Research Article

Finding of principle square root of a real number by using interpolation method

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Abstract

There are various methods of finding the square roots of positive real number. This paper deals with finding the principle square root of positive real numbers by using Lagrange's and Newton's interpolation method. The interpolation method is the process of finding the values of unknown quantity (y) between two known quantities.

Keywords: Interpolation, real number, square root

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Introduction

The square root of a number is value that when multiplied by itself, gives the number. All positive real number has two square roots, one positive square root and one negative square root. The positive square root is sometimes referred to as the principle square root. A square root is written with a radical symbol $\sqrt{\ }$ and the number or expression inside the radical symbol, below denoted a called radicand $\sqrt{a}$. Negative numbers don't have real square roots since a square is either positive or 0. If the square root of an integer is another integer then the square is called perfect square. For example: 36. It is perfect square $\sqrt{36} = \pm 6$.

If the radicant is not perfect square i.e the square root is not a whole number then we have to approximate the square root $\pm\sqrt{3} = \pm 1.73205 \approx \pm 1.7$

The square roots of numbers that are not a perfect square are members of the irrational numbers.

The process of finding the value of $y(x_i)$ for corresponding value of $x_i \in (x_0, x_n)$ is called interpolation.

The process of finding the value of $y(x_i)$ for corresponding value of $x_i \notin (x_0, x_n)$ is called extrapolation.

To obtain the value of unknown quality by using Newton's interpolation method we use three types of formula namely, forward difference, backward differences and central difference. Forward difference: The first order forward difference of f(x) is the change in f(x) when x is increased by a positive difference h. The operation is carried out through the notation Δ. Backward difference: The first order backward difference of f(x) is the change in f(x) when x decreases by a positive difference h. The operation is carried out through the notation ∇. Central difference*:* The forward and backward differences are mainly useful in interpolating the values near the beginning and the end of the table respectively. Central differences are particularly useful in interpolating for value of x in the interior of the table.

For equal interval we use the following method to find the interpolation:
1. Newton's forward difference interpolation formula.
2. Newton's backward difference interpolation formula.

For unequal interval we use the following methods to find the interpolation:
1. Newton's divided difference formula. 2. Stiring central difference formula.
3. Gauss central difference formula. 4. Bessel's interpolation formula.
5. Lagring's interpolation formula

Table 1
Newton's Divided Difference Table

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
x_0	f_0				
		$\Delta f_0 = \dfrac{f_1 - f_0}{x_1 - x_0}$			
x_1	f_1		$\Delta^2 f_0 = \dfrac{\Delta f_1 - \Delta f_0}{x_2 - x_0}$		
		$\Delta f_1 = \dfrac{f_2 - f_1}{x_2 - x_1}$		$\Delta^3 f_0 = \dfrac{\Delta^2 f_1 - \Delta^2 f_0}{x_2 - x_0}$	

x_2	f_2		$\Delta^2 f_1 = \dfrac{\Delta f_2 - \Delta f_1}{x_3 - x_1}$		$\Delta^4 f_0 = \dfrac{\Delta^3 f_1 - \Delta^3 f_0}{x_4 - x_0}$
		$\Delta f_2 = \dfrac{f_3 - f_2}{x_3 - x_2}$		$\Delta^3 f_1 = \dfrac{\Delta^2 f_2 - \Delta^2 f_1}{x_3 - x_1}$	
x_3	f_3		$\Delta^2 f_2 = \dfrac{\Delta f_3 - \Delta f_2}{x_4 - x_2}$		
		$\Delta f_3 = \dfrac{f_4 - f_3}{x_4 - x_3}$			
x_4	f_4				

Table 2

Newton's Forward Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\Delta y_0 = y_1 - y_0$			
x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$		
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
x_3	y_3		$\Delta^2 y_2 = \Delta y_2 - \Delta y_1$		
		$\Delta y_3 = y_4 - y_3$			
x_4	y_4				

Table 3

Newton's Backward Difference Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
x_0	y_0				
		$\nabla y_0 = y_1 - y_0$			
x_1	y_1		$\nabla^2 y_0 = \nabla y_1 - \nabla y_0$		
		$\nabla y_1 = y_2 - y_1$		$\nabla^3 y_0 = \nabla^2 y_1 - \nabla^2 y_0$	
x_2	y_2		$\nabla^2 y_1 = \nabla y_2 - \nabla y_1$		$\nabla^4 y_0 = \nabla^3 y_1 - \nabla^3 y_0$
		$\nabla y_2 = y_3 - y_2$		$\nabla^3 y_1 = \nabla^2 y_2 - \nabla^2 y_1$	
x_3	y_3		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$		

		$\nabla y_3 = y_4 - y_3$
x_4	y_4	

Result and Discussion

To find the principle square root of real number by interpolation method it is necessary to know the Lagrange's interpolation formula as well as Newton's interpolation formula.

Lagrange's interpolation formula

Suppose y=f(x) be function with $f(x_0)$, $f(x_1)$, $f(x_2)$ .... $f(x_n)$ corresponding to the values x_0, x_1, x_2 .... x_n. then $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$ .... $y_n = f(x_n)$. then Lagrange's interpolation formula is given by

$$y = f(x) = \frac{(x-x_1)(x-x_2).........(x-x_n)}{(x_0-x_1)(x_0-x_2).......(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2).........(x-x_n)}{(x_1-x_0)(x_1-x_2).......(x_1-x_n)} \times y_1$$

$$+ .... + \frac{(x-x_0)(x-x_1).........(x-x_{n-1})}{(x_n-x_0)(x_n-x_1).......(x_n-x_{n-1})} \times y_n$$

Newton's divided difference formula

Suppose y=f(x) be function with $f(x_0)$, $f(x_1)$, $f(x_2)$ .... $f(x_n)$ corresponding to the values x_0, x_1, x_2 .... x_n. then $y_0 = f(x_0)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$ .... $y_n = f(x_n)$. then Newton's divided difference formula is given by

$$y = y_0 + (x-x_0)\Delta f_0 + (x-x_0)(x-x_1)\Delta^2 f_0 + ... + (x-x_0)(x-x_1)........................(x-x_n)\Delta^n f_0$$

Newton's forward difference formula

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + ...$$

Where $p = \dfrac{x_p - x_0}{h}$

x_p = value of which interpolation is to be found

x_0 =initial value related to x_p

h =interval of x.

Newton's backward difference formula

$$y_p = y_n + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n + \dots\dots\dots\dots$$

Where $p = \dfrac{x_p - x_n}{h}$

x_p = value of which interpolation is to be found

x_n =finall value related to x_p

h =interval of x.

Example: Given that

X	1	4	9	16	25
y	1	2	3	4	5

Calculate the approximate principle square root of 10.56 by using Lagrange's interpolation formula.

$Here,\ x_0 = 1, x_1 = 4,\ x_2 = 9, x_3 = 16, x_4 = 25$

$Here,\ y_0 = 1, y_1 = 2,\ y_2 = 3, y_3 = 4, y_4 = 5$

X=10.56

Lagrange's interpolation formula is

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots\dots\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots\dots\dots(x_0-x_n)}\times y_0 + \frac{(x-x_0)(x-x_2)\dots\dots\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots\dots\dots(x_1-x_n)}\times y_1$$

$$+\dots\dots\dots\dots\dots\dots\dots\dots + \frac{(x-x_0)(x-x_1)\dots\dots\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots\dots\dots(x_n-x_{n-1})}\times y_n$$

$$or,\ y = f(10.56\ ) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3$$

$$= \frac{(10.56-4)(10.56-9)(10.56-16)(10.56-25)}{(1-4)(1-9)(1-16)(1-25)} \times 1$$

$$+ \frac{(10.56-1)(10.56-9)(10.56-16)(10.56-25)}{(4-1)(4-9)(-16)(9-25)} \times 2$$

$$+ \frac{(10.56-1)(10.56-4)(10.56-16)(10.56-25)}{(9-1)(9-4)(9-16)(9-25)} \times 3$$

$$+ \frac{(10.56-1)(10.56-4)(10.56-9)(10.56-25)}{(16-1)(16-4)(16-9)(16-25)} \times 4$$

=0.09304-0.61985+3.298+0.49831

=3.23.

Therefore the approximate principle square root of 10.56 is 3.23.

Example: Given that

X	7	10	13	16	19
y	2.64	3.16	3.60	4	4.35

Calculate the approximate principle square root of 10.56 by using Newton's forward difference formula.

Table 4

Newton's Forward Difference Formula is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
x_0	y_0				
		$\Delta y_0 = y_1 - y_0$			

x_1	y_1		$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$		
		$\Delta y_1 = y_2 - y_1$		$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$	
x_2	y_2		$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$		$\Delta^4 y_0 = \Delta^3 y_1 - \Delta^3 y_0$
		$\Delta y_2 = y_3 - y_2$		$\Delta^3 y_1 = \Delta^2 y_2 - \Delta^2 y_1$	
x_3	y_3		$\Delta^2 y_2 = \Delta y_2 - \Delta y_1$		
		$\Delta y_3 = y_4 - y_3$			
x_4	y_4				

Or.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
7	2.64				
		3.16-2.64=0.52			
10	3.16		0.44-0.52=-0.08		
		3.60-3.16=0.44		-0.04+0.08=0.04	
13	3.60		0.4-0.44=-0.04		-0.01-0.04=-0.05
		4-3.60=0.4		-0.05+0.04=-0.01	
16	4		0.35-0.4=-0.05		
		4.35-4=0.35			
19	4.35				

x_p = 10.56 lies between 10 and 13

$x_0 = 10,\ y_0 = 3.16$

h =3

$$p = \frac{x_p - x_0}{h} = \frac{10.56 - 10}{3} = 0.18666$$

y_p is value of x when x=10.56

Newton's forward difference formula is

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \ldots\ldots\ldots\ldots$$