General Article

Application of differential equation in L-R and C-R circuit analysis by classical method.

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Abstract

The paper deals with the analysis of L-R and C-R circuit by using linear differential equation of first order. A circuit containing an inductance L or a capacitor C and resistor R with current and voltage variable given by differential equation. The general solution of differential equation have two parts complementary function (C.F) and particular integral(P.I) in which C.F. represent transient response and P.I. represent steady response. The general solution of differential equation represent the complete response of network .In this connection, this paper includes L-R, C-R circuit and ordinary differential equation of first order and its solution,.

Key words: circuit analysis, classical method, L-R and C-R circuit, ordinary differential equation, ,.

Introduction

An equation which involves differential coefficient is called differential equation. A differential equation involving derivatives with respect to single independent variable is called ordinary differential equation and involving partial derivatives with respect to more than one independent variable is called partial differential equation. The inter-connection of simple electric device in which there is at least one closed path for current to flow is called electric circuit. The circuit is switch from one condition to another by change in the applied source or a change in the circuit elements there is a transition period during which the branch current and voltage changes from their former values to new ones. This period is called transient. After the transient has passed the circuit is said to be steady state. The linear differential equation that describes the circuit will have two parts to its solution the complementary function corresponds to the transient and the particular solution corresponds to steady state.

The *v-i* relation for an inductor or capacitor is a differential. A circuit containing an inductance L or a capacitor C and resistor R with current and voltage variable given by differential equation of the same form. It is a linear first order differential equation with constant coefficient when the value of R,L,C are constant. L and C are storage elements. Circuit has two storage elements like one L and one C are referred to as second order circuit.

Therefore, the series or parallel combination of R and L or R and C are first order circuit and RLC in series or parallel are second order circuit.

The circuit changes are assumed to occur at time t=0 and represented by a switch. The switch may be supposed to closed (on) and open (off) at t=0.

The order of differential equation represent derivatives involve and is equal to the number of energy storing elements and differential equation considered as ordinary. the differential equation that formed for transient analysis will be linear ordinary differential equation with constant coefficient.

The value of voltage and current during the transient period are known as transient response. The C.F. of differential equation represents the transient response.

The value of voltage and current after the transient has died out are known as steady state response. The P.I. of differential equation represents the steady state response. The complete or total response of network is the sum of the transient response and steady state response which is represented by general solution of differential equation.

The value of voltage and current that result from initial conditions when input function is zero are called zero input response. The value of voltage and current for the input function which is applied when all initial condition are zero called zero state response.

Table 1

Elements symbol and units of measurements

S.No.	element	symbol	unit
1	charge	q	Coulomb
2	current	i	Ampere
3	resistance	R	Ohm
4	inductance	L	Henry
5	Capacitance	C	Farad
6	voltage	V	volt

Data and Methods

The paper uses secondary sources and table where necessary. The published journal and books related to differential equation, circuit and systems mathematical physics and electrical engineering and electricity from various publishers are the secondary sources as indicated in reference section.

Results and Discussion

To study the transients and steady state in electric circuit, it is necessary to know the mathematical concept of differential equation and its solution by classical method.

First order homogenous differential equation.

$$\frac{dy(t)}{dt} + py(t) = 0$$

$$\Rightarrow \frac{dy(t)}{yt} = -pdt$$

Integrating, lny(t)=-pt+lnk

$$\therefore y(t) = ke^{-pt}$$

First order non homogenous differential equation

$$\frac{dy(t)}{dt} + py(t) = Q$$

The equation is not altered by multiplying e^{pt}

$$e^{pt}\frac{dy(t)}{dt} + e^{pt}py(t) = e^{pt}Q$$

$$\Rightarrow \frac{d\left\{e^{pt}y(t)\right\}}{dt} = \int Qe^{pt}dt + k$$

$$\Rightarrow y(t).e^{pt} = \int Qe^{pt}dt + k$$

$$\Rightarrow y(t). = e^{-pt}\int Qe^{pt}dt + ke^{-pt}$$

The first term of above solution is known as particular Integral and second is known as complementary function. Particular Integral does not contains any arbitrary constant and C.F. does not depend on the forcing function Q. If Q is constant. Then

$$\Rightarrow y(t). = e^{-pt} Q \frac{e^{pt}}{p} + ke^{-pt}$$

$$\therefore y(t). = \frac{Q}{p} + ke^{-pt}$$

The formation of differential equation for an electric circuit depends upon the following laws.

i) i= $\frac{dq}{dt}$

ii) Voltage drop across resistance (R) = Ri

iii) Voltage drop across inductance (L) = L $\frac{di}{dt}$

iv) Voltage drop across capacitance (C) = $\frac{q}{c}$

Kirchhoff's law: the algebraic sum of the voltage drop around any closed circuit is equal to resultant emf in the circuit.

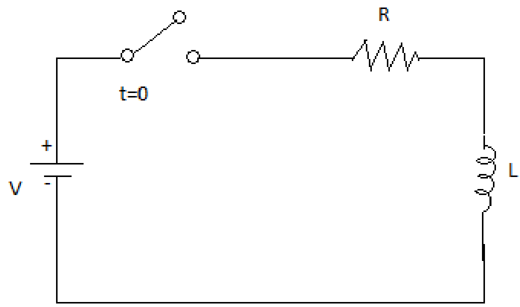
Current law: at a junction current coming is equal to current going.

L-R series circuit: Ri+ L $\frac{di}{dt}$ =E $\Rightarrow \frac{di}{dt}$ + i $\frac{R}{L}$ = $\frac{E}{L}$

C-R series circuit: Ri+ $\frac{q}{c}$ =E $\Rightarrow$ R $\frac{dq}{dt}$ + $\frac{q}{c}$ =E

L-R circuit analysis

The switch s is closed at time t=0. Find the current i(t) through the voltage across the resister and inductor.



Here, the voltage across resistance= Ri(t)

Voltage drop across inductance= L $\dfrac{di(t)}{dt}$

From Kirchhoff's law, L $\dfrac{di(t)}{dt} + Ri(t) = V$

$$\Rightarrow \frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V}{L}$$

Which is first order linear differential equation.

I.F= $e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$

General solution is, i(t) $e^{\frac{R}{L}t} = \int \frac{V}{L} e^{\frac{R}{L}t} dt + k = \frac{V}{L} . \dfrac{e^{\frac{R}{L}t}}{\frac{R}{L}} + k = \frac{V}{R} . e^{\frac{R}{L}t} + k$

$$\therefore i(t) = \frac{V}{R} + k.e^{-\frac{R}{L}t} \quad ....(1)$$

Since the inductor behaves as a open circuit.

$$\therefore i(0^{+}) = 0$$

$$from\,(1) \therefore 0 = \frac{V}{R} + k. \Rightarrow k = -\frac{V}{R}$$

$$\therefore i(t) = \frac{V}{R} - \frac{V}{R}.e^{-\frac{R}{L}t} = \frac{V}{R}(1 - e^{\frac{-R}{L}t})$$

The voltage across the resistor and inductor are given as

$$V_R(t) = i(t).R \therefore i(t) = V(1 - e^{\frac{-R}{L}t})$$

$$V_L(t) = L\frac{di(t)}{dt}. = L.\frac{V}{R}\left[0 - (\frac{-R}{L})e^{\frac{-R}{L}t}\right]$$

$$\therefore V_L(t) = V\left[e^{\frac{-R}{L}t}\right]$$

At t=0, i(t)=0 and $\therefore V_L(t) = V, V_R(t) = 0$

At t= ∞, i(t)= $\frac{V}{R}$ and $\therefore V_L(t) = 0, V_R(t) = V$

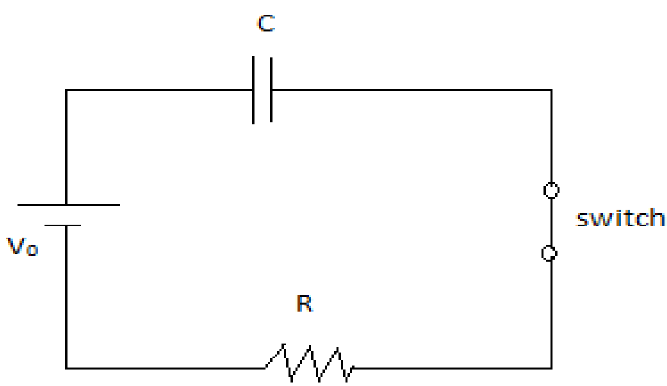
At t= $\frac{L}{R} = \tau$, i(t)= $\frac{V}{R}\left(1 - e^{-1}\right) = 0.632\frac{V}{R}$ and

$\therefore V_L(t) = Ve^{-1} = 0.368V, V_R(t) = 0.632V$

$\frac{L}{R} = \tau$ is known as the time constant of the circuit and is defined as the interval after which current or voltage changes 63.2 percent of its total change.

C-R circuit analysis

A condenser of capacity C farads with V_0 is discharged through a resistance R ohms. Show that if q coulomb is the charge on the condenser, i ampere the current and V the voltage at time t, q=cV , V=Ri and $i = \frac{dq}{dt}$. Then V=$V_0 e^{\frac{1}{RC}}$



Here, the voltage across resistance=Ri

Voltage drop across capacitance= $\frac{q}{C}$

From krichhoff's law, L $\frac{di(t)}{dt} + Ri(t) = V \Rightarrow \frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V}{L}$

When after release of key the condenser gets discharged and at that time voltage across the battery gets zero. So $V_0 = 0$.

The differential equation of above circuit is $Ri + \frac{q}{C} = 0 \Rightarrow R\frac{dq}{dt} + \frac{q}{c} = 0$

$$\Rightarrow \frac{dq}{dt} = -\frac{q}{Rc} \Rightarrow \frac{dq}{q} = -\frac{1}{Rc}dt$$

Integrating, $\log q = -\frac{1}{Rc}t + A$ ....(1)

But at t=0, the charge at condenser is q_0. Therefore $\log q_0 = A$

From (1)

$$\log q = -\frac{1}{Rc}t + \log q_0 \Rightarrow \frac{q}{q_0} = e^{-\frac{1}{Rc}t} \Rightarrow q = q_0 e^{-\frac{1}{Rc}t} \Rightarrow \frac{q}{c} = \frac{q_0}{c}e^{-\frac{1}{Rc}t}$$

$$\Rightarrow V = V_0 e^{-\frac{1}{Rc}t}$$

Conclusion

By using first order ordinary differential equation in L-R and C-R circuit we can find the current(i) and voltage (v) in the circuit when inductance (L) or capacitance(C) and resistance (R) are given.

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