Reciprocal Learning Mathematics through Honeycomb Patterns

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ABSTRACT

The purpose of this paper is to examine the mathematical ideas, problems, and solutions from the honeycomb patterns. This is a qualitative research design under an interpretive paradigm. I used interview, participant observation, small group discussions, photos, notes taken, and personal reflection for the data collection tools. The information was analyzed by participants generated text, activities, and experience of learning mathematics through the honeycomb patterns. The major themes that generated were algebra and students knowing of in-depth processes and solutions; algebraic and geometrical interpretation of 2D shapes with formulas; and geometrical knowledge of honeycomb patterns. The study showed that honeycomb patterns are the source for reciprocal learning of multidimensional area contents like algebra (quadratic and arithmetic sequence, hexagonal congruent tessellation, and geometry parallel lines, angles, and similar and congruent shapes together). What the students and the teachers need in reciprocal learning is the skill of observing patterns, redrawing, imagining, estimating, and logically intervening and shearing in them. The findings of the study can be helpful to teachers, students, and educators for reciprocal ways of learning, teaching, and training mathematics, connecting such patterns interestingly and meaningfully.

Keywords: School Mathematics, Learning, Honeycomb Patterns

Introduction

Johnson et al. (2020); Pradhan (2017); and Rosa & Orey (2011) have focused on learning school mathematics linked with students' lives and their real-world experiences, like bee comb patterns, which triggered us in the direction of transformative pedagogies. The first attraction to our eyes was how beehive patterns can be applied by students and trainee teachers to learn and teach algebra and geometry. How do students and trainee teachers link various areas of mathematics concepts to the patterns made by honeybees when building their houses? How can students and trainee teachers generate local knowledge into global knowledge with proper understanding, thinking, and communicating mathematically on honeycomb patterns with reasoning in a creative way? The articles (for example, Acharya et al., 2021; D'Ambrosio, 2006) also reminded us that his kind of pedagogy practice and teacher training have played an important role in student-centered learning, where students engage and observe themselves to learn new approaches and techniques. For this, a teacher is also a key person, and she/he should be able to support students and provide a meaningful way of learning to build the various skills to solve problems, generate big ideas, and connect individual ideas (Mawadah, 2015).

Many people have said that mathematics is abstract, it's difficult to study, and they think and believe it to be a hard subject (Lamichhane & Belbase, 2017). However, kinds of literature on ethnomathematics (for example, Mauluah & Marsigit, 2019; Parajuli & Koirala, 2022; Rosa & Orey, 2015; Rosa & Orey, 2018; Tarman, 2016) advocate that if mathematics teaching and learning are practiced linked with students' real-life activities, it will be a great response to so-called difficult mathematics and the resource for innovating mathematics learning as the complete package against spoon feeding (Chahine, 2021; D'Ambrosio, 2020). But by observing the practices, experiences, classroom and training activities, and classroom presentations of students, many questions aroused in our minds. Various literates on ethnomathematics have claimed that teachers can teach and students can learn mathematics easily by linking it with the contexts and nature in which they are already familiar with and have experienced in their life. Other pieces of Nepali and abroad pieces of literature (Pradhan, 2021; Pradhan et. al.; Sharma & Sharma, 2021; Widodo, 2019) say that against teacher-cantered pedagogies, students learn mathematics long-term in meaningful understanding by engaging themselves in nature under teachers' guidelines through an ethnomathematics perspective.

The various geometrical shapes, their properties, the quadratic sequence patterns, arithmetic patterns, algebra formulae geometrical exploration, and many more parallel lines can be learned by doing through observing honeycomb patterns as an example but such teachers training and students classroom practices making differences in the real-world problems with the process of solving them self hardly be seen in the context of Nepalese classroom practices. Learning by doing mathematics linked with honeycomb patterns and activities is very rare in the context of Nepal. So what mathematics content can students and trainee teachers learn by embedding honey bees' hexagonal patterns? How do students and trainee teachers explore their experiences and views in the process of learning various areas of mathematics linked to honeycomb patterns? We were very interested in researching that area.

This study aims to explore the answer to the research questions, such as: a) What mathematics contents can trainee teachers and ninth-grade students learn by using honey bee's hexagonal patterns? b) How do ninth-grade students and trainee teachers explore their experiences and views in the process of learning mathematics (algebra, geometry, and mensuration) through honeycomb patterns in the context of Nepal?

Research Paradigm, Methodology, and Methods

Literature has suggested that quality qualitative research attends to a research paradigm, methodology, and methods (data collection and data analysis procedures) and should demonstrate internal consistency between them (Mukhles and Al-Ababneh, 2020; Saunders et al., 2009; Zaidi & Larsen, 2018). Therefore, in this paper, we have used an interpretive research paradigm under relativist ontology, subjective epistemology, and naturalist methodological procedures. This paradigm helps us to understand and capture our trainee teachers and students' subjective and multiple worldviews, thoughts, realities, and experiences in depth on how they learn and respond to mathematics (Chowdhury, 2014; Kelliher, 2011) through honeycomb patterns. It helps us to evolve and continue to observe their activities very closely, time and again, for the collection of sources of knowledge rather than rigid frameworks (Mukhles & Al-Ababneh, 2020; Zaidi & Larsen, 2018). The ontological stance of this study was that student participants constructed mathematical knowledge from various perspectives. They also interpret and make meaning in an inter-subjective way (Clark et al., 2022), exploring various ways of thinking about honeycomb structures by drawing diagrams and writing the steps of solutions.

Similarly, research methodology is the strategy, plan of action, process, or design and use of particular methods, linking the choice of methods to the desired outcomes(Crotty, 1998, p. 77). For this, we have used a qualitative research design to generate valid and scientific knowledge from the field (Korstjens & Moser, 2018). The research participants were the public school teachers in Pokhara Metropolitan City who are teaching in the basic label and nineth-grade students' who studied in private school and who can link mathematics with honeycomb creatively. Therefore, the reality is that how students and trainee teachers have experienced learning mathematics interaction among friends and teachers is linked with such patterns (Zaidi & Larsen, 2018). Therefore, this methodology allowed us to capture participants' individual and group experiences and world views on learning mathematical activities connecting honeycombs in the inductive process of knowing (Tomaszewski et al., 2020; Hartas, 2010; Willig & Rogers, Eds., 2017).

Research Method

Research method is the technique or procedure of data collection that is used to gather information related to research questions and analyse the data opined by Al-Ababneh (2020; Johnson et al., 2020). Therefore, as the choice of research method, we used qualitative research for the collection and thematic analysis of the field data. To capture the multi-dimensional realities, it was necessary to choose the appropriate research sites, areas, schools, and potential research participants. We conducted the research with four students in grade nine of the academic year 2022 at the private school. Similarly, four teachers among the 45 who participated in the capacity-building training for basic-level teachers conducted by Pokhara Metropolitan City were also consulted for their observation. The result of the project work with the 9th graders was also incorporated into it.

Tools and Procedure of Data Collection

We used multiple sources for data generation (Denzin & Lincoln, 2011; Denzin & Lincoln, 2005). We also followed Fox, and Alldred (2023) for selecting an appropriate method and sources of relevant data and did data analysis accordingly. As they suggested, we adopted multiple data sources during the field visit such as interviews, participant observation, individual and group discussion, and informal conservation. We collected photos of the mathematics-generated text on the honeycomb. At times, we were careful and engaged them to capture the participants' realities on the issue (Forsey, 2010). We also took key points in the diary after observing their activities. In other words, these tools were administered carefully exploring the issues associated with the research topic and ideas collection (Al-Ababneh, 2020). We also participated and observed students' activities time and again very closely and how they explored, wrote, and presented the solutions sequentially in different rounds. We also built trust on collected information from individuals in a group. We took field notes and reflective notes as well. Besides, we accessed these students for the maximum time and frequently shared my ideas with them through face-to-face and digital communication. We used participants' observation, informal conservation, and observation of students 'and trainee teachers' activities and their generated text and diagrams on honeycomb patterns. It has brought the students 'and trainee teachers' experiences in learning mathematics through honeycomb (Korstjens & Moser, 2022; Saunder et al., 2009). As a qualitative researcher, we observed very closely and continuously how the students and trainee teachers learned mathematics through the honeycomb. As proposed by (Moser & Korstjens, 2018). We frequently engaged with my students and trainee teachers in a three-day training program in my role as a facilitator. We took photos of their activities as well.

Data Analysis

As opined by Braun et al. (2016), thematic analysis for qualitative research identifies organizing large data and patterns ('themes') in a dataset and describing and interpreting the meaning. In the process of analyzing the field data, facts, figures, and diagrams produced by mathematics were cleaned up, reduced, sorted, coded, and processed to provide usable information. Additionally, we interpreted the data as qualitative research demands, developing the themes as we went along (Korstjens & Moser, 2022). In this study, we followed the six-phase model of thematic analysis suggested by Korstjens & Moser (2022). These are transcribing, familiarization of the contents of the field by reading and re-reading all data items, making notes, coding (theme development, revision, and naming as the core analytic work), organizing and categorizing codes, coding data, reviewing and revising those candidate themes, and developing a rich analysis of the data and making finalized themes and writing up.

Each research participant's experience was transcribed and coded with more attention to constructing meaningful categories for learning mathematics by linking honeycombs. The coding of the transcribed data was focused on the final grouping of the codes and categories by merging similar categories into a broader, overarching final theme. Thus, we produced thick, evocative descriptions of personal and interpersonal experiences. Individual reflection is required for data analysis since it requires an awareness of the researcher's contribution to the formation of meaning throughout the research process (Johnson et al., 2020). Therefore, we reflected on our teaching, training, learning, and research experiences, beliefs, and knowledge of contextual activities. We were aware that our interpretation might contribute to this process of exploration of participants' meaning while constructing mathematical knowledge. In this manner, the connection between different areas of mathematical knowledge using nature honeycomb patterns was generated by analysing and interpreting individual students' drawn diagrams, working practices, written text in papers, and their views regarding the issue. While interpreting the data, we considered that there were multiple truths about social realities based on the participants' opinions.

Results and Discussion

Honeycomb patterns that the students and trainee teachers observed and indicated the algebra, translation, symmetry, geometry parallel lines, and similar and congruent concepts of various areas of mathematics. Varghese & McCusker (2006) have suggested that they noted the support for learning and teaching mathematics based on local knowledge.

From it, students developed creative thinking skills, developing questions, communicating skills, and reasoning skills. Through social constructivist perspectives like group interaction, coordination, presentation, and sharing, collaborate for the construction of knowledge as an active learner by learning and doing. Then they figured out various areas of mathematics in the honeycomb. Such activities helped students to promote an environment where mathematics is applied in their career and life rather than just solving problems in books (Honegger, 2020). The ethnomathematics researchers (for example, Arisetyawan et al., 2014; D'Ambrosio, 2012; Mesquita et al., 2012; Rosa et al., 2016) revealed that learning this way, students did not only learn mathematics; they also learned various life skills that show the symbol of academic success (Honegger, 2020). In this light, we have presented below the original bee hive as seen in Fig. 1.

Figure 1 : Honeycomb Patterns and Communities Activities

Honeycomb Patterns and Students' Mathematical Learning

Algebra is the generalization of arithmetic. In some cases, algebra can be shown through geometrical interpretation. Beehive is a geometrical shape with wonderful hexagonal patterns. However, we practice very little link with contextual activities. But some of the girl students noted algebra in the beehive patterns and learned to make questions with solutions in detail. The informants have generated algebra, some from banana leaf, some from kitchen arrangements of glasses, some from bamboo handicrafts, and some from honeycomb patterns.

Figure 2 : Activity Based Learning Mathematics in Holistic Approach Through Honeycomb Patterns

One of the girls, Sarita Sharma, in class nine, has done total mathematical activities like algebra, arithmetic, and geometry on honeycomb patterns. She generated a quadratic sequence, a sequence of terms using hexagonal patterns, its general terms and process of generating patterns, and a geometrical interpretation of the algebraic formula, which can be seen in figures 2, 3, and 4. However, two girls, Samjhauta and Sarita, and some boys found different congruent hexagonal patterns in triangular and rectangular shapes. They took out the portion with triangular shapes. Figs. 3 and 4 were drawn figures and field text that were generated. Boys informants made congruent hexagonal patterns of holes in a honeycomb. However, girl informants learned and redrew by cutting the small portions and rearranging them in sequential patterns. They also claimed the quadratic sequence formed by hexagonal patterns in Pascal's triangular shapes formed by holes is 1, 3, 6, 10, 13,... with a detailed process with the first term (a) = 1 and the second common difference (d) = 1, which can be seen in detail in Fig. 3 and Fig. 4.

Quadratic Sequence with Solutions on Successive Hexagonal Patterns of Honeycomb

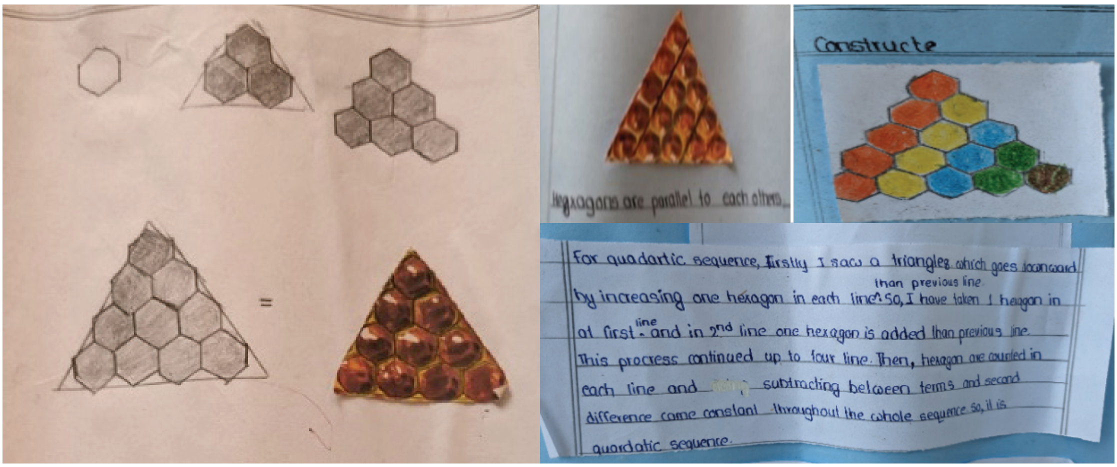


Figure 3 : Text Generated by Sarita, 9th February 2023.

The girl students Sarita and Samjhauta argued that "we did not only see the quadratic sequence in patterns; we also brought the overall process and the steps of the solution with the process." When we observed their generated text, Sarita constructed the patterns of sequence, formed the triangular shapes, and found the quadratic sequence of regular hexagonal as 1, 3, 6, 10, and 15 with the second common difference of 1". In the same way, Samjhauta reflected on the overall process, presented steps, and derived its general term on the whiteboard through discussion with her group and classroom (see Fig. 4).

Quadratic Sequence on Honeycomb Patterns

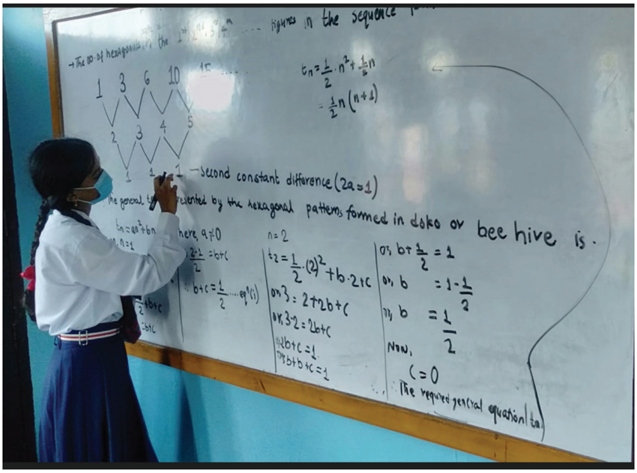


Figure 4 : Text Generated by Samjhauta, 12th February 2023.

We analysed all the writing texts of the students on the white board and charts of the students. From the analysis of their writing and drawing patterns, we came to know that they drew the patterns of diagrams of regular hexagons, wrote the sequence in specific orders, took the difference, and found the first difference and second constant difference, 2 a = 1. They also expressed the detailed process and got its general terms on a chart and whiteboard, as well as any number of terms, by using the following process: We knew from their written text that they let a = second common difference = 1, and they had written its general term (tn) = an + b ....(i). Such activities go on from time to time, often in pairs or small groups. We noticed that the participants practiced, interacted, and perceived shared ideas in a small group for the construction of knowledge, listed and interpreted as presented above.

In this light, the above patterns, and below text are the reconstruction of the first Sarita and second detail process for a general term by Samjhauta of class 9 of a private school in sequence and series. They said:

> *There are many congruent regular hexagonal patterns made by the bee in the beehive. They reconstructed using their patterns. After the reconstruction of the figure, they saw the different parallel and vertical sequences and series patterns of congruent hexagonal. They found the general term below. Apart from Sarita, Samjhauta also made the sigma notation. This notation can be connected with the figures of geometry, arithmetic, and algebra as well (Reading the text of Sarita, 11th February 2023).*

We tried to compare and contrast the participants' ways of conceiving and producing mathematics. As we found, Sarita has seen complex and abstract mathematical patterns generated in 'Honeycomb patterns', but Samjhauta (see Fig. 4) wrote an in-depth process step by step, making the sequence of hexagons sequentially in horizontal order. She also took the first and second differences. From the observation of several regular hexagonal pattern sequences that were generated by Sarita and Samjhauta, the process of knowing and generating information about sequences and series. They gave detailed patterns, problems, and solutions of combined proving questions of sequence with the patterns below:

- Search any number of sequence patterns and be confirmed that hold arithmetic/ quadratic/ geometric/ any and construct any one in your copy and the beehive.
- Add one more similar pattern either in your copy or in the original shape of honey comb
- Construct the number of sequences that were constructed by honey to make its house stay in each pattern.
- Decide by mental test whether it is holding arithmetic sequence or geometry sequence
- Find its general term with process
- Estimate it's any number of terms.
- Express the sequence in its corresponding series and express it in sigma notation too.
- List other possibilities of patterns with solutions steps.

Both the girls and other two boys were able to address all the solutions with steps of the above 8 questions answering them through honeycomb patterns in copy and presenting and writing on the whiteboard with right process using regular hexagonal patterns.

First, we saw that both Sarita and Samjhauta had expressed the no. of dots in the pattern in this way: **(can see Fig.2, 3 and 4)**.

1, 3, 6, 9,12, 15, 18,…….. Both had shown the first difference and second difference of the above redrew figures on no. patterns in the above sequence. They mentioned the sequence as quadratic as she justified that as the second difference is the same i.e. 1. However teachers who enrolled in mathematics training said and listed arithmetic patterns after observing the honeycomb with out process **(can see Fig 6 right teachers activities)**. However, girls and boys students expressed with the process patterns generated general term of the quadratic sequence $(t_n) = an^2 + bn + c, a \neq 0$........... (i). We found as they assumed 2a = second difference = 1 by taking the help of ideas from the reference book.

or, 2a= 1

∴ a = . Also, he put as n=1 in (i), then he found (t1) = a $\times 1^2$ + b ×1 +c,

1 = + b +c

or, 2= 1+ 2b+ 2c

or, 1 = 2(b+c) ....(ii)

Also, he kept, n=2, (t_2) = a $\times 2^2$ + b ×2 +c

3 = × 2^2 + (b +c) +b [∴ b= -c

Or, 1 = + b [they took help from (ii) as well

∴ b = . They substituted the value of ‘b’ in eqn (ii), ∴ c = 0

They concluded the general term as (tn) = = n^2 + n + 0 = in the formulae as algebra in a honeycomb pattern.

Geometrical Interpretation Through Honeycomb Patterns

Usually,the Geometrical interpretation of algebraic formulae of two-dimensional can be seen in a mathematical textbook in school. Students can interpret the areas relations of $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2= a^2 - 2ab +b^2$ considering squares and rectangles considering the length of one hexagonal = 1 units with the help of measurement of unit squares which we can see Fig. 5.

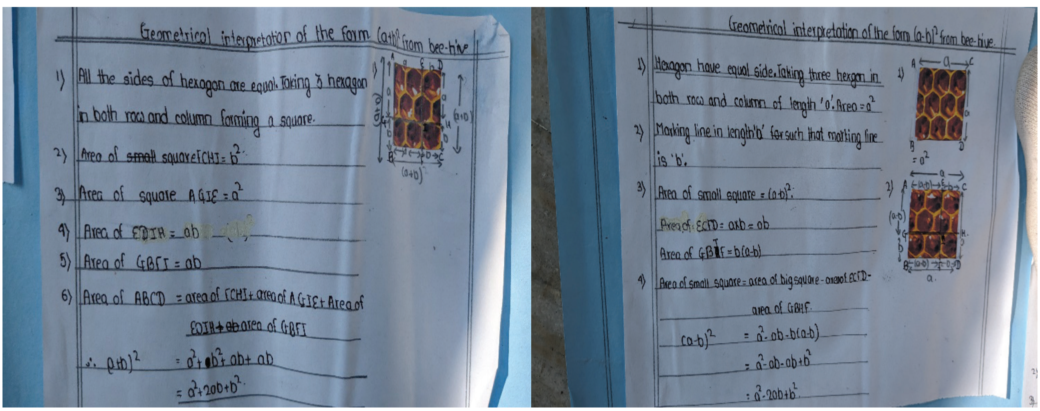


Figure 5 : Students Generated Formula Interpretation: Beehive Hexagonal Patterns

Geometrical Knowledge from Honeycomb Patterns

Usually, when the teacher is teaching, it is customary to teach guided by all bookish patterns, problems, and processes of solving. On the other hand, very few teachers and students only try to connect to understand and learn practically the various contents of mathematics. However, our students in class nine perceived and generated the separate pattern of lines, segments, parallelogram properties, angles, and problems with processes and solutions found in different areas of mathematics in Honeycomb. They were all focused on generating the two different areas of geometry and algebra through patterns and shapes formed on honeycombs and holes. In our role as a mathematics teacher and researcher, we encouraged students and other teachers to create newer ways of teaching and learning. The diagrams, problems, and in-depth solutions they generated are shown in Fig. 6.

Reciprocal Approaches of Mathematics Learning Through Honeycomb Patterns

Figure 6 : Students and trainee teachers field activities dated 2nd February 2023, 6th July 2023

Students and Teachers on Reciprocal Learning Mathematics

Reciprocal learning is an approach in which students, teachers, and facilitators can work separately or in groups to acquire knowledge. This type of environment allows both instructors and learners to have control over the learning material. This approach facilitates conversation regarding the goals (Mafarja et al., 2023). When observation of trainee teachers' activities in training and students' activities in the classroom with patterns generated mathematics learning approaches, teachers were also found actively engaged with honeycomb patterns. They generated mathematical constants like sequence hexagonal patterns, angles, and congruent hexagonal shapes through different approaches, like poems, singing and dancing with music, and presentations in activities. They presented math into songs by adding music and dance (as you can see in Fig. 6 on the right side). Below is the field text from the group discussion.

Students Group Discussion:

> *One of the boys' student of class nine said: I saw congruent rectangles and hexagonal patterns everywhere in honeycomb patterns which exist in the translation and tessellation shift rule as well, congruent and similar triangles can be constructed. But Sarita and Samjhauta tested the parallel lines by drawing the lines on top and bottom of the hexagonal shape of the honeycomb. They said and shared their ideas in open conservation among classroom friend like that such a way of learning forces us to think creatively using our ideas and knowledge first (Group Discussion generated and observed field text dated 2ndFebruary 2023).*

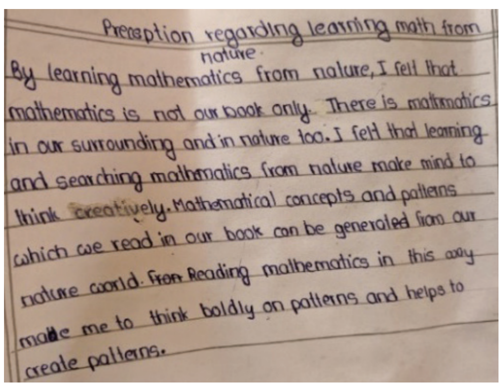
Suman with in group discussions to other friends also viewed the importance of home activities to learn mathematics creatively sharing and presenting with each other. They said after playing with honeycomb patterns for connecting school mathematics and sharing, discussing and interacting, and presenting in the group with this tool as follows:

> *This way of learning changed our view on mathematics understanding and the way of writing, reading, and perceiving. We are happy and aware to read and learn mathematics from a broader perspective rather than just following problems and techniques in books individually. We felt that learning mathematics was practically connected to surrounding (15th February2023).*

Jasmin:

> *I found that mathematics is not only possible to learn following the exact procedures in the book. I came to know from the group and individual engagement on bee comb. I am confident that bee comb patterns exist in algebra graphic patterns as well. However, one of the girls drew patterns, sequences, and solutions steps in detail on honeybee comb patterns considering one hexagon-covered area = 1 unit which can be seen in the original text of fig (Jasmin and Suman generated text, observation, and interview dated 18th February 2023).*

Sarita :


Preception regarding learning math from nature

By learning mathematics from nature, I felt that mathematics is not our book only. There is mathmatics in our surrounding and in nature too. I felt that learning and searching mathematics from nature make mind to think creatively. Mathematical concepts and patterns which we read in our book can be generated from our nature world. From Reading mathematics in this way make me to think boldly on patterns and helps to create patterns.

Figure 7 : Sarita Generated Field text dated 2nd February 2023

Group Conservation with Students and Trainee Teaches:

> *We need to understand that both teachers and students should also make us understand the mathematics concept well by relating our natural and societal activities visiting the real context They also claimed that learning mathematics using such activities helps us to construct new knowledge and increase our creativity making we are active rather than passive speaker and listeners. Such activities also thinking and reading and practicing broadly and help to create more mathematical information and patterns (Informal Conservation in Group dated, 20th February 2022).*

The analysis shows that honeycomb patterns can be used to learn almost plane geometry shapes, symmetry, and their properties, as well as an in-depth quadratic sequence with solutions, translation, and tessellation patterns and algebraic formulas. Similarly, the contents of quadratic sequences in hexagonal patterns are seen in honeycomb. While playing with patterns of honeycomb with my participants, we also saw the horizontal and vertical arithmetic sequence of hexagon patterns and types of angles and parallelograms. While observing profoundly, we can see many regular hexagonal square patterns. They were able to link the mathematics contents of algebra and geometry addressed by the CDC of the school curriculum with clear information through hexagonal honeycomb patterns.

The informants also felt much time was needed to think mathematically. They realized that this kind of learning approach demands teachers and students' brainstorming capacity through researching, learning, and doing. However, students feel that teachers need an additional facilitator to guide them in new directions. Fenstermacher and Richardson's (2005) understanding that ‘learners are not passive learners and they don't come with a blank mind" fits well with this process. They said that such practices help to promote creative thinking, mathematical reasoning, and understanding math from a broader perspective. Maryati et. al. (2019) noted that learning through honeycomb patterns will be more productive and make meaning, promoting necessary discussion to work for knowledge creation rather than knowledge transfer. The Cultural Relevance of Mathematics Teaching (NCTM Standards) (2014) also suggested a similar argument: if students and teachers internalize mathematical knowledge in creative ways, they will be critical thinkers and hope to enhance critical awareness to link students' prior knowledge of cultural math with school math. This is why Lim, Tan, and Saito (2019) advocated that teachers act as the mediators of learning, which would provide students with opportunities to think and reflect by exploring mathematical ideas and justifying answers to problems that are capable of verifying their reasoning and thinking. How our students' participants constructed the knowledge as Lewis (2018) stated that teachers need to ignore spoon-feeding teaching and use problem-solving to expose students to cultural math.

Trainee teachers and boys' participants also claimed that they observed a linear arithmetic sequence of regular hexagons. They also noted that honeycomb hexagonal patterns are seen in parallel and perpendicular tessellation designs in the reciprocal learning approach. They said that they learned geometry, reflection, and the enlargement of similar and congruent concepts in a beehive. They also claimed intersecting lines, vertically opposite angles, and other types of angles by measuring protractors, which are shown in **Figs. 2–6.**

From the informal conservation and student participant observation, we knew that learning mathematics of linear and quadratic sequence, formulae expansion in plane, and geometry together from honeycomb patterns is easy for the students. The finding above truly matches D'Entremont's (2015) views. He envisioned that the design of meaningful patterns illustrated how to link mathematics knowledge to the learners' surroundings, which may be beneficial for students as learning resources in mathematics that can provide individuals with "big picture" explanations on how to link academic mathematics to the learners' living situations (Fitrianawati et al., 2020).

Focusing on the subject matter, Sunzuma and Maharaj (2019) and Albanese and Perales (2015) advocated that students' mathematics learning strategies that build on prior knowledge and experience enable them to improve their mathematical understanding and creative thinking with the right reasoning. Ernest & Albert (2018), Fouze& Amit (2018), and Rosa & Orey (2013) also stated that the social constructivist theory might have worked there in helping students learn mathematics that way. It also helped students learn brainstorming activities, draw real figures with reasonable representation, develop self-confidence, self-esteem, and a sense of belonging, as well as respect for communities' activities through mathematical thinking" (D'Entremont, 2015).

Conclusion

The first conclusion of this study is that students can do research, go for brainstorming and collaborative undertaking, generate patterns, formulae, and processes of solving through accurate steps from honey bees' comb patterns. The second can also be concluded that students' learning, teachers teaching, and trainers processes of academic mathematics could be made easier through this reciprocal process. The third conclusion is that students know algebra, arithmetic, and geometry concepts through beehives at one stroke. Besides, they can develop, interpret, and test the given formulas by linking them with their bookish knowledge. The implications of this study are that students, teachers, and teacher educators can use honeycombs like hexagonal patterns to teach students, train teachers, and learn mathematics from the things around them.

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