section decreases due to the occurrence of Compton scattering at low energy levels. Additionally, with higher photon power, pair production takes place in small quantities, while Compton scattering remains dominant at energies below 1-5MeV. Consequently, Compton scattering is the primary factor in these energy ranges, leading to a high cross-section. The appropriate atomic numbers for Al₂O₃, PbO, and Fe₂O₃ compounds are determined as 15, 66, and 24, respectively. Consequently, by calculating the mass attenuation, cross-section, and appropriate atomic number, one can select the optimal radiation shielding material for various protection purposes.

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