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Modification of SCE-UA genetic algorithm for runoff-erosion modelling

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ABSTRACT

In order to provide a robust tool to be used in runoff-erosion modelling, the present paper introduces new evolution steps in the SCE-UA genetic algorithm, which is based on the simplex theory. The new evolution steps were conceived in order to improve the efficiency of such an algorithm. Thus, they will theoretically expand the simplex in a direction of more favourable conditions, or contract it if a move is taken in a direction of less favourably conditions. Hence, these new evolution steps enable the simplex both to accelerate along a successful track of improvement and to home in on the optimum conditions. Therefore, it will usually reach the optimum region quicker than the previous version and pinpoint the optimum levels more closely. The new proposed algorithm is tested with special mathematical functions, as well as in the optimisation of the erosion parameters presented in a physically-based runoff-erosion model. On the basis of these simulation results, the mean erosion parameter values are given, which agree with previous values reported to the same area. Thus, the new algorithm can be considered as a promising tool to optimise physically-based models as well as other kinds of model.

INTRODUCTION

The difficulties involved in calibration of physically-based erosion models have been partly attributed to the lack of robust optimisation tools, hence new robust techniques have always been investigated. The evolutionary algorithms have been proving to be robust in optimisation process. As Fogel (1994) described, natural evolution is a population based optimisation process, then simulating this process on computer results in optimisation techniques that can often outperform classical methods of optimisation when applied to difficult real-world problems. The evolutionary algorithms are presented in a variety of ways: genetic algorithm, evolutionary programming, evolution strategies, classifier systems, and genetic programming. The latter three algorithms are the current main lines of investigation. Genetic algorithms stress chromosomal operators whereas evolutionary programming stresses behavioural changes at the level of the species, and evolution strategies emphasise behavioural changes at the level of the individual.

Thus, evolutionary algorithms are stochastic search methods that mimic the metaphor of natural biological evolution. They operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation.

A genetic algorithm named the Shuffled Complex Evolution (SCE-UA) developed by Duan et al. (1992) proved to be useful to optimise conceptual rainfall-runoff models. The SCE-UA also applies a simplex downhill search scheme (Nelder and Mead 1965); thus, in order to improve its efficiency in terms of how to reach the global minimum, new evolution steps are introduced. These new evolution steps are intended to enable the simplex both to accelerate along a successful track of improvement and to home in on the optimum conditions. Therefore, it will usually reach the optimum region quicker and pinpoint the optimum levels more closely.

Finally, in order to test if the modified method is also capable in finding the global minimum of test mathematical functions, the paper will describe the application of such method to these functions.

THE MODIFIED SCE-UA METHOD

The typical optimisation problems that characterise the problems encountered in physically-based erosion model calibration are (1) global convergence in the presence of multiple regions of attraction; (2) ability to avoid being trapped by small pits and bumps on the objective function surface; (3) robustness in the presence of differing parameter sensitivities and parameter interdependence; (4) non-reliance on the availability of an explicit expression for the objective function or the derivatives; and (5) capacity of handling high-parameter dimensionality.

The SCE-UA method embodies the desirable properties described above and is based on a synthesis of four concepts: (1) combination of deterministic and probabilistic approaches; (2) systematic evolution of a 'complex' of points spanning the parameter space, in the direction of global improvement; (3) competitive evolution; (4) complex shuffling. The synthesis of these elements makes the SCE-UA method effective and robust, and also flexible and efficient. The steps of the SCE-UA method are (a) generate randomly a sample of s points $x_1, \ldots, x_s$ in the feasible space $\Omega \subset \mathbb{R}^n$, compute the function value f_i at each point x_i, rank the points according to the order of increasing criterion, and partition of the sample into p complexes $A^1, \ldots, A^p$, each containing m points where the first point in the first complex represents the point with the smallest function value, the second smallest value is in the second complex and so on (Fig. 1a); (b) evolve each complex (community) independently according to the Competitive Complex Evolution (CCE) algorithm (Fig. 1b); (c) shuffle the complexes (Fig. 1c); (d) check if any of the pre-specified convergence criteria are satisfied, if so stop (Fig. 1d), otherwise, check the reduction in the number of complexes and continue to evolve.

The CCE algorithm, based on the Nelder and Mead (1965) simplex downhill search scheme, used by the original SCE-UA presents three evolution steps: reflection, contraction and mutation. The simplex methods are based on an initial design of $n + 1$ trials, where n is the number of variables. Then, a geometric figure in a n-dimensional space is called a simplex, thus a simplex defined by three different trial conditions for two control variables has a shape of a triangle. The shapes of the simplex in a one and three variable search space are a line and a tetrahedron, respectively. A geometric interpretation is difficult with more variables, but the basic mathematical approach can handle the search for optimum conditions. In order to improve the evolution process and to make the algorithm reach the optimum region quicker and pinpoint the optimum levels more closely, new evolution steps were introduced in this present paper. The Modified Competitive Complex Evolution (MCCE) algorithm required for the evolution of each complex is presented below and is illustrated in Fig. 2:

1. To initialise the process, select q, α, and β, where $2 \le q \le m$, $\alpha \ge 1$ and $\beta \ge 1$.
2. Assign weights as follows. Assign a trapezoidal probability distribution to A^k, i.e.,

$$\rho_i = \frac{2(m+1-i)}{m(m+1)},\ i = 1, \ldots, m \tag{1}$$

   The point x_1^k has the highest probability $\rho_1 = 2/m + 1$. The point x_m^k has the lowest probability $\rho_m = 2/m(m+1)$.
3. Select parents by randomly choosing q distinct points $u_1, \ldots, u_q$ from A^k according to the probability distribution specified above. The q points define a "subcomplex", which functions like a pair of parents, except that it may comprise more than two members. Store them in array $B = \{u_j, v_j, j = 1, \ldots, q\}$, where v_j is the function value associated with point u_j. Store in L the locations of A^k which are used to construct B.

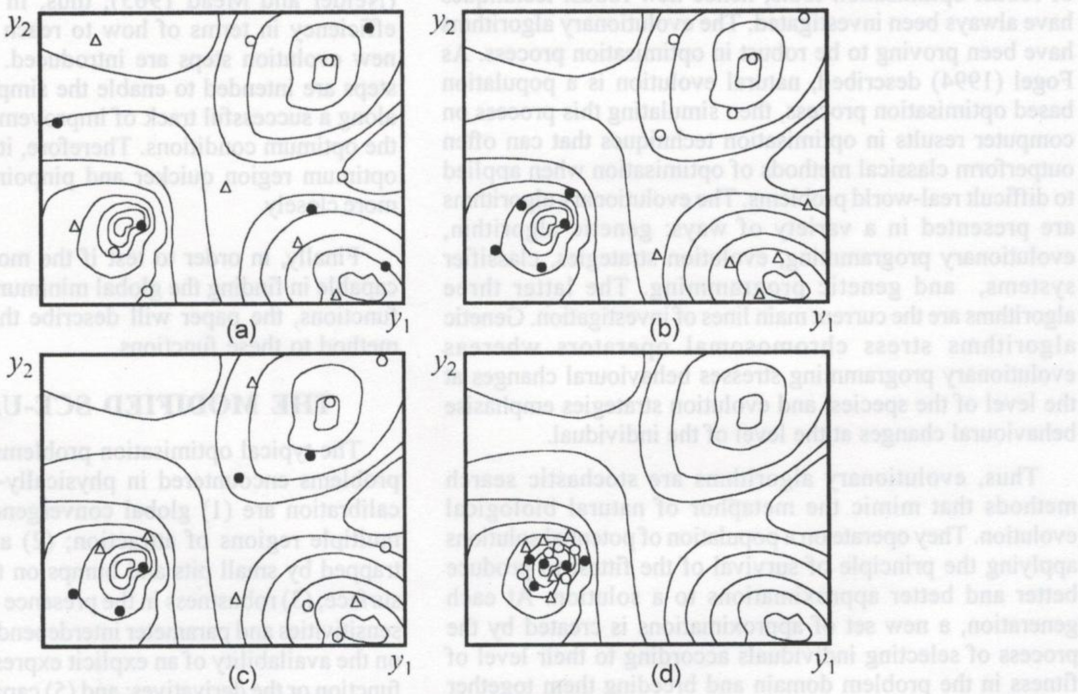

Fig. 1: Illustration of the shuffled complex evolution (SCE-UA) method

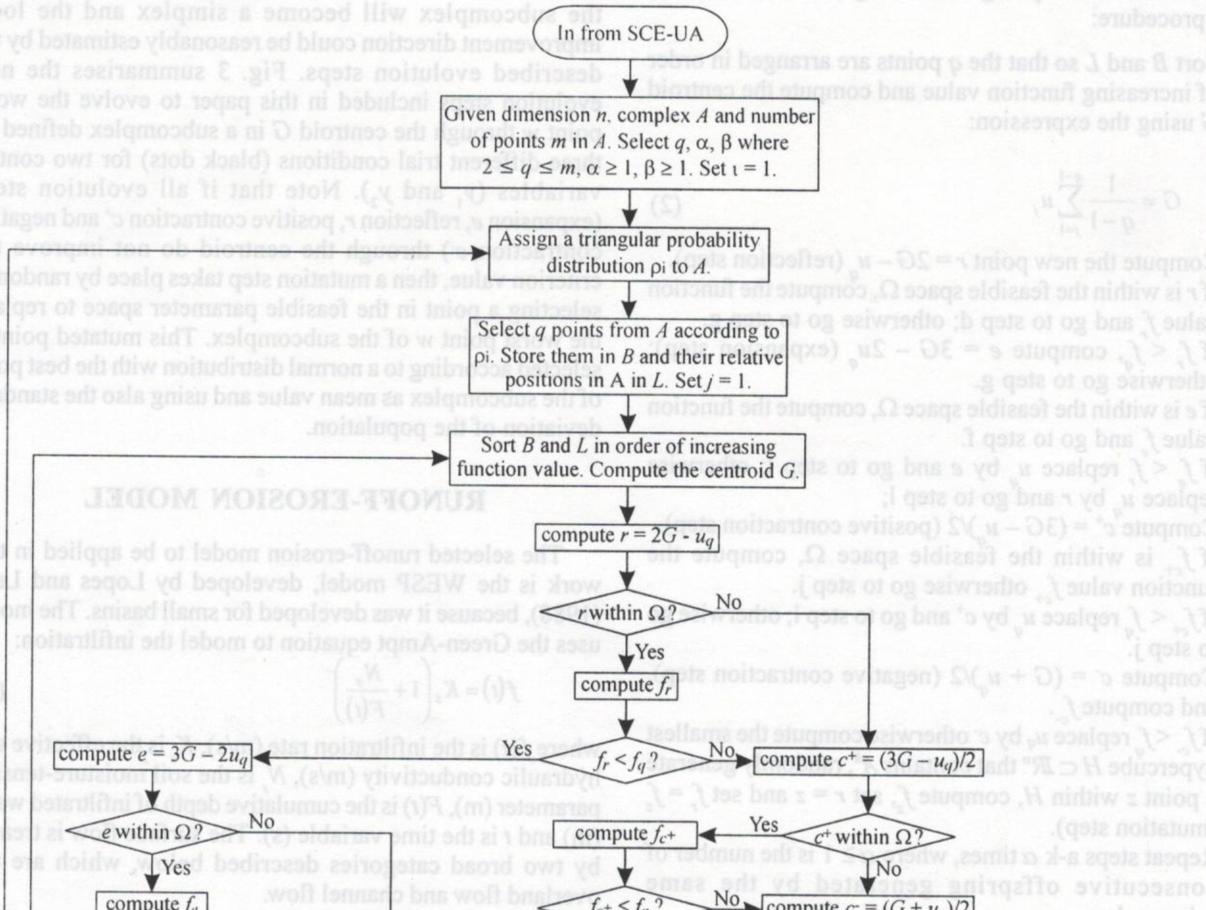

Fig. 2: Flowchart of the MCCE for the SCE-UA algorithm

4. Generate offspring according to the following procedure:

(a) Sort B and L so that the q points are arranged in order of increasing function value and compute the centroid G using the expression:

$$G = \frac{1}{q-1}\sum_{j=1}^{q-1} u_j \quad (2)$$

(b) Compute the new point $r = 2G - u_q$ (reflection step).
(c) If r is within the feasible space Ω, compute the function value f_r and go to step d; otherwise go to step g.
(d) If $f_r < f_q$, compute $e = 3G - 2u_q$ (expansion step); otherwise go to step g.
(e) If e is within the feasible space Ω, compute the function value f_e and go to step f.
(f) If $f_e < f_r$ replace u_q by e and go to step l; otherwise replace u_q by r and go to step l;
(g) Compute $c^+ = (3G - u_q)/2$ (positive contraction step).
(h) If f_{c+} is within the feasible space Ω, compute the function value f_{c+} otherwise go to step j.
(i) If $f_{c+} < f_q$ replace u_q by c^+ and go to step l; otherwise go to step j.
(j) Compute $c^- = (G + u_q)/2$ (negative contraction step), and compute f_{c-}.
(k) If $f_{c-} < f_q$ replace u_q by c^- otherwise compute the smallest hypercube $H \subset \mathbb{R}^n$ that contains A^k, randomly generate a point z within H, compute f_z, set $r = z$ and set $f_r = f_z$ (mutation step).
(l) Repeat steps a-k α times, where $\alpha \geq 1$ is the number of consecutive offspring generated by the same subcomplex.

5. Replace parents by offspring as follows: Replace B into A^k using the original locations stored in L. Sort A^k in order of increasing function value.
6. Iterate by repeating steps 2–5 β times, where $\beta \geq 1$ is the number of evolution steps taken by each complex before complexes are shuffled; i.e., how far each complex should evolve.

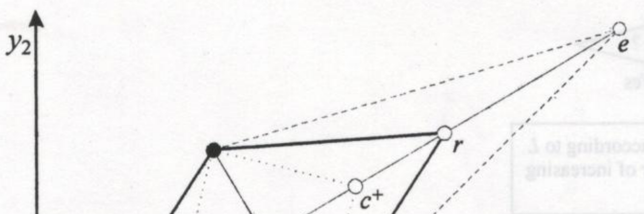

Fig. 3: Example of the evolution steps that can be taken by each complex in a two-variable control space (y_1 and y_2).

Thus, if the dimension of the subcomplex is set to $n + 1$, the subcomplex will become a simplex and the local improvement direction could be reasonably estimated by the described evolution steps. Fig. 3 summarises the new evolution steps included in this paper to evolve the worst point w through the centroid G in a subcomplex defined by three different trial conditions (black dots) for two control variables (y_1 and y_2). Note that if all evolution steps (expansion e, reflection r, positive contraction c^+ and negative contraction c^-) through the centroid do not improve the criterion value, then a mutation step takes place by randomly selecting a point in the feasible parameter space to replace the worst point w of the subcomplex. This mutated point is selected according to a normal distribution with the best point of the subcomplex as mean value and using also the standard deviation of the population.

RUNOFF-EROSION MODEL

The selected runoff-erosion model to be applied in this work is the WESP model, developed by Lopes and Lane (1988), because it was developed for small basins. The model uses the Green-Ampt equation to model the infiltration:

$$f(t) = K_s\left(1 + \frac{N_s}{F(t)}\right) \quad (3)$$

where $f(t)$ is the infiltration rate (m/s), K_s is the effective soil hydraulic conductivity (m/s), N_s is the soil moisture-tension parameter (m), $F(t)$ is the cumulative depth of infiltrated water (m) and t is the time variable (s). The surface flow is treated by two broad categories described below, which are the overland flow and channel flow.

Overland flow

The overland flow is considered one-dimensional. Manning's turbulent flow equation is given by:

$$\bar{u} = \frac{1}{n_p} R_H^{2/3} S_f^{1/2} \quad (4)$$

where $\bar{u}$ is the local mean flow velocity (m/s), $R_H(x,t)$ is the hydraulic radius (m), S_f is the friction slope and n_p is the Manning friction factor of flow resistance for the plane. Thus, the local velocity equation for planes can be obtained considering the hydraulic radius equal to the depth of flow ($R_H = h$) and using the kinematic approximation that the friction slope is equal to the plane slope ($S_0 = S_f$):

$$\bar{u} = \alpha' h^{m'-1} \quad (5)$$

where h is the depth of flow (m), α' is a parameter related to surface roughness, equal to $(1/n_p)S_0^{1/2}$, and m' is a geometry parameter whose value is set to 5/3.

Putting Equation (5) into the equation of continuity for the flow, the following equation for h is obtained:

$$\frac{\partial h}{\partial t} + \alpha' m' h^{m'-1}\frac{\partial h}{\partial x} = r_e \quad (6)$$

From Equations (5) and (6), the overland flow (u, h) can be calculated with a given excess rainfall r_e.

Sediment transport is considered as the erosion rate in the plane reduced by the deposition rate within the reach. The erosion occurs due to raindrop impact as well as surface shear. Thus, the continuity equation for sediment transport is given by:

$$\frac{\partial(ch)}{\partial t} + \frac{\partial(cuh)}{\partial x} = e_I + e_R - d \quad (7)$$

where c is the plane sediment concentration in transport (kg/m³), e_I is the rate of sediment by rainfall impact (kg/m²/s), e_R is the rate of sediment by shear stress (kg/m²/s), and d is the rate of sediment deposition (kg/m²/s). Each one of the components in the net sediment flux expression is given as follows: the rate of sediment by rainfall impact e_I is obtained from the relationship:

$$e_I = K_I I r_e \quad (8)$$

in which K_I is the soil detachability parameter (kg.s/m⁴). The rate of sediment by shear stress e_R is expressed by the relationship:

$$e_R = K_R \tau^{1.5} \quad (9)$$

where K_R is a soil detachability factor for shear stress (kg m/N$^{1.5}$s), and τ is the effective shear stress (N/m²), which is given by $\tau = \gamma h S_f$, where γ is the specific weight of water (N/m³). Finally, the rate of sediment deposition d in Equation (7) is expressed as:

$$d = \varepsilon V_s c \quad (10)$$

where ε is a coefficient that depends on the soil and fluid properties, set to 0.5 in the present study, and V_s is the particle fall velocity (m/s) computed by Rubey's equation:

$$V_s = F_o \sqrt{\frac{(\gamma_s - \gamma)}{\gamma} g d_s} \quad (11)$$

and

$$F_o = \sqrt{\frac{2}{3} + \frac{36\nu^2}{g d_s^3\left(\frac{\gamma_s}{\gamma} - 1\right)}} - \sqrt{\frac{36\nu^2}{g d_s^3\left(\frac{\gamma_s}{\gamma} - 1\right)}} \quad (12)$$

where γ_s is the specific weight of sediment (N/m³), ν is the kinematic viscosity of water (m²/s), d_s is the mean diameter of the sediment (m), and g is the acceleration of gravity (m/s²).

From Equation (7), sediment transport rate (cuh) can be calculated under the overland flow (h, u) given by Equation (6).

Channel flow

The concentrated flow in the channels is also described by continuity and momentum equations. The momentum equation can be reduced to the discharge equation with the kinematic approximation:

$$Q = \alpha' A R_H^{\;m'-1} \quad (13)$$

where Q is the discharge (m³/s), A is the area of flow (m²) and α' is the same as described in Equation (5), however, the Manning friction factor of flow resistance must be the value set to the channel, n_c. The continuity equation for the channel flow is given by:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_A \quad (14)$$

where q_A is the lateral inflow per unit length of channel. From Equations (13) and (14), the surface flow for the channel flow (A, Q) can be calculated.

Since the effect of rainfall impact is neglected in the channel, the continuity equation for the sediment is expressed without the rainfall impact component by:

$$\frac{\partial AC}{\partial t} + \frac{\partial CQ}{\partial x} = q_s + e_r - d_c \quad (15)$$

where $C(x,t)$ is the sediment concentration in transport in the channel (kg/m²), q_s is the lateral sediment inflow into the channel (kg/m/s), d_c is the rate of sediment deposition into the channel (kg/m/s), and e_r is the erosion rate of the channel bed material (kg/m/s). The components of the net sediment flux expression for the channel segment are given as follows: the erosion rate of the channel bed material e_r is obtained from the relation:

$$e_r = a(\tau - \tau_c)^{1.5} \quad (16)$$

in which a is the sediment erodibility parameter, and τ_c is the critical shear stress for sediment entrainment (N/m²), which is given by $\tau_c = \delta(\gamma_s - \gamma) d_s$, where δ is a coefficient, set to 0.047 in the present study, γ_s is the specific weight of sediment (N/m³), and d_s is the mean diameter of sediments (m). This critical shear stress component has been neglected in Equation (9) for the planes due to the difficulty of its evaluation on a field scale or for small watershed applications. Also, values suggested for critical shear stress from studies conducted in single rills are not applicable to the broad sheet-flow approach in overland flow erosion modelling. Furthermore, there are always fine particles of sediment detached by the action of wind or other elements between rainfall events, which will be available to be transported by sheet flow as soon as rainfall exceeds infiltrability on the soil surface, without any resistance to removal.

The rate of sediment deposition within the channel d_c (kg/m/s) in Equation (15) is expressed by:

$$d_c = \varepsilon_c T_W V_s C \quad (17)$$

in which ε_c is the deposition parameter for channels, considered as unity in the present case, and T_W is the flow top width (m). From Equation (15), sediment transport rate (CQ) can be calculated under the overland flow (A, Q) given by Equation (14).

AREA DESCRIPTION

The physically-based runoff-erosion model is applied to a bare micro-basin, which is one of the four micro-basins of the Sumé Experimental Watershed, in northeastern Brazil at Paraíba State. The micro-basin mean slope, area, and perimeter are 7.1%, 0.48 ha, and 302 m, respectively. This experimental watershed was operated since 1972 (Cadier and Freitas 1982) by SUDENE (Superintendency of Northeast Development, Brazil), ORSTOM (French Office of Scientific Research and Technology for Overseas Development), and UFPB (Federal University of Paraíba, Brazil).

Based on the work of Santos et al. (1998), 45 events were selected between 1987 and 1991, because during this period there was no vegetation cover. The runoff and erosion data were measured after each rainfall event and the rainfall data were obtained from a recording rain gauge installed close to the selected micro-basin.

APPLICATION OF THE MODIFIED SCE-UA TO THE WESP MODEL

Selection of SCE-UA algorithm parameters

The SCE-UA method contains many probabilistic and deterministic components that are controlled by some algorithmic parameters. For the method to perform optimally, these parameters must be chosen carefully. The first one is m, the number of points in a complex ($m \geq 2$), which should be not too small to avoid that the search proceeds like an ordinary simplex procedure, neither too large to avoid that an excessive use of the computer processing time with no certain in effectiveness is taken. Then the default value, $m = 2n + 1$, was selected, in which n means the number of parameters to be optimised on as explained previously. For the number of points in a subcomplex q ($2 \leq q \leq m$), the value of $n + 1$ was selected because it would make the subcomplex a Simplex; this defines a first-order approximation (hyperplane) to the objective function surface, and since the evolution steps are based on the simplex method it will consequently give a reasonable estimate of the local improvement direction. The number of consecutive offspring generated by each subcomplex α ($\alpha \geq 1$), was set to one to avoid that the search becomes more strongly biased in favour of local search of the parameter space. The number of evolution steps taken by each complex β ($\beta > 0$) was set to $2n + 1$ to avoid that the complexes would be shuffled frequently if set to a small value or to avoid that they would shrink into a small cluster if a great value is used. The number of complexes p was set to 2 based on the nature of the problem, and the minimum number of complexes required in the population $p_{\min}$ ($1 \leq p_{\min} \leq p$) was set to p because it gave the best overall performance in terms of effectiveness and efficiency.

Optimisation of the WESP erosion parameters

In order to start the calibration process, the micro-basin had to be represented as a scheme of planes and channels. The authors have discussed which schematisation is the best to represent the area (Santos et al. 1994), and the schematisation in 10 elements was chosen here. The parameters which could be determined from a priori information are the Manning friction factor, which was assumed as 0.02 for planes (n_p), and 0.03 for channels (n_c) based on the soil type, its grain size composition, and surface characteristics, the specific weight of water assumed as 9.8 kN/m^3, and the specific weight of sediment assumed as 2.6×10^4 kN/m^3. However, there are some parameters that are specific for this area which should be determined by field tests such as the effective soil hydraulic conductivity K_s whose value was assumed equal to 5.0 mm/h, and the mean diameter of sediments d_s whose value was assumed to be equal to d_{50} which is 0.5 mm; however, other parameter values should be set according to the literature or determined by calibration or optimisation process.

The first parameter to be calibrated in the WESP model was the soil moisture-tension parameter N_s and it could be calibrated by a simple optimisation method because it was necessary just to fit the computed runoff depth with the observed value. However, after this step the WESP model contains more three erosion parameters (a, K_R, and K_I) which should be calibrated; thus, the Modified SCE-UA method was used for such task.

The initial values of the erosion parameters were $a = 0.0144$ kg m^2, $K_R = 2.174$ kg m/N$^{1.5}$ s and $K_I = 5.0 \times 10^8$ kg s/m^4, and the following objective function to be minimised was used:

$$J = \left| \frac{E_o - E_c}{E_o} \right| \qquad (18)$$

where E_o is the observed sediment yield (kg) and E_c is the calculated one (kg). The optimisation for the 45 events gave the mean values of the erosion parameters as $a = 0.008$ kg m^2, $K_R = 2.524$ kg m/N$^{1.5}$ s, and $K_I = 5.632 \times 10^8$ kg s/m^4. The values were used then to run new simulations, and Fig. 4 shows the simulation results for the sediment yield.

TESTING THE MODIFIED SCE-UA WITH MATHEMATICAL FUNCTIONS

This section describes a number of test functions applied for use with the Modified SCE-UA Algorithm. These functions are drawn from the literature on genetic algorithms, evolutionary strategies, and global optimisation. The genetic algorithm parameters used for these tests are as follows based on the previous discussion. Since there are two control variables, n is equal to 2 and the number of points in a complex m is equal to 5 because $m = 2n + 1$. The number of points in a subcomplex q is equal to $n + 1$, thus $q = 3$. The number of consecutive offspring generated by each subcomplex α is set to 1. The number of evolution steps taken by each complex β is equal to 5 because $\beta = 2n + 1$. The number of complexes p is set to 2 thus the population becomes equal to 10, and finally the minimum number of complexes required in the population $p_{\min}$ is set to p.

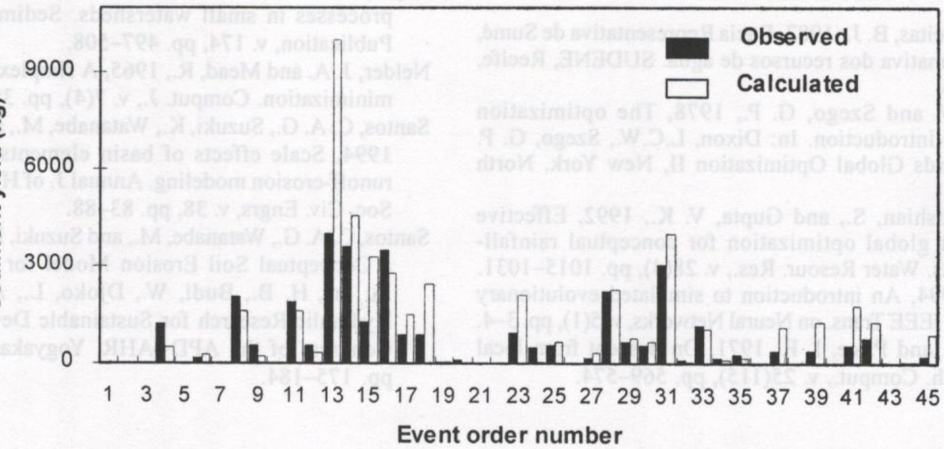

Fig. 4: Observed and simulated sediment yield

Three test functions were selected to perform the tests: The Rosenbrock, Goldstein-Price (Goldstein and Price 1971), and Six-Hump Camel-Back functions.

Rosenbrock's valley is a classic optimisation problem, also known as Banana function. The global optimum is inside a long, narrow, parabolic shaped flat valley. To find the valley is trivial, however convergence to the global optimum is difficult and hence this problem has been repeatedly used in assess the performance of optimisation algorithms.

$$f_{\text{Rosen}} = 100\left(y_2 - y_1^2\right) + \left(1 - y_1\right)^2 \tag{19}$$

in which the control variables are as $-2.048 \leq y_1 \leq 2.048$ and $-2.048 \leq y_2 \leq 2.048$.

The global minimum is located at $(y_1, y_2) = (1, 1)$ where the function value is $f_{\text{Rosen}}(y_1, y_2) = 0$. The Goldstein-Price function f_{Gold} is also a global optimisation test function used to test global optimisation techniques, which is defined as:

$$f_{\text{Gold}}(y_1, y_2) = \text{Term}_1 \times \text{Term}_2 \tag{20}$$

$$\text{Term}_1 = 1 + \left(y_1 + y_2 + 1\right)^2 \left(19\text{-}14y_1 + 3y_1^2\text{-}14y_2 + 6y_1y_2 + 3y_2^2\right) \tag{21}$$

$$\text{Term}_2 = 30 + \left(2y_1\text{-}3y_2\right)^2 \left(18\text{-}32y_1 + 12y_1^2 + 48y_2\text{-}36y_1y_2 + 27y_2^2\right) \tag{22}$$

in which the control variables are as $-2 \leq y_1 \leq 2$ and $-2 \leq y_2 \leq 2$. The global minimum is located at $(y_1, y_2) = (0,-1)$ where the function value is $f_{\text{Gold}}(y_1, y_2) = 3$. The 2-Dimensional Six-hump camel back function (Dixon and Szego 1978) is another global optimisation test function. Within the bounded region are six local minima, two of them are global minima.

$$f_{\text{Sixh}}(y_1, y_2) = \left(4 - 2.1y_1^2 + y_1^{4/3}\right)y_1^2 + y_1y_2 + \left(-4 + 4y_2^2\right)y_2^2 \tag{23}$$

in which the control variables are as $-3 \leq y_1 \leq 3$ and $-2 \leq y_2 \leq 2$. The global minimum is located at $(y_1, y_2) = (-0.0898, 0.7126)$ or $(y_1, y_2) = (0.0898, -0.7126)$ where the function value is $f_{\text{Sixh}}(y_1, y_2) = -1.0316$.

In spite of the difficulty involved in finding these function global minima, the Modified SCE-UA showed a promising performance in terms of effectiveness (the ability to locate global optimum) and efficiency (the speed to locate global optimum) to locate each global minimum.

CONCLUSIONS

In order to improve the effectiveness and efficiency of the SCE-UA genetic algorithm, new evolution steps were introduced in the competitive complex evolution (CCE) algorithm, which is based on the simplex downhill search scheme. This Modified SCE-UA proved to be useful to calibrate the distributed physically-based erosion models and despite the difficulty involved in finding the global minima of Rosenbrock, Goldstein-Price, and Six-Hump Camel-Back functions, it showed a promising performance in terms of effectiveness (the ability to locate global optimum) and efficiency (the speed to locate global optimum) to locate each global minimum.

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