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Journal of Nepal Physical Society
Volume 8, No 2, 2022
(Special Issue: ANPA Conference 2022)
ISSN: 2392-473X (Print), 2738-9537 (Online)

JNPS, **8** (2), 37-41 (2022)
DOI: http://doi.org/10.3126/jnphyssoc.v8i2.50147

Published by: Nepal Physical Society
P.O. Box: 2934
Tri-Chandra Campus
Kathmandu, Nepal
Email: nps.editor@gmail.com

ANPA Conference 2022
ISSN : 2392-473X(Print), 2738-9537 (Online)

Research Article ©Nepal Physical Society
DOI:http://doi.org/10.3126/jnphyssoc.v8i2.50147

Comparative Study of Magnetic Levitation Models

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Abstract. Levitation refers to free flotation, where the levitated object is suspended freely, against gravity without any physical contact. Among many levitation, magnetic levitation due to a finite-sized type-I superconductor was demonstrated and characterized. Here, we have developed a model by extending the two-loop method to calculate the levitation height for magnetic levitation within the superconducting microwave cavity and is compared with widely used mirror and finite-size superconductor method. The models were used to calculate the levitation height from the center and edge of the superconductor for magnet with strength 0.1 - 2.0 T. We observed a large discrepancy between the models for the edge levitation where our model underestimate the levitation height by 40-95%. Furthermore, in contrast to other models, our model has shown a superior capacity to calculate the levitation height at any location on the superconductor.

Received: 31 August 2022; **Accepted**: 18 November 2022; **Published**: 31 December 2022

Keywords: Superconductor; Levitation; Mirror method; Two-loop model

INTRODUCTION

An object levitates when the upward pushing levitation force balance out the downward pulling gravitational force [1]. The gravitational force depends only on the mass of the object.The source of the levitation force is related more with the microscopic property of the material [2]. For example, in the optical levitation, levitating train, and superconducting levitation the sources of levitation force are, respectively, angular momentum of the input field, repulsive magnetic field, and the Meissner effect [3]. The Meissner effect is associated with the complete expulsion of the magnetic flux from the interior of the superconductor [4]. Here, the necessary condition being an applied magnetic field is less than the critical field of the superconductor [5] and the boundary condition is that the perpendicular component of the magnetic field is zero on the superconductor's surface [6]. Based on this boundary condition, different models have been developed to calculate levitation height.

The mirror method is a simple and commonly used model. It considers the magnet and its diamagnetic image as a point dipole [7]. However, it does not consider the dimension of either the superconductor or the magnet. Another model developed by Lugo *et al.* [8] considers the finite size of the superconductor where the limitation may be the size of the superconductor. Experimental demonstration of Meissner-levitated ferromagnet within the superconducting lead trap (Type-I superconductor) has already shown promising results [9]. Superconducting cavities have proven to achieve the high quality factor performance goals [10], [11]. Magnetic levitation within superconducting cavities could be platform for the study of quantum mechanical system [12], [13], [14]. We have reported the demonstration and characterization of magnetic levitation within a superconducting coaxial quarter-wave cavity in Refs. [15], [16]. A permanent magnet is levitated from the edge of the superconductor [17] where the axis of the superconductor does not coincide with the axis of the magnet [18]. The two methods discussed above are limited to the co-axial case and over estimate the levitation height. In this study, a model is developed to calculate the levitation height using two-loop method [19]. This model take into account of the size of both the magnet and superconductor. We found that our method calculates levitation height more accurately than previously proposed methods.

MIRROR METHOD

Figure 1 shows the schematic of the mirror method. For a magnet above the Type-I superconductor, this model assumes that it has its diamagnetic image inside the superconductor. Both magnets (real and image) move in the opposite direction. The potential energy (or levitation force) is calculated between the dipole [20]. It depends upon the magnetic moment ($\vec{m}$) and magnetic field ($\vec{B}$). Mathematically the potential energy can be written as

$$\vec{U} = \frac{1}{2}(\vec{m} \cdot \vec{B}) \tag{1}$$

The magnetic field due to the dipole at the distance z from its center is given by:

$$B(0,0,z) = \frac{\mu_0 m}{2\pi z^3} \tag{2}$$

Substituting the above expression into Eq. (1), we get an expression for the potential energy between the real and mirror image as

$$U(0,0,z) = \frac{\mu_0 m^2}{4\pi} \frac{1+sin^2\theta}{(2z)^3}, \tag{3}$$

here, θ is an angle between the magnet and the superconductor and (2z) is the distance between the magnets. Due to this coeffiecient, the potential energy due to the radially magnetized magnet is half of the axially polarized magnet. Now, the total potential energy is the sum of Eq. (3) and the gravitational (Mgz) given by:

$$U_{total}(0,0,z) = \frac{\mu_0 m^2}{4\pi} \frac{1+sin^2\theta}{(2z)^3} + Mgz. \tag{4}$$

The above expression assumes the magnetic field is completely expelled from the superconductor and the superconductor as an infinite plane. The magnet levitates at the point with the least potential above the superconductor [19]. Eq. (4) is plotted for an axially magnetized ($\theta = 90°$) N52 (remanence = 1.47 T) permanent neodymium magnet of a radius and height of 0.5 mm in Fig. 2. The total potential energy near the superconductor is high because of the large repulsion between the magnet and its diamagnetic image. As the magnet moves farther away from the superconductor's surface, the potential energy quickly falls off. Its value becomes minimum at 3.8 mm above the superconductor. Hence, the magnet levitates at this minimum energy point.

The levitation force then can be calculated as [19]:

$$F_{Lev}(0,0,z) = -\Delta U_{Lev}, \tag{5}$$

$$F_{Lev}(0,0,z) = \frac{6\mu_0 m^2}{4\pi} \frac{1+sin^2(\theta)}{(2z)^4} \tag{6}$$

The vertical stiffness can be derived from Eq. (6) as:

$$K_z = -\frac{\partial F_{Lev}(0,0,z)}{\partial z} \tag{7}$$

$$F_{Lev}(0,0,z) = \frac{48\mu_0 m^2}{4\pi} \frac{1+sin^2(\theta)}{(2z)^5} \tag{8}$$

This will lead to the resonance frequency of:

$$\omega_z = \sqrt{\frac{K_z}{M}}, \tag{9}$$

$$\omega_z = m\sqrt{\frac{3\mu_0}{8\pi M} \frac{1+sin^2(\theta)}{(z_0^5)}}, \tag{10}$$

$$\omega_z = \sqrt{\frac{4g}{z_0}}. \tag{11}$$

For the N52 neodymium magnet of mass (M) 2.75 milligram and magnetic moment (m) 0.46 mAm^2, levitation height and f_z will be, respectively, 3.8 mm and $\sim$ 11 Hz, respectively.

FINITE-SIZE SUPERCONDCUTOR

One of the main drawbacks of the mirror method is that it considers the superconductor as an infinite plane. A method developed by Lugo *et al.* [8] includes the size of the superconductor in their calculations. In this model, the magnet is supposed as a point dipole and the superconductor a continuous array of point dipoles. The levitation force (or energy) is then obtained by integrating the dipole-dipole interaction between the real and an image magnet over the volume of the superconductor [8], [21]. The levitation force is given by:

$$F_{Lev}(0,0,z) = \frac{3\mu_0 m^2}{4\pi} \frac{(1+sin^2\theta)}{32}[f(a) - f(a+t)] \tag{12}$$

where R and t are radius and thickness of the superconductor respectively. The difference in Eq. (12) from Eq. (8) is the term [f(a)-f(a+t)], which includes dimension of the superconductor given by:

$$f_z = \frac{1}{z^4} - \frac{5R^2+3z^2}{3(R^2+z^2)^3}. \tag{13}$$

Fig. 3 shows levitation force calculated using Eq. (12) and (13). The dimension and strength of the magnet is kept same as that used in Fig. 2. The levitation is achieved for the height z = 2.75 mm, 28 % less than the height predicted by mirror method.

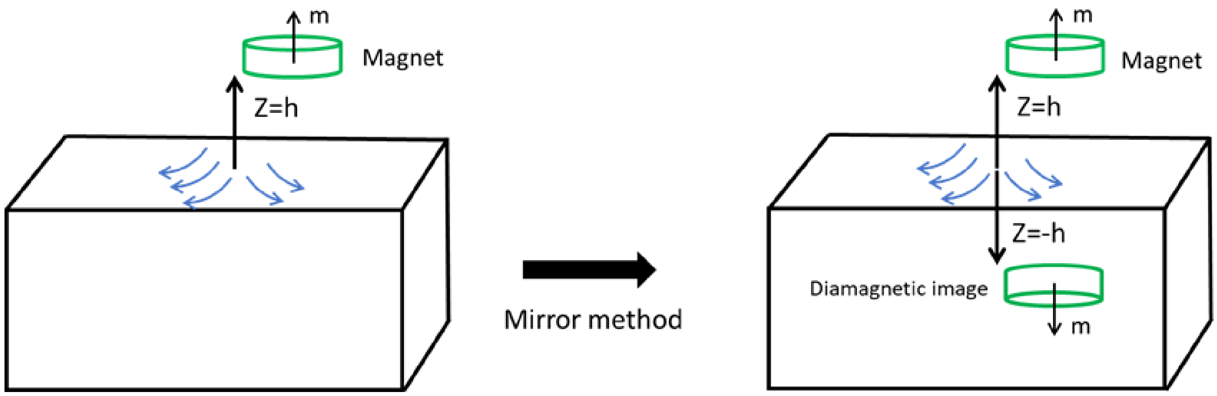

FIGURE 1. The mirror method's schematic view of a magnet at a height h above a superconductor.

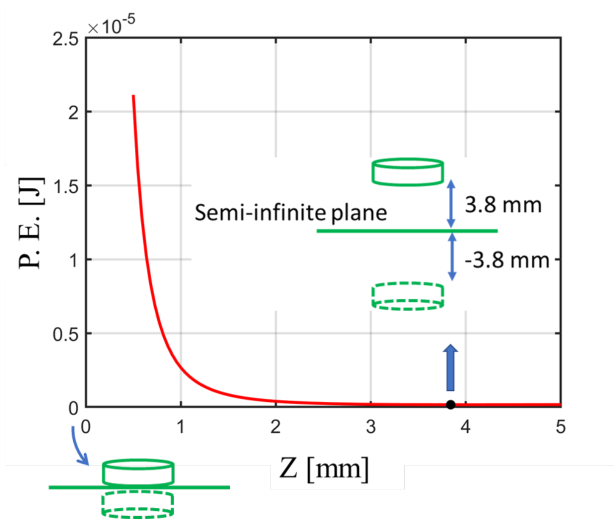

FIGURE 2. Total potential energy as a function of the vertical position of the magnet. The graph is generated from 0.5 mm vertical height to avoid large repulsion between the real and image magnets.

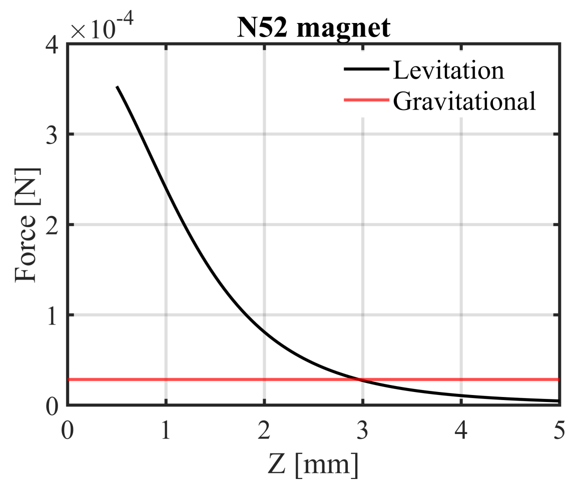

FIGURE 3. Levitation force for a finite-size superconductor. The upward levitation force balances the downward gravitational force at 2.75 mm.

IMPROVED MODEL

In the levitation experiment, the axis of the magnet might not necessarily coincide with the axis of the superconductor. In addition, the size of the magnet might be comparable to the size of the superconductor. In that case, the above two methods give an inaccurate levitation height calculations [22]. The two-loop method is used to calculate magnetic field between the two non-coaxial and opposite current carrying loops. We have implemented the same concept for the magnetic levitation above a finite-sized superconductor. Figure 4 shows the schematic of the levitation and concept of the model. Here, the magnet and its image are considered as two loops of current. Their distance is taken from the center of mass of the magnet. Consider a magnet with radius, R_M and height h place on the superconducting stub of radius R_S and thickness t. In the two-loop model, the magnet is replaced by a loop of current with the same radius as the magnet. Also, a loop of current replaces the image magnet with a radius equal to the radius of the superconductor. The distance between the two loops is now 2Z+h instead of 2Z.

Let's calculate magnetic field due to the two loops. The vector potential only has an azimuthal component, which is given by the equation:

$$A_\phi = \frac{\mu_0}{4\pi}[(R_s+r)^2+z^2]^{\frac{1}{2}}[(1-\frac{1}{2}k^2).\kappa(k)-E(k)] \quad (14)$$

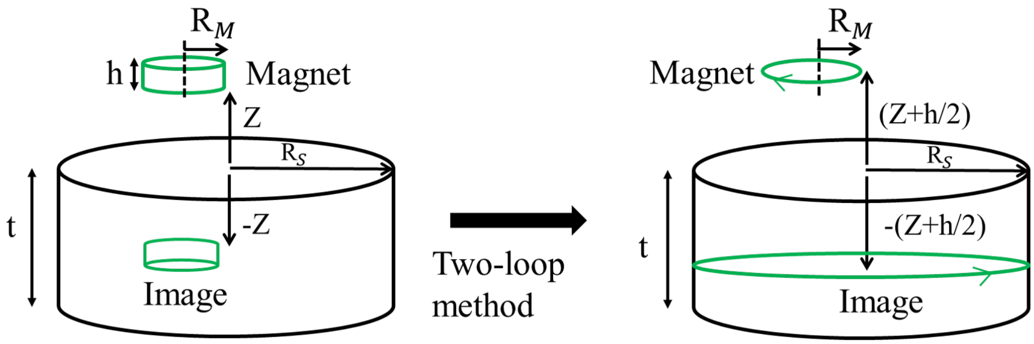

FIGURE 4. Two-loop representation of the magnet and its image. Here, the magnet and its image are replaced by current-carrying loops in the opposite direction.

Where:

$$k(r) = \frac{4R_s r}{(R_s+r)^2+z^2}, \tag{15}$$

$$r = \sqrt{(R_M cos_{\phi 2}+y)^2+(R_M sin_{\phi 2})^2}, \tag{16}$$

$$R = \sqrt{R_s^2+r^2+z^2-2R_s r cos_{\phi 1}} \tag{17}$$

Using relation $\vec{B} = \vec{\Delta} \times \vec{A}$, we get:

$$B_z = \frac{\mu_0 I}{4\pi[(R_s+r)^2+z^2]^{\frac{1}{2}}}[\frac{R_s^2-r^2-z^2}{(R_s-r)^2+z^2}.E(k)+\kappa(k)] \tag{18}$$

$$B_r = \frac{\mu_0 z I}{4\pi[(R_s+r)^2+z^2]^{\frac{1}{2}}}[\frac{R_s^2+r^2+z^2}{(R_s-r)^2+z^2}.E(k)-\kappa(k)] \tag{19}$$

From the frame of reference of the magnet, the components of the magnetic field will be:

$$B_{r|x'-y'-z'} = B_{r|x-y-z} cos(tan^{-1}(\frac{R_M sin_{\phi 2}}{y+R_M cos_{\phi 2}})-\phi_2) \tag{20}$$

$$B_{\phi|x'-y'-z'} = B_{\phi|x-y-z} sin(tan^{-1}(\frac{R_M sin_{\phi 2}}{y+R_M cos_{\phi 2}})-\phi_2) \tag{21}$$

where ϕ_1 and ϕ_2 are angles between y-axis and a point on the superconducting and magnetic coil, respectively. Potential energy is the dot product of the magnetic moment of the magnet (m) and response field from the image magnet. Using Eq. (1) the potential energy yields:

$$U = \frac{\mu_0 I m}{4\pi\sqrt{(R_s-r)^2+z^2}}[\frac{R_s^2-r^2-z^2}{(R_s-r)^2+z^2}.E(k)+\kappa(k)] \tag{22}$$

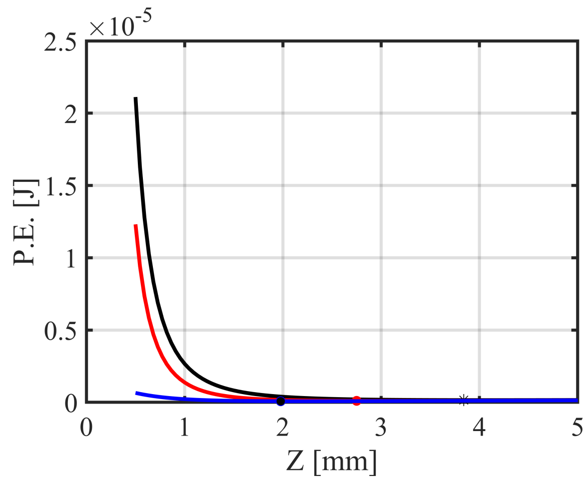

FIGURE 5. Comparison of the potential energy calculated at the center of the superconductor using three models: the mirror, finite-size superconductor, and two-loop model.

Potential energy calculated from our model is compared with the potential energy calculated by using the mirror method and finite-size superconductor in Fig. 5. The model give the levitation height of 2.65 mm lower than the height predicted by mirror and finite superconductor methods.

Figure 6 shows the levitation height calculated using all three methods as a function of remanence field. Calculations of the mirror and finite-SC method are independent to the position of the magnet on the superconductor. Our model has capacity to calculate levitation height at any location on the superconductor. Importantly, Fig. 6 shows a large deviation in the levitation height for the magnetic levitation from the edge of the superconductor. Levitation height is reduced as the magnet moves from the center to the edge of the superconductor. For example, according to our model, for a magnet with a strength of 1.5 T, the levitation height is reduced by 26 % (from 2.7 mm to 2 mm) as we go from the center to the edge of the superconductor likely due to the reduction of response supercurrent at the edge of the superconductor.

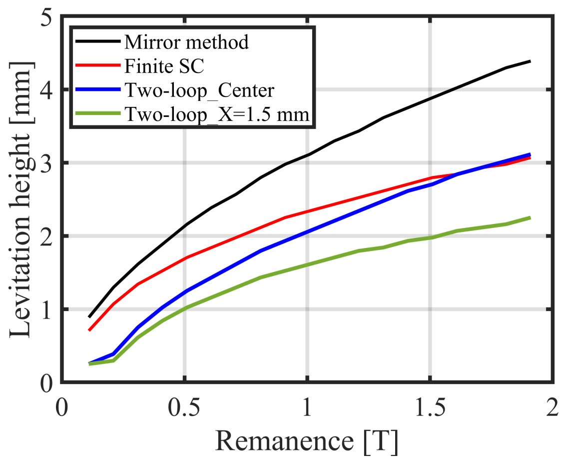

FIGURE 6. Levitation height as a function of remanence of the magnet. Three models (mirror, finite SC, and two-loop) are compared at the center and edge of the superconductor.

CONCLUSION

Comparative study between three models, the mirror method, finite-size superconductor, and our model, has been done. For the coaxial case, discrepancy between three models is relatively less especially for the weaker magnets. In the experiment, magnetic levitation is observed from the edge of the superconductor. In such non-coaxial case of levitation, inaccuracy in the mirror and finite-superconductor method calculations increased drastically.

ACKNOWLEDGMENTS

I would like to thank Jefferson Lab for giving me time for the preparation of this manuscript. Appreciation goes to Prof. Chih-Chun Chien, Prof. Michael Scheibner, and Dr. Pashupati Dhakal for all help and support.

EDITORS' NOTE

This manuscript was submitted to the Association of Nepali Physicists in America (ANPA) Conference 2022 for publication in the special issue of Journal of Nepal Physical Society.

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