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Analytical assessment of subsidence due to Aquifer Storage and Recovery (ASR) applications to multi-aquifer systems

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ABSTRACT

The present paper emphasises concerns of land subsidence or compression of clay confining beds caused by periodic withdrawal and injection of water from or into the adjacent aquifers. An analytical solution for a one-dimensional case based on a sandwich model is found so that analysis of potential risk of aquifer system deformation due to the technology of Aquifer Storage and Recovery (ASR) can be conducted. A governing equation expressed directly in terms of displacement is employed to describe the one-dimensional subsidence. For simplicity, saturated aquifer systems are assumed to behave like poroelastic material. A cyclic loading function with a triangle pattern is assumed at boundaries to simulate effective stress induced by changes in hydraulic head at boundaries. The both compression and swelling of clay due to the periodic and linear loads at the boundaries are considered in this model. The two aquifers (one above the confining bed and the other beneath) can be pumped independently of each other. The results from the analytical solution are applied to estimate and predict potential risk of land subsidence due to ASR activity and to provide a first-estimate type of guideline for city or regional development and management of water resources.

INTRODUCTION

Today, Aquifer Storage and Recovery (ASR), a water management technology, has been widely employed to meet the demand of water supply or to improve water quality (Pyne et al. 1995). For example, in the United States, more than forty ASR systems currently in more than twelve states are operational or in development. The ASR technology is used for different applications. For instance, in Florida the ASR method is used to improve water quality of contaminated surface water, while in Los Angeles, California, the ASR method is applied to impede sea water intrusion by injecting lightly treated sea water into aquifers. In Texas, the ASR technology is employed to improve water quality and to remove harmful cancer-causing by-products from disinfectants by recharging aquifers with treated wastewater. In contrast, in Nevada aquifers are employed as buffers with utilisation of the ASR approaches so that groundwater can meet high demand in the summers (i.e., to recharge surface water into aquifers in the winters and discharge water from aquifers in the summers). Such activities, however, cause fluctuation of the hydraulic head. In turn, change in pore water pressure is related to change in effective stress on the soil skeleton of the aquifer through the principle of effective stress (Terzaghi 1943). Effective stress fluctuations control the displacement field and cause deformation of the aquifer system (Helm 1972). Eventually, any accumulated compression of an aquifer system due to groundwater withdrawal is reflected at the land surface as land subsidence (Poland et al. 1975). In the present paper, it is the attempt to find an analytical solution in the displacement field. On boundaries, it is considered that the displacement periodically varies due to the injecting-pumping activity. The governing equation in terms of displacement (Helm 1987) for a one-dimensional problem is introduced with consideration of specific boundary conditions. At the boundaries, an assumption of cyclic and linear variations in pore water pressure is made. Accordingly, the boundary conditions are related to the strain of the soil skeleton. An alternating change in effective stress with a triangular pattern that is induced by the fluctuation of pore water pressure p is assumed to simulate the situation of regular injecting or pumping water into or from an aquifer. At the same time the average mean of the fluctuating pore water pressure is assumed to decrease or increase linearly with time or to be constant. Thus, the long-term recharge into the two aquifers can be less than, more than, or equal to the overall discharge from the aquifers. Poroelastic material is assumed in this paper for the clay separator. Thus, the case of the long-term recharge into the two aquifers can be less than, more than, or equal to the overall discharge from the aquifers.

ANALYTICAL SOLUTION

A model of two aquifers with two assumed wells for withdrawal and injection of water is drawn in Fig. 1.

In Fig. 1, the two wells are located in the top aquifer (sand) and the bottom aquifer (sand). They pump and inject water from or into each aquifer independently of each other in pumping-injecting rates Q_{w1} and Q_{w2}.

If one assumes:

1) that the hydraulic separator between the two aquifers is poroelastic,
2) that the fluctuation of pore water pressure within each aquifer approximately changes as a triangular function, and that the average mean of water pressure changes linearly (Fig. 2),

3) that the effective stress principle holds, and
4) that total stresses within the boundary layers do not change very much with time,

then a governing equation written in terms of displacement *u* (Helm 1987) is applicable for a one-dimensional vertical case, initial and boundary conditions based on the model shown in Fig. 1, namely:

$$\frac{\partial u}{\partial t} - c_v \frac{\partial^2 u}{\partial z^2} = B \quad (1)$$

$$u(z,0) = 0, -H \le z \le H \quad (2)$$

$$\frac{\partial u}{\partial z} = [a_1 t / E_{1c} + \sigma_{01} f_1(t) / E_{1s}] \quad z = H \quad (3a)$$

$$\frac{\partial u}{\partial z} = [a_2 t / E_{2c} + \sigma_{02} f_2(t) / E_{2s}], \quad z = -H \quad (3b)$$

where t is time, c_v is the consolidation coefficient of the clay layer and is assumed to be a constant in the present paper, z is the coordinate in the vertical direction, H stands for the half of the thickness of the clay layer in Fig. 1, and B is a function of time and space (Helm 1987). The term σ_0 denotes the amplitude of induced effective stress due to the same value of pore water pressure that changes triangularly at the interface between sand and clay layers, E represents the elastic modulus of the clay layer. The term a is a coefficient related to the slope of the average mean water pressure that linearly decreases or increases with time. For the cases of recharge larger than ($a > 0$), less than ($a < 0$) and equal to ($a=0$) discharge within an aquifer system can be analysed. The term $f(t)$ is a triangular function of time shown in Fig. 2 and is introduced because of fluctuation of water pressures due to pumping-injecting water into or from each of the two aquifers (Fig. 2). Subscripts *1* and *2* denote the upper and lower aquifers respectively, and subscripts s and c stand for swelling and compression of the clay layer (Fig. 3).

In Fig. 3, σ_i denotes the initial stress. From Fig. 3, it is apparent that for the case of a periodic function without a linear trend, one can use E_s except for the first loading (from location 1 to 3). However, for the case of a periodic function with a linear trend, one has to use E_s for swelling under the periodic loading and E_c for compression along the virgin line due to the linear loading (see Fig. 3b). For a case of the decreasing linear trend, the path will be the same as that

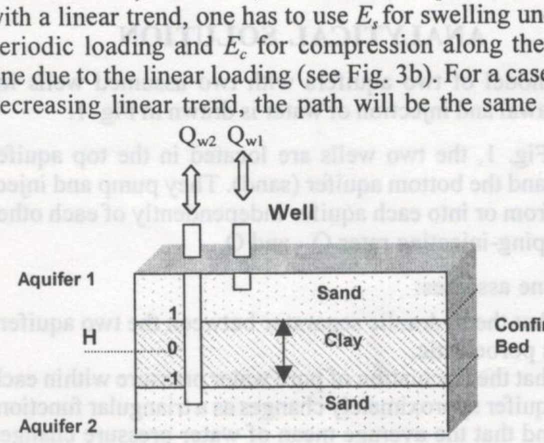

Fig. 1: A diagram for an aquifer system

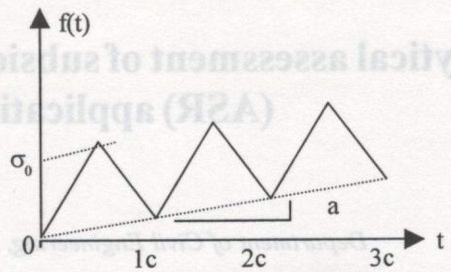

Fig. 2: A diagram for function f(t)

shown in Fig. 3a except for the point at 2 will move towards the left along the slope E_s. Thus only E_s will need to be applied to (3) rather than E_s and E_c. For calculational convenience, the linear poroelastic material is assumed (i.e., using the constant elastic parameters E_s or E_c instead of the non-linear elastic parameters C_s or C_c in this paper).

If one takes Laplace transform (Spiegle 1965) of governing equation (1), and the initial and boundary conditions (2) and (3), one has the following expressions in the Laplace transform space:

$$su^*(z,s) - u(z,0) = c_v \frac{\partial^2 u^*(z,s)}{\partial z^2} + \frac{B}{s} \quad (4)$$

$$u(z,0) = 0 \quad (5)$$

$$\frac{\partial u^*(H,s)}{\partial z} = [\frac{\sigma_{01} \tanh(c_1 s/2)}{c_1 s^2 E_{1s}} + \frac{a_1}{s^2 E_{1c}}], \quad (z = H) \quad (6a)$$

$$\frac{\partial u^*(-H,s)}{\partial z} = [\frac{\sigma_{02} e^{t_0 s} \tanh(c_2 s/2)}{c_2 s^2 E_{2s}} + \frac{a_2}{s^2 E_{2c}}], (z = -H), \quad (6b)$$

where, u^* is the Laplace transform of u, and s is the Laplace transform parameter. The lower case c_1 and c_2 in (6) are related

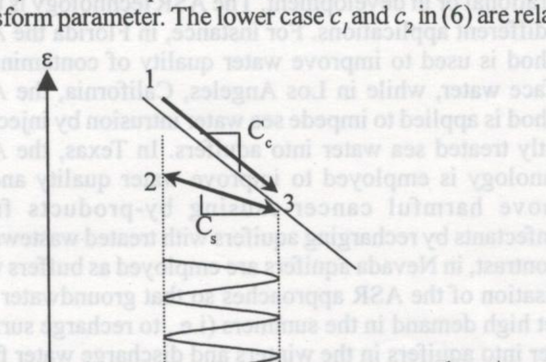

Fig. 3a: A periodic function without a linear trend

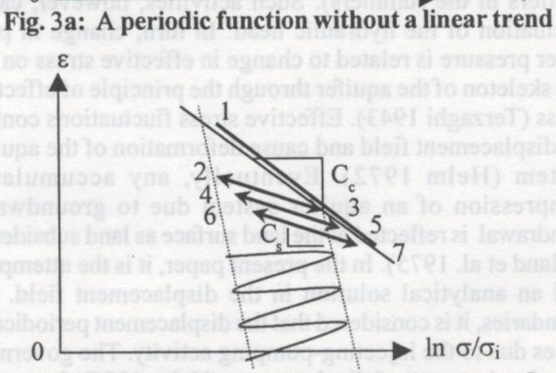

Fig. 3b: A periodic function with a linear trend

to the periodic function (Fig. 2). The term c_v and B are assumed to be constant so (1) can be represented by equation (4).

Keeping initial condition (5) in mind and solving equation (4) for u^*, with an assumption that B is not a function of z, one finds the solution in Laplace transform space to be:

$$u^* = C_1 \cosh[(\frac{s}{c_v})^{1/2} z] + C_2 \sinh[(\frac{s}{c_v})^{1/2} z] + \frac{B}{s^2} \tag{7}$$

where upper case C_1 and C_2 are constant coefficients. With help of boundary conditions [i.e., (6a) and (6b)], C_1 and C_2 are found to be:

$$C_1 = \{[\frac{\sigma_{01}\tanh(\frac{c_1 s}{2})}{c_1 s^2 E_{1s}} + \frac{a_1}{s^2 E_{1c}}] - [\frac{\sigma_{02} e^{t_0 s}\tanh(\frac{c_2 s}{2})}{c_2 s^2 E_{2s}} + \frac{a_2}{s^2 E_{2c}}] / \{2(\frac{s}{c_v})^{1/2} \sinh[(\frac{s}{c_v})^{1/2} H]\} \tag{8}$$

$$C_2 = \{[\frac{\sigma_{01}\tanh(\frac{c_1 s}{2})}{c_1 s^2 E_{1s}} + \frac{a_1}{s^2 E_{1c}}] + [\frac{P_2 e^{t_0 s}\tanh(\frac{c_2 s}{2})}{c_2 s^2 E_{2s}} + \frac{a_2}{s^2 E_{2c}}]\} \times \{2(\frac{s}{c_v})^{1/2} \cosh[(\frac{s}{c_v})^{1/2} H]\} \tag{9}$$

Substituting equations (8) and (9) into (7) and taking the inverse Laplace transform of each term in (7), the solution can be written as a function of non-dimensional variables T and Z with five parts:

$$u\,(Z,T) = u_I + u_{II} - u_{III} + u_{IV} + u_V \tag{10}$$

The variables Z, T and ω are defined as non-dimensional space $Z(=z/H)$ and time $T(=tc_v/H^2)$ as well as angular frequency $\omega\,[=2\pi(2n-1)/c_i,\ i = 1,2]$. The five components of the solution (u_i, $i = I \ldots V$) are given by the following expressions:

$$u_I = Q_1\{\frac{T}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin(\varpi_{1n}T)}{\varpi_{1n}} + \sum_{n=1}^{\infty}\frac{(-1)^n \cos(N_n Z)(1-e^{-N_n^2 T})}{N_n^{\ 2}} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n \cos(N_n Z)[\cos(\varpi_{1j}T - \phi_{11}) - e^{-N_n^2 T}\cos(\phi_{11})]}{\sqrt{(N_n^2)^2 + \varpi_{1j}^{\ 2}}} \tag{11a}$$

$$u_{II} = Q_1\{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sin(M_n Z)(1-e^{-M_n^2 T})}{M_n^{\ 2}} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n \sin(M_n Z)[\cos(\varpi_{1j}T - \phi_{12}) - e^{-M_n^2 T}\cos(\phi_{12})]}{\sqrt{(M_n^2)^2 + \varpi_{1j}^{\ 2}}} \tag{11b}$$

$$u_{III} = -Q_2\{\frac{T}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin(\varpi_{2n}T)}{\varpi_{2n}} + \sum_{n=1}^{\infty}\frac{(-1)^n \cos(N_n Z)(1-e^{-N_n^2 T})}{N_n^{\ 2}} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n \cos(N_n Z)[\cos(\varpi_{2j}\Delta T - \phi_{21}) - e^{-N_n^2 T}\cos(\varpi_{2j}T_0 + \phi_{21})]}{\sqrt{(N_n^2)^2 + \varpi_{2j}^{\ 2}}} \tag{11c}$$

$$u_{IV} = Q_2\{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sin(M_n Z)(1-e^{-M_n^2 T})}{M_n^{\ 2}} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n \sin(M_n Z)[\cos(\varpi_{2j}\Delta T - \phi_{22}) - e^{-M_n^2 T}\cos(\varpi_{2j}T_0 + \phi_{22})]}{\sqrt{(M_n^2)^2 + \varpi_{2j}^{\ 2}}}\} \tag{11d}$$

$$u_V = -Q_3[\frac{T^2}{4} + \sum_{n=1}^{\infty}(-1)^n \cos(NZ)[\frac{T}{N^2} - \frac{(1-e^{-N^2 T})}{N^4}] + Q_4[\sum_{n=1}^{\infty}(-1)^{n-1}\sin(MZ)[\frac{T}{M^2} - \frac{(1-e^{-M^2 T})}{M^4}] + \frac{TBH^2}{c_v} \tag{11e}$$

in which $\Delta T\,(=T-T_0)$ is incremental non-dimensional time, T_0 represents the initial T, N and M are parameters, and defined as functions of n by $n\pi$ and $(2n-1)\pi/2$ respectively, coefficients Q_i $(i = 1 \ldots 4)$, ϕ_{ji} $(j = 1, 2$ and $i = 1, 2)$ and ϖ_i $(i = 1, 2)$ are defined as follows:

$$Q_j = \sigma_{0j} H / 2E_{js}, \quad (j = 1, 2) \tag{12}$$

$$Q_3 = (a_2 / E_{2c} - a_1 / E_{1c}) H^3 / c_v \tag{13}$$

$$Q_4 = (a_2 / E_{2c} + a_1 / E_{1c}) H^3 / c_v \tag{14}$$

in which ϖ and ϕ are given by:

$$\varpi_i = \omega_i H^2 / c_v, (i = 1,2) \tag{15}$$

$$\phi_{i1} = \tan^{-1}(\varpi_i / N^2), (i = 1, 2) \tag{16}$$

$$\phi_{i2} = \tan^{-1}(\varpi_i / M^2), (i = 1, 2) \tag{17}$$

In the above formulas, one should keep it in mind that ϕ is a function of *n*.

Based on the solution [i.e., (10) and (11)], it is not difficult to find the strain field by taking the derivative with respect to normalised space *Z*, namely:

$$\varepsilon(Z,T)= \varepsilon_I + \varepsilon_{II} + \varepsilon_{III} + \varepsilon_{IV} + \varepsilon_V \tag{18}$$

where $\varepsilon_i(i = I \ldots V)$ are found from (11) to be:

$$\varepsilon_I = Q_1\{-\sum_{n=1}^{\infty}\frac{(-1)^n \sin(N_n Z)(1-e^{-N_n^2 T})}{N_n}\} + \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n N_n \sin(N_n Z)[\cos(\varpi_{1j}T-\phi_{11}) - e^{-N_n^2 T}\cos(\phi_{11})]}{\sqrt{(N_n^2)^2+\varpi_{1j}^2}} \tag{19a}$$

$$\varepsilon_{II} = Q_1\{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cos(M_n Z)(1-e^{-M_n^2 T})}{M_n} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n M_n \cos(M_n Z)[\cos(\varpi_{1j}T-\phi_{12}) - e^{-M_n^2 T}\cos(\phi_{12})]}{\sqrt{(M_n^2)^2+\varpi_{1j}^2}}\} \tag{19b}$$

$$\varepsilon_{III} = -Q_2\{-\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sin(N_n Z)(1-e^{-N_n^2 T})}{N_n} + \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n N_n \sin(N_n Z)[\cos(\varpi_{2j}\Delta T - \phi_{22}) - e^{-N_n^2 T}\cos(\varpi_{2j}T_0+\phi_{22})]}{\sqrt{(N_n^2)^2+\varpi_{2j}^2}}\} \tag{19c}$$

$$\varepsilon_{IV} = Q_2\{\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\cos(M_n Z)(1-e^{-M_n^2 T})}{M_n}] - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n M_n \cos(M_n Z)[\cos(\varpi_{2j}\Delta T-\phi_{22}) - e^{-M_n^2 T}\cos(\varpi_{2j}T_0+\phi_{22})]}{\sqrt{(M_n^2)^2+\varpi_{2j}^2}}\} \tag{19d}$$

$$\varepsilon_V = Q_3[\sum_{n=1}^{\infty}(-1)^n \sin(NZ)[\frac{T}{N^1}-\frac{(1-e^{-N^2 T})}{N^3}]] + Q_4[\sum_{n=1}^{\infty}(-1)^n\cos(MZ)[\frac{T}{M^1}-\frac{(1-e^{-M^2 T})}{M^3}]] \tag{19e}$$

Accordingly, from the solution (10), the accumulated deformation can be calculated from the difference $u(Z, T) - u(Z_0, T)$, where Z_0 can be any element elevation at which no displacement (or no vertical movement) is occurring, say, at non-deformed bedrock or at a point that serves *-H/H* a datum.

In order to estimate the impact of pumping-injecting water within aquifers, some simplified cases based the found solution are discussed in the next section.

DISCUSSION OF THE SOLUTION

From equations (10) and (11), one can see the analytical solution comprises three parts, namely, periodic variation, exponential reduction and quadratic change with non-dimensional time *T*. For convenience of analysis, if one chooses $Z = 0$ (i.e., $z = 0$), the solution (10) reduces to:

$$u(0,T) = u_I(0,T) + u_{III}(0,T) + u_V(0,T) \tag{20}$$

where:

$$u_I = Q_1\{\frac{T}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin(\varpi_{1n}T)}{\varpi_{1n}} + \sum_{n=1}^{\infty}\frac{(-1)^n(1-e^{-N_n^2 T})}{N_n^{\ 2}} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n[\cos(\varpi_{1j}T-\phi_{11}) - e^{-N_n^2 T}\cos(\phi_{11})]}{\sqrt{\varpi_{1j}^{\ 2}+(N_n^2)^2}} \tag{21a}$$

$$u_{III} = -Q_2\{\frac{T}{2} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin(\varpi_{2n}T)}{\varpi_{2n}} + \sum_{n=1}^{\infty}\frac{(-1)^n(1-e^{-N_n^2 T})}{N_n^{\ 2}} - \frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^n[\cos(\varpi_{2j}\Delta T-\phi_{21}) - e^{-N_n^2 T}\cos(\varpi_{2j}T_0+\phi_{21})]}{\sqrt{(N_n^2)^2+\varpi_{2j}^{\ 2}}} \tag{21b}$$

$$u_V(0,T) = -Q_3\{\frac{T^2}{4} + \sum_{n=1}^{\infty}[(-1)^n[\frac{T}{N^2} - \frac{1}{N^4}(1 - e^{-N^2T})]]\} + \frac{TBH^2}{c_v} \tag{21c}$$

If one assumes that pumping and injecting water in the upper and lower aquifers start simultaneously with the same period, namely $\varpi_1 = \varpi_2 = \varpi$ and $\varpi T_0 = 0$ (i.e., $\omega_1 = \omega_2 = \omega$ and $\omega t_0 = 0$), then (20) reduces to:

$$u(0,T) = (Q_1 + Q_2)\{\frac{T}{2} + \sum_{n=1}^{\infty}(-1)^n(1 - e^{-N^2T})/N^2 - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin\varpi T}{\varpi}$$
$$-\frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}(-1)^n[\cos(\varpi T - \phi) - e^{-N^2T}\cos\phi]/\sqrt{(N^2)^2 + \varpi^2}\}$$
$$-Q_3\{T^2/4 + \sum_{n=1}^{\infty}[(-1)^n[T/N^2 - (1 - e^{-N^2T})/N^4]\} + TBH^2/c_v \tag{22}$$

where, ϕ is defined in the same form as (1) except for $\omega_i = \omega$ (i = 1, 2). If a sign convention is applied in (22), namely, that downward displacement (e.g., u_I) is assumed to be positive and upward displacement (e.g., u_{II}) is negative. If one assumes the pumping-injecting activity in both the upper and lower sandy layers has the same period but different phase, say a phase-lag p, namely, $\varpi_1 = \varpi_2 = \varpi$ and $\varpi T_0 = \pi$ (i.e., $\omega_1 = \omega_2 = \omega$ and $\omega t_0 = \pi$), then (22) reduces to:

$$\mathrm{u}(0,\mathrm{T}) = (\mathrm{Q}_1 + \mathrm{Q}_2)\{\frac{\mathrm{T}}{2} + \sum_{n=1}^{\infty}(-1)^n(1 - e^{-N^2T})/N^2 - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\sin\varpi T}{\varpi}$$
$$-\frac{8}{\pi^2}\sum_{j=1}^{\infty}\sum_{n=1}^{\infty}(-1)^n[\cos(\varpi T - \phi) - e^{-N^2T}\cos\phi]/\sqrt{(N^2)^2 + \varpi^2}\}$$
$$-Q_3\{T^2/4 + \sum_{n=1}^{\infty}[(-1)^n[T/N^2 - (1 - e^{-N^2T})/N^4]\} + TBH^2/c_v \tag{23}$$

It is interesting that (22) shows a case in which a maximum displacement may occur at $Z = 0$ if the pumping-injecting activity has the same feature of periodic changes in pore water pressure at both the top and bottom sandy layers. In contrast, (23) shows a case in which a minimum displacement may take place at $Z=0$ if the pumping-injecting activity is the same except that a phase-lag p exists between these two sandy layers. This means that one can reduce the potential displacement at $Z = 0$ or in a zone near the centre of the clay layer by controlling the pumping-injecting activities with a proper phase-lag between the upper and lower sandy areas. In fact, it is true for the entire cross-section of compressible confining bed. Furthermore if the sum between Q_1 and Q_2 is assumed to be negligibly small (i.e., $Q_1 \approx -Q_2$), which means that water is discharged from the aquifer above the confining layer and is recharged into the one below aquifer, (20) can be further simplified to:

$$u(0,T) = -Q_3\{T^2/4 + \sum_{n=1}^{\infty}[(-1)^n[T/N^2 - (1 - e^{-N^2T})/N^4]\} + TBH^2/c_v \tag{24}$$

If one recalls the definition of Q_3 in (13) and keeps the sign convention in mind, the first term in the relation (24) will become negligibly small with assumptions of $a_1 \approx -a_2$ and $E_{c1} \approx E_{c2}$. In addition, (24) will simplify further when B is negligibly small as assumed by Helm (1987).

In brief, if the amplitude of periodic stress s_0 and the swelling elastic modulus E_s are to be the same at the upper and lower boundaries of the clay layer (i.e., $E_{s1} \approx E_{s2}$ and $\sigma_{01} \approx \sigma_{02}$, or their ratios), then the displacement can possibly reduced to a minimum value. In other words, the risk of subsidence due to APR activities can be reduced. Moreover, if the overall discharge and recharge can be controlled so that the mean water pressure either remains stable ($a \approx 0$) or changes along a similar but opposite slope (namely, $a_1 \approx -a_2$), then from (24), displacement can reduced (i.e., $Q_3 \approx 0$). Well-controlled phase lag by planning pumping-injecting seasons between the top and bottom aquifers will reduce the risk of land subsidence dramatically. It is apparent that if the pumping-injecting activity takes place in a single aquifer only (i.e., either the top or the bottom aquifers, but not both), one may be not able to reduce the risk of subsidence. Finally, it should be pointed out that though a simplified procedure is demonstrated for a special case of $Z = 0$, the above conclusions are true for a general case (i.e., $-1 < Z < 1$) as well.

Based on the solution in (10), the displacement along Z direction for both short and long terms are drawn in Fig. 4a and 4b. Fig. 4a and 4b are drawn with the following parameters: the pumping and injecting period to be 356 day; $H = 5$ m; $\sigma_{01} = \sigma_{02} = 10$ KPa; $a_1 = a_2 = 1E\text{-}4$ Kpa/day; E_c =100 and 1200 Kpa; E_s = 500 and 5000 Kpa; c_v=0.001 m²/day; and $B = 1E\text{-}7$ and $1E\text{-}5$ m/day. Two families of curves in Fig. 3a and 3b are illustrated with different periods of time (i.e., 150 – 400 day, and 10 –50 year). For simplicity, $E_1 = E_2$, $\sigma_{01} = \sigma_{02}$ and $\varpi_1 = \varpi_2$ are assumed (i.e., $c_1 = c_2$). From the above figures, one can concluded that the accumulative displacement or subsidence in Fig. 4a is dominated by repeated loading during a short period of time, and controlled by linear loading in a long term.

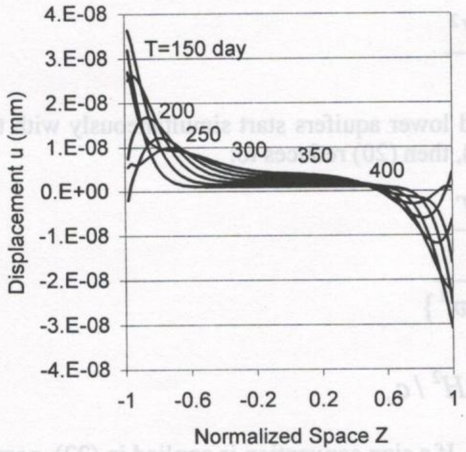

Fig. 4a: Displacement u versus Z for a short period

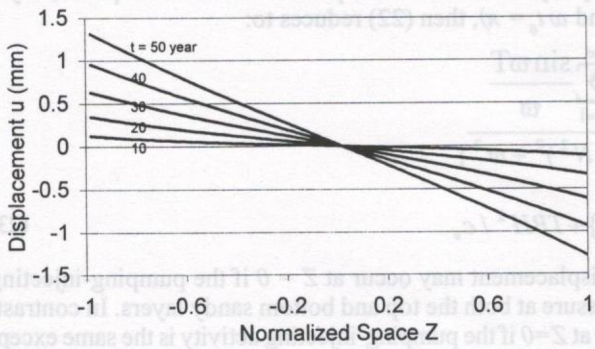

Fig. 4b: Displacement u versus Z for a long term

CONCLUSIONS

In brief, the following conclusions can be drawn. First, a one-dimensional analytical solution has been found. This analytical solution is based on a model featured by a two-aquifer sandwich pattern. Boundary conditions are set according to the triangular pumping-injecting activity at the interfaces of two sand and one clay layers. Second, the found solution in the displacement field is composed of three components, namely, the components of periodic fluctuation, exponential reduction, and quadratic variation with time. Third, displacement due to the ASR activities is dominated by the repeated loading during a short period of time and controlled by the linear loading in a long term. Fourth, a special case, namely $Z = 0$, has been discussed. Based on the model in Fig. 1, the following factors may affect the displacement u at a zone near the centre of the clay layer (compressible bed):

1) The phase-lag ϖT_0 of the pumping and injecting activities between the upper and lower aquifers. When the phase-lag equals p, the potential risk of deformation may be reduced when compared to a case without a phase-lag (i.e., $\varpi T_0 = 0$),

2) Similarity of pumping-injecting periods of the upper and lower aquifers. Identical periods (i.e., $\varpi_1 = \varpi_2$) reduce the potential displacement of the clay layer when $\varpi T_0 = 0$ is considered.

3) Similarity of parameters Q_i $(i = 1, 2)$ related to E, σ_0 and v at boundaries as well as their ratio. In other words, if $Q_1 \approx Q_2$, the displacement of the clay layer at Z may lessen when $\varpi T_0 = 0$ and $\varpi_1 \approx \varpi_2$ are assumed.

4) Negligibly small Q_3 related to the parameter a/E_c. Namely, if $(a_1/E_{c1} \approx -a_2/E_{c2})$, risk of subsidence can decrease.

The facts listed above are true for a general case ($-1 < Z < 1$) as well. Finally, pumping-injecting water in two or more aquifers (e.g., the sandwich model: *s-c-s* in Fig. 1 or multiple sandwich model: *s-c-s-c-s*) has better chance, when compared to the case of pumping-injecting in a single aquifer, at reducing the displacement.

It should be pointed out that in order to allow an analytical approach [i.e., equations (1), (2) and (3)], some factors are not taken into account in the present paper. For instance, effects of stress history (e.g., current stress status related to over, under and normally consolidated states). These factors, however, can be considered by using a numerical approach as was done by Helm (1975).

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