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General expression for the critical seepage gradient in an infinite slope

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ABSTRACT

With reference to the ideal case of an infinite slope of granular (c' =0) homogeneous and isotropic soil subject to uniform seepage with a vertical component directed upwards, the possibility, in principle, of a slope instability due to static liquefaction (quick condition) is examined. Such instability takes place when the seepage gradient is the critical one (i_c). In this paper, a general expression for i_c is obtained.

INTRODUCTION

The potential of physical effects of groundwater flow on slope instability is well known. In this context, although the assumption of parallel-to-the-slope seepage is realistic in most natural situations whose analysis can be amenable to that of an infinite slope, it is interesting to consider the general case where water flow direction is arbitrary and flow occupies the entire slope. The note examines the ideal case of an infinite slope of saturated granular soil (c' =0) under a uniform seepage with a vertically-directed upward component. This situation might occur in slope-foot areas, where the seepage forces have an upward component. Leaving out the canonical shear failure analysis, slope instability due to static liquefaction (quick condition) is considered.

A general expression for the critical hydraulic gradient is derived. Nevertheless, it provides a theoretical limit, given that simple considerations show that shear failure precedes liquefaction and can eventually trigger it.

THE CRITICAL GRADIENT

Fig. 1 shows an infinite slope of saturated granular soil, assumed homogeneous and isotropic, in which seepage is assumed uniform. The slope and flow line inclinations with respect to the horizontal are β and ϑ respectively. Seepage gradient is:

$$i = \frac{\Delta H}{\overline{RP}},$$

where, $\Delta H = \overline{RQ}\sin\beta$ and $\overline{RP} = \overline{RQ}\cos(\beta + \vartheta)$. We have:

$$i = \frac{\sin\beta}{\cos(\beta + \vartheta)} \tag{1}$$

With the sign convention in Fig. 1, the previous expression becomes:

$$i = \frac{\sin\beta}{\cos(\beta - \vartheta)} \tag{1'}$$

Since the flow is uniform and soil homogeneous and isotropic, the seepage forces are distributed and dissipated uniformly throughout the slope and their direction is parallel to the flow lines. Considering a typical soil element ABCD (with a volume $V=1\cdot 1\cdot z$), the overall seepage force F acting on it is:

$$F = jV = \gamma_w iz, \tag{2}$$

where, j and γ_w are, respectively, the seepage force per unit volume and the unit weight of water.

From the equilibrium of the slope element ABCD in the direction normal to BC, one obtains the expression for the resultant N' of the effective normal stresses at the bottom of the element, i.e.:

$$N' = \gamma_b z\cos\beta - \gamma_w iz\sin(\beta - \vartheta), \tag{3}$$

where, γ_b is the submerged unit weight of soil.

Flow, considered in the analysis, is upward emergent. One can therefore consider the case where the seepage force component normal to the base of the slope element is such as to cancel the component N' of the effective weight of the element itself. In this case, since $N'=\sigma'=0$ (viz. $u=\sigma$), the soil mass which has been assumed cohesionless would, in principle, statically liquefy to a quick condition. From Equation (3), N' is zero when seepage gradient assumes a critical value i_c, i.e. when

$$i = i_c = \frac{\gamma_b \cos\beta}{\gamma_w \sin(\beta - \vartheta)} \tag{4}$$

Equation (4) has been derived ignoring the influence of shear stresses on the vertical sides of the element. In the special case β=0 and ϑ=−π/2, i.e. horizontal ground level and vertically ascendant seepage, Equation (4) takes on the familiar form (Taylor 1948):

$$i_c = \frac{\gamma_b}{\gamma_w} \tag{5}$$

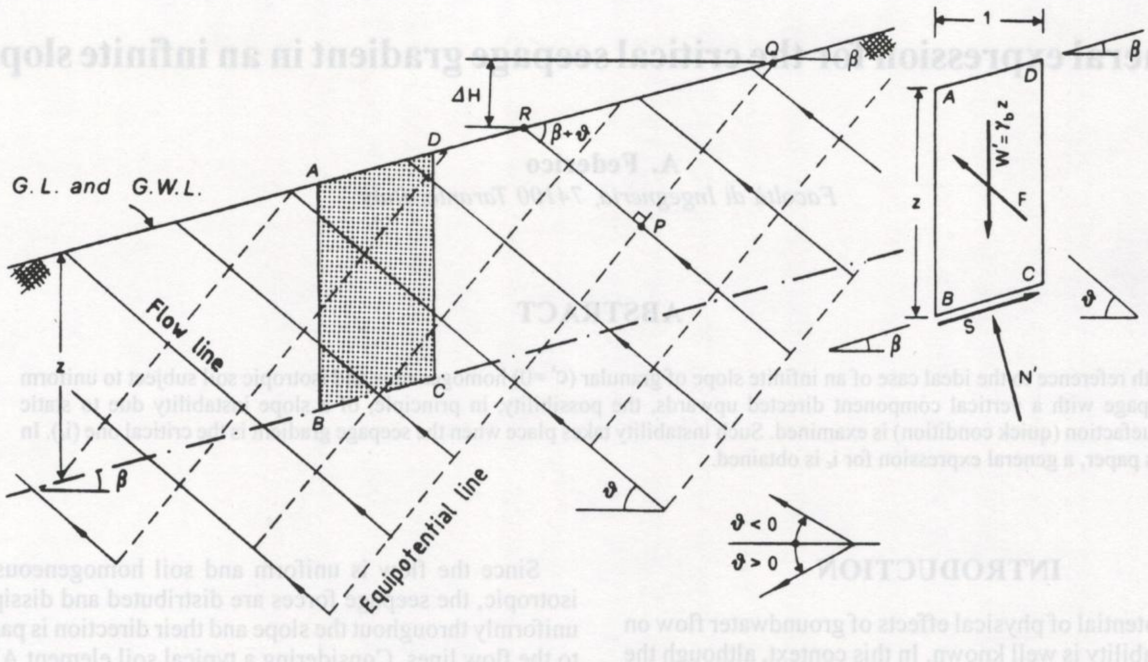

Fig. 1: Infinite slope with uniform upward emergent seepage

The safety factor against static liquefaction failure is given by

$$F_{LIQ} = \frac{i_c}{i} = \frac{\gamma_b}{\gamma_w}\cot\beta\cot(\beta - \vartheta) \quad (6)$$

The limit condition $F_{LIQ}=1$ can be written as

$$\tan(\beta - \vartheta) = \frac{\gamma_b}{\gamma_w}\cot\beta, \quad (7)$$

or,

$$\tan\vartheta = \frac{\gamma_w}{\gamma}\sec\beta\,\mathrm{cosec}\beta - \cot\beta, \quad (7')$$

where $\gamma=\gamma_{sat}$ is the total unit weight of soil.

Equation (7) or (7') expresses the critical combinations of flow line and slope inclinations, given the values of unit weights of soil and water. Fig. 2 gives the critical pair (ϑ, β) for several values of γ_b/γ_w ratio.

It is possible to provide an expression for the critical gradient in which the inclination ϑ of the flow lines does not appear explicitly. In fact, by using the following trigonometrical relationships

$$\sin(\beta - \vartheta) = \frac{2\tan\frac{(\beta-\vartheta)}{2}}{1+\tan^2\frac{(\beta-\vartheta)}{2}} \quad \text{and} \quad \tan(\beta - \vartheta) = \frac{2\tan\frac{(\beta-\vartheta)}{2}}{1-\tan^2\frac{(\beta-\vartheta)}{2}},$$

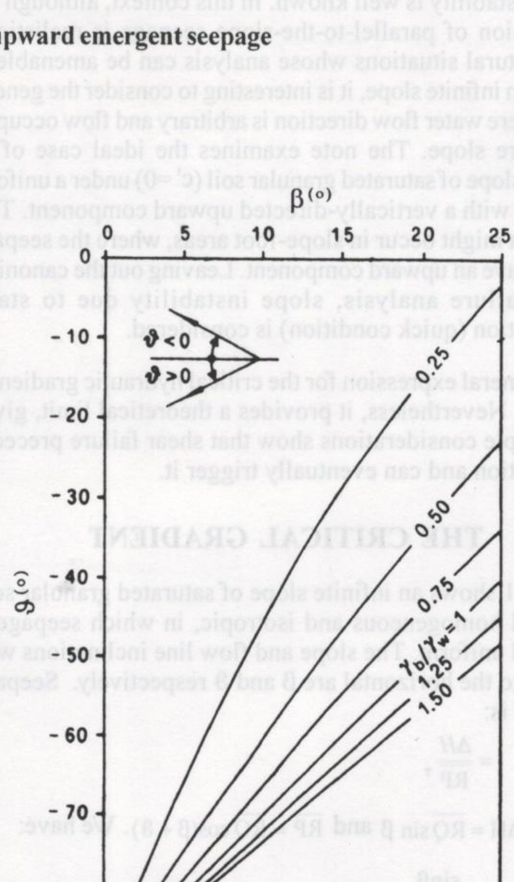

Fig. 2: Critical pair (ϑ, β) as a function of γ_b/γ_w ratio

the argument of sine function in Equation (4) can be written in terms of the second member of Equation (7). This leads to the following expression for i_c:

$$i_c = [\sin^2\beta + (\frac{\gamma_b}{\gamma_w})^2 \cos^2\beta]^{1/2} \quad (8)$$

Again, Equation (8) reduces to Equation (5) in the limiting case β=0 and, furthermore, if $\gamma_b/\gamma_w=1$, the critical gradient is not dependent on β.

Fig. 3 shows plots of i_c versus β for three different values of the γ_b/γ_w ratio.

A COMPARISON OF STATIC LIQUEFACTION WITH SHEAR FAILURE

The concept of critical gradient for a granular soil is associated to the zeroing of the soil effective weight that leads to the quick condition or to static liquefaction. In literature, this phenomenon is described with reference to a cohesionless soil bounded by a horizontal plane (β=0) under

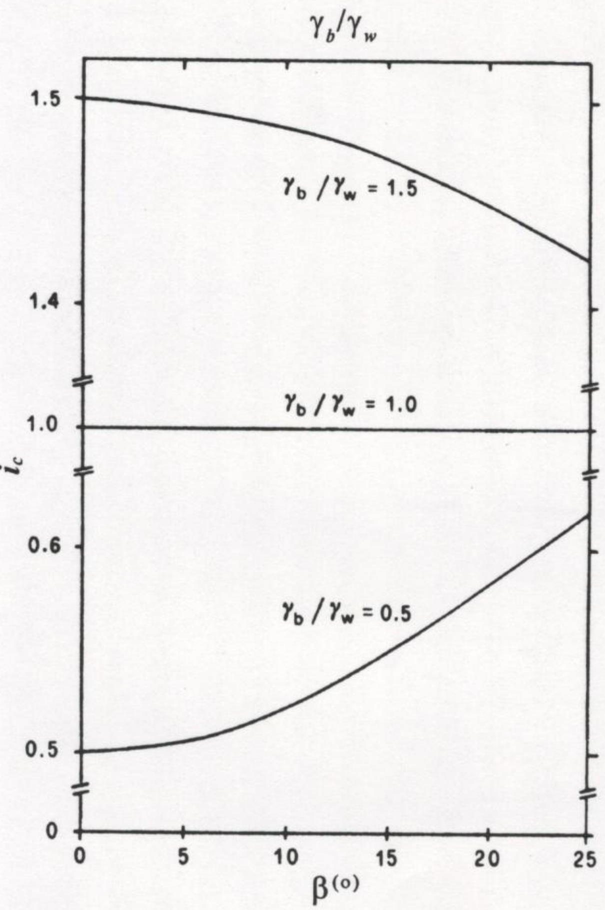

Fig. 3: Critical gradient versus slope angle for selected values of γ_b/γ_w ratio

a vertical ascendant seepage with gradient $i_c = \gamma_b/\gamma_w$. However, it is worth noting that in a slope of cohesionless soil under upward seepage, slope stability is conditioned by shear (or Coulomb) failure rather than static liquefaction failure. This can be seen by comparing the hydraulic gradients or the inclinations of flow lines or the pore pressures on the failure surface related to both situations, or, also, in terms of limiting angle of shear strength. For example, dealing with pore pressures, if c' =0, safety factor F against a shear failure is:

$$F = \frac{\tau_F}{\tau} = \frac{(\sigma - u)\tan\phi'}{\tau}, \quad (9)$$

where, τ_F is the available shear strength, $\sigma=\gamma z\cos^2\beta$ is the total normal stress, $\tau=\gamma z\sin\beta\cos\beta$ is the shear stress or mobilized shear strength, u is the pore pressure, ϕ' is the angle of shear strength (all the quantities refer to the potential sliding surface). At failure (F=1), the pore pressure value on the sliding surface is:

$$u = u_F = \sigma - \gamma z \frac{\sin\beta\cos\beta}{\tan\phi'} = \sigma\left[1 - \frac{\tan\beta}{\tan\phi'}\right] \quad (10)$$

An excess of pore pressure Δu is required to move to a liquefaction condition (u=σ):

$$\Delta u = \gamma z \frac{\sin\beta\cos\beta}{\tan\phi'} \quad (11)$$

so that shear failure precedes liquefaction failure. Moreover, for a loose soil, contraction during shear failure can cause a transient liquefaction. As a consequence, an initially sliding mass of saturated granular soil can be rapidly transformed into a deforming flow (debris flow) (Casagrande 1979; Sassa 1984; Poulos et al. 1985; Fleming et al. 1989).

Note that, from Equation (10), liquefaction and shear failure are simultaneous when β=0; note also that the mobilised angle of shear strength in the liquefaction condition is π/2, that is to say $\tan\phi'_{mob}=\infty$.

CONCLUSIONS

An analysis of stability for an infinite slope of cohesionless soil under seepage of arbitrary direction has been carried out using zero effective stress condition on planes parallel to the slope as a criterion of instability. Seepage gradient required for determining this condition (i.e. soil liquefaction) is the critical gradient i_c and its general expression has been obtained. The expression derived reduces to the well-known Taylor's expression in the special case of upward vertical flow and horizontal ground level.

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