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Research Article

SOME GENERALIZED DIFFERENCE SEQUENCE SPACES OVER BI-COMPLEX SCALARS

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ABSTRACT

In this article, we have introduced the notion of generalized difference sequence spaces within the framework of bi-complex numbers. Further we have established their algebraic, topological and geometric properties including some inclusion relations.

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Keywords: Banach space, Bi-complex numbers, Convexity, Difference sequence space

INTRODUCTION

Classical sequence spaces l_∞, c and c_0 are linear spaces of bounded, convergent and null sequences of real or complex numbers respectively, which are defined by

$$l_\infty = \{z = (z_k) \in \omega : \sup_{k \in \mathbb{N}} |z_k| < \infty\},$$

$$c = \{z = (z_k) \in \omega : \lim_{k \to \infty} z_k = l \text{ for some } l \in \mathbb{R} \text{ or } \mathbb{C}\},$$

$$c_0 = \{z = (z_k) \in \omega : \lim_{k \to \infty} z_k = 0\},$$

where ω denotes the space of all sequences.

These spaces play a significant role in the advancement of functional analysis because of their huge application in various branches of science and engineering.

H. Kizmaz (Kizmaz, 1981) introduced the following difference sequence spaces based on the classical sequence spaces l_∞, c and c_0.

$$l_\infty(\Delta) = \{z = (z_k) : \Delta z_k \in l_\infty\},$$

$$c(\Delta) = \{z = (z_k) : \Delta z_k \in c\},$$

$$c_0(\Delta) = \{z = (z_k) : \Delta z_k \in c_0\}, \text{ where } \Delta z_k = z_k - z_{k+1},\ k \in \mathbb{N}, \text{ the set of positive integers.}$$

These spaces are Banach spaces under the norm $\|z\|_\Delta = |z_1| + \sup_k \|\Delta z_k\|$.

The concept of difference sequence spaces was later extended by (Et M., 1993), (Tripathy & Esi, 2006) and many others. (Et & Colak, 1995) subsequently defined the following generalized difference sequence spaces over the field of real or complex numbers.

$$l_\infty(\Delta^m) = \{z = (z_k) : \Delta^m z_k \in l_\infty\},$$

$$c(\Delta^m) = \{z = (z_k) : \Delta^m z_k \in c\},$$

$$c_0(\Delta^m) = \{z = (z_k) : \Delta^m z_k \in c_0\}, \text{ where } \Delta^m z_k = \Delta^{m-1} z_k - \Delta^{m-1} z_{k+1}.$$

For m = 2, $\Delta^2 z_k = \Delta z_k - \Delta z_{k+1} = (z_k - z_{k+1}) - (z_{k+1} - z_{k+2})$

$$= z_k - 2 z_{k+1} + z_{k+2}$$

Hence, in general $\Delta^m z_k = \sum_{s=0}^{m}(-1)^s \binom{m}{s} z_{k+s}$.

Later, (Ghimire & Pahari, 2022) introduced the following class of difference sequence spaces generated by $\varphi -$ function which are the extension of sequence spaces studied by (Herawati & Gulton, 2019).

$$W_0(\Delta, f) = \{ z = (z_k) \in \omega : \frac{1}{m}\sum_{k=1}^{m} f\left(\frac{|\Delta z_k|}{\rho}\right) \to 0 \text{ as } m \to \infty \text{ and } \rho > 0\}$$

$$W(\Delta, f) = \{ z = (z_k) \in \omega : \exists\ l > 0 : \frac{1}{m}\sum_{k=1}^{m} f\left(\frac{|\Delta z_k - l|}{\rho}\right) \to 0 \text{ as } m \to \infty \text{ and } \rho > 0\}$$

$$W_\infty(\Delta, f) = \{ z = (z_k) \in \omega : \exists\ \rho > 0 : \sup_m \frac{1}{m}\sum_{k=1}^{m} f\left(\frac{|\Delta z_k|}{\rho}\right) < \infty \}$$

where f is a $\varphi -$function which is even, non- decreasing on $[0, \infty)$ and f(t) = 0 if and only if t = 0. It is closely related to an Orlicz function.

To broaden sequence spaces within the bi-complex framework, we first recall the idea of bi-complex numbers. These numbers arise by introducing two independent imaginary units, which extend the traditional complex number system. The earliest account of this construction was given by (Segre, 1892), and later it was systematically analyzed by (Price, 1991). Recently, (Alpay et al., 2014) formulated a functional analytic structure based on bi-complex scalars. Building on these foundations, several researchers such as (Bera & Tripathy, 2023; Degirmen & Sagir, 2022; Kumar & Tripathy, 2024; Parajuli et al., 2025; Rochan & Shapiro, 2004; Sager & Sagir, 2020; Wagh, 2014) have significantly advanced the field by exploring its algebraic, topological and geometric structures as well as applications to sequence spaces.

Definition 1. A bi-complex number can be expressed in the form

$\xi = x_1 + i\,x_2 + j\,x_3 + ij\,x_4 = z_1 + j\,z_2$, where $z_1 = x_1 + i\,x_2$, $z_2 = x_3 + i\,x_4$, with $x_1, x_2, x_3, x_4 \in \mathbb{R}$, $z_1, z_2 \in \mathbb{C}_1$, where $\mathbb{R}$ and $\mathbb{C}_1$ denote the set of real and complex numbers respectively (Segre, 1892).

The imaginary units i and j satisfy the following relations.

$i^2 = -1$, $j^2 = -1$, and $i\,j = ji = k$ where $k^2 = 1$ and k is called hyperbolic unit.

The set of all bi-complex numbers is denoted by $\mathbb{C}_2$ and is defined by $\mathbb{C}_2 = \{z_1 + j\,z_2 : z_1, z_2 \in \mathbb{C}_1\}$

Besides 0 and 1, there exist exactly two non-trivial idempotent elements in $\mathbb{C}_2$ which are defined by

$$e_1 = \frac{1 + i\,j}{2} \text{ and } e_2 = \frac{1 - i\,j}{2}$$

Every bi-complex number $\xi = z_1 + j\,z_2$ has unique idempotent representation as $\xi = \eta_1 e_1 + \eta_2 e_2$, where $\eta_1 = z_1 - i\,z_2$ and $\eta_2 = z_1 + i\,z_2$ are the idempotent components of ξ.

The norm of bi-complex number $\xi = x_1 + i\,x_2 + j\,x_3 + k\,x_4$ is defined by

$$\| \xi \|_{C_2} = \sqrt{{x_1}^2 + {x_2}^2 + {x_3}^2 + {x_4}^2} = \sqrt{|z_1|^2 + |z_2|^2} = \sqrt{\frac{|\eta_1|^2 + |\eta_2|^2}{2}} \text{ which is a Euclidean norm.}$$

Definition 1.2. (Price, 1991) A sequence (ξ_j) in $\mathbb{C}_2$ is said to converge to a limit $\xi \in \mathbb{C}_2$ if, for each $\varepsilon > 0$, there corresponds an integer $n_0(\varepsilon) \in \mathbb{N}$ for which $\| \xi_j - \xi \|_{\mathbb{C}_2} < \varepsilon$, $\forall\ j \geq n_0$. In this situation, we denote the limit by $\lim_{j\to\infty} \xi_j = \xi$.

Definition 1.3. (Price, 1991) A sequence (ξ_j) in $\mathbb{C}_2$ is called Cauchy if, for each $\varepsilon > 0$, there corresponds an integer $n_0(\varepsilon) \in \mathbb{N}$ for which

$$\| \xi_j - \xi_k \|_{\mathbb{C}_2} < \varepsilon, \text{ whenever } j,\ k \geq n_0.$$

Some fundamental sequence spaces defined in the bi-complex setting are listed below.

$\omega(\mathbb{C}_2) = \{ \xi = (\xi_k) : \xi_k \in \mathbb{C}_2 \text{ for all } k \in \mathbb{N}\}$,

$l_\infty(\mathbb{C}_2) = \{ \xi = (\xi_k) : \xi_k \in \omega(\mathbb{C}_2) : \sup_k \| \xi_k \|_{\mathbb{C}_2} < \infty\}$,

$c(\mathbb{C}_2) \quad = \{ \xi = (\xi_k): \xi_k \in \omega(\mathbb{C}_2), \exists\, l \in \mathbb{C}_2: \lim_{k\to\infty} \xi_k = l\}$,

$(\mathbb{C}_2) \quad = \{ \xi = (\xi_k): \xi_k \in \omega(\mathbb{C}_2): \lim_{k\to\infty} \xi_k = 0\}$,

$l_p(\mathbb{C}_2) \quad = \{ \xi = (\xi_k): \xi_k \in \omega(\mathbb{C}_2): \sum_{k=1}^{\infty} ||\xi_k||_{\mathbb{C}_2}^p < \infty\}$.

The set of all bi-complex numbers $C_o\mathbb{C}_2$ forms a commutative ring with zero divisors. Due to the existence of non-invertible elements and idempotent representation, bi-complex sequence spaces have very rich algebraic and topological properties.

Using the idea of (Kizmaz, 1981), (Parajuli et al., 2025) explored difference sequence spaces of bi-complex numbers over the classical spaces l_∞ , c and c_0 as follows.

$l_\infty(\mathbb{C}_2)(\Delta) = \{ \xi = (\xi_k): \Delta\xi_k \in l_\infty(\mathbb{C}_2) \}$,

$c(\mathbb{C}_2)(\Delta) = \{ \xi = (\xi_k): \Delta\xi_k \in c(\mathbb{C}_2) \}$,

$c_0(\mathbb{C}_2)(\Delta) = \{ \xi = (\xi_k): \Delta\xi_k \in c_0(\mathbb{C}_2) \}$.

In this article, we have further extended this study by defining a more general class of difference sequence spaces in the bi-complex setting as follows:

$l_\infty(\Delta^m)(\mathbb{C}_2) = \{ \xi = (\xi_k) : \Delta^m\xi_k \in l_\infty(\mathbb{C}_2)\}$,
$c(\Delta^m)(\mathbb{C}_2) = \{ \xi = (\xi_k) : \Delta^m\xi_k \in c(\mathbb{C}_2)\}$,
$c_0(\Delta^m(\mathbb{C}_2) = \{ \xi = (\xi_k) : \Delta^m\xi_k \in c_0(\mathbb{C}_2)\}$.

Difference sequence spaces of bi-complex numbers gain dimensional richness by extending classical real and complex difference sequence spaces. They are very useful in signal and image processing. They can represent signal with two frequency bands.

RESULTS

This section is devoted to the formulation and proof of the principal results underlying the present study.

Theorem 1. The sequence spaces $l_\infty(\Delta^m)(\mathbb{C}_2)$, $c(\Delta^m)(\mathbb{C}_2)$, and $c_0(\Delta^m)(\mathbb{C}_2)$ are normed linear space with norm defined by

$$\|\xi\|_{\Delta^m} = \sum_{i=1}^{m}\|\xi_i\| + \sup_k\|\Delta^m\xi_k\| \text{ where } \xi = (\xi_k).$$

Proof.

Let $\xi = (\xi_k)$, $\eta = (\eta_k) \in l_\infty(\Delta^m)(\mathbb{C}_2)$ and α, β are scalars.

Then, $\sup_k \|\Delta^m\xi_k\| < \infty$ and $\sup_k \|\Delta^m\eta_k\| < \infty$.

$$\begin{aligned}\text{Now, } \sup_k \|\Delta^m(\alpha\xi_k + \beta\eta_k)\| &= \sup_k \|\Delta^m(\alpha\xi_k) + \Delta^m(\beta\eta_k)\| \\ &\le \sup_k \|(\Delta^m(\alpha\xi_k)\| + \sup_k \|(\Delta^m(\beta\eta_k)\| \\ &\le |\alpha| \sup_k \|\Delta^m\xi_k\| + |\beta|\sup_k \|\Delta^m\eta_k\| \\ &< \infty.\end{aligned}$$

Hence, $\alpha\xi_k + \beta\eta_k \in l_\infty(\Delta^m)(\mathbb{C}_2)$, which shows that it is a linear space. The same argument applies to the spaces $c(\Delta^m)(\mathbb{C}_2)$ and $c_0(\Delta^m)(\mathbb{C}_2)$.

Next, we verify the norm structure on $l_\infty(\Delta^m)(\mathbb{C}_2)$.

(i) Let $\xi = (\xi_k)$, then $\|\xi\|_{\Delta^m} = \sum_{i=1}^{m}\|\xi_i\| + \sup_k\|\Delta^m\xi_k\| \ge 0$.

(ii) Let $\xi = \theta$, where θ is the zero element of $l_\infty(\Delta^m)(\mathbb{C}_2)$, then $\|\theta\|_{\Delta^m} = 0$.

Conversely, suppose $\|\xi\|_{\Delta^m} = 0$, then, $\sum_{i=1}^{m}\|\xi_i\| + \sup_k\|\Delta^m\xi_k\| = 0$.

This implies $\xi_i = 0$ for $i = 1,2,\dots,m$ and $\|\Delta^m\xi_k\| = 0$ for $k \in \mathbb{N}$.

For, k = 1, $\| \Delta^m \xi_1 \| = 0$, this implies $\| \xi_1 - \xi_{m+1} \| = 0$

$\therefore \xi_{m+1} = 0$.

Continuing this argument, it follows that $\xi_k = 0$ for every $k \in \mathbb{N}$.

Thus, $\| \xi \|_{\Delta^m} = \theta$ if and only if $\xi = \theta$.

(iii) Let $\xi = (\xi_k) \in l_\infty (\Delta^m)(\mathbb{C}_2)$ and α is any scalar.

Then, $\| \alpha\xi \|_{\Delta^m} = \sum_{i=1}^{m} \| \alpha\xi_i \| + \sup_k \| \Delta^m (\alpha\xi_k) \|$

$= |\alpha| \left[\sum_{i=1}^{m} \| \xi_i \| + \sup_k \| \Delta^m \xi_k \| \right]$

$= |\alpha| \| \alpha\xi \|_{\Delta^m}$.

$\therefore \| \alpha\xi \|_{\Delta^m} = |\alpha| \| \xi \|_{\Delta^m}$

(iv) Let, $\xi = (\xi_k)$, $\eta = (\eta_k) \in l_\infty (\Delta^m)(\mathbb{C}_2)$.

Then, $\| \xi + \eta \|_{\Delta^m} = \sum_{i=1}^{m} \| \xi_i + \eta_i \| + \sup_k \| \Delta^m (\xi_k + \eta_k) \|$

$\leq \left[\sum_{i=1}^{m} \| \xi_i \| + \sup_k \| \Delta^m \xi_k \| \right] + \left[\sum_{i=1}^{m} \| \eta_i \| + \sup_k \| \Delta^m \eta_k \| \right]$

$= \| \xi \|_{\Delta^m} + \| \eta \|_{\Delta^m}$.

$\therefore \| \xi + \eta \|_{\Delta^m} \leq \| \xi \|_{\Delta^m} + \| \eta \|_{\Delta^m}$.

Thus, the mapping $\|.\|_{\Delta^m}$ is a norm on the spaces $l_\infty (\Delta^m)(\mathbb{C}_2)$, $c (\Delta^m)(\mathbb{C}_2)$, and $c_0 (\Delta^m)(\mathbb{C}_2)$. Hence, they are normed linear spaces.

Theorem 2. The class of sequences $l_\infty (\Delta^m)(\mathbb{C}_2)$, $c (\Delta^m)(\mathbb{C}_2)$, and $c_0 (\Delta^m)(\mathbb{C}_2)$ are Banach spaces equipped with the norm

$$\| \xi \|_{\Delta^m} = \sum_{r=1}^{m} \| \xi_r \| + \sup_k \| \Delta^m \xi_k \|.$$

Proof.

Suppose, $\xi = (\xi^n)$ is a Cauchy sequence in $l_\infty (\Delta^m)(\mathbb{C}_2)$ where each term of the given sequence is expressed as

$(\xi^n) = (\xi_k{}^n) = \{ \xi_1{}^n, \xi_2{}^n, \xi_3{}^n, \dots \}$ for each $n \in \mathbb{N}$. Then,

$$\| \xi^n - \xi^i \|_{\Delta^m} = \sum_{r=1}^{m} \| \xi_r{}^n - \xi_r{}^i \| + \sup_k \| \Delta^m \xi_k{}^n - \Delta^m \xi_k{}^i \| \to 0 \text{ as } n, i \to \infty.$$

$$\| \xi_k{}^n - \xi_k{}^i \| \to 0 \text{ as } n, i \to \infty, \text{ for each } k \in \mathbb{N}.$$

Therefore, $(\xi_k{}^i)$ is a Cauchy sequence in $\mathbb{C}_2$. As $\mathbb{C}_2$ is complete, $(\xi_k{}^i)$ converges to ξ_k (say).

$\therefore \lim_{i \to \infty} \xi_k{}^i = \xi_k$ for each $k \in \mathbb{N}$.

Because (ξ^n) is a Cauchy sequence, for any $\epsilon > 0$, one can find an integer $N \in \mathbb{N}$ for which

$$\| \xi^n - \xi^i \|_{\Delta^m} < \varepsilon \text{ for all } n, i \geq N.$$

Hence, $\sum_{r=1}^{m} \| \xi_r{}^n - \xi_r{}^i \| \leq \varepsilon$

and $\| \Delta^m (\xi_k{}^n - \xi_k{}^i) \| \leq \varepsilon$ for all $k \in \mathbb{N}$ and $n, i \geq N$.

So, we have, $\lim_{i \to \infty} \sum_{r=1}^{m} \| \xi_r{}^n - \xi_r{}^i \| = \sum_{r=1}^{m} \| \xi_r{}^n - \xi_r \| \leq \varepsilon$

And $\lim_{i \to \infty} \| \Delta^m (\xi_k{}^n - \xi_k{}^i) \| = \| \Delta^m (\xi_k{}^n - \xi_k) \| \leq \varepsilon$ for all $n \geq N$.

This implies that $\| \xi^n - \xi \|_{\Delta^m} \leq 2\varepsilon$ for all $n \geq N$.

Thus, $\xi_n \to \xi$ as $n \to \infty$, where $\xi = (\xi_k)$.

Finally, we show that $\xi = (\xi_k) \in l_\infty(\Delta^m)(\mathbb{C}_2)$.

Since we have, $\|\Delta^m \xi_k\| = \left\| \sum_{s=0}^{m} (-1)^s \binom{m}{s} \xi_{k+s} \right\|$

$$= \left\| \sum_{s=0}^{m} (-1)^s \binom{m}{s} (\xi_{k+s} - \xi_{k+s}{}^N + \xi_{k+s}{}^N) \right\|$$

$$\leq \left\| \sum_{s=0}^{m} (-1)^s \binom{m}{s} \left(\xi_{k+s}{}^N - \xi_{k+s}\right) \right\| + \left\| \sum_{s=0}^{m} (-1)^s \binom{m}{s} \left(\xi_{k+s}{}^N\right) \right\|$$

$$\leq \| \xi^N - \xi \| + \left\| \Delta^m \xi_k{}^N \right\| = O(1).$$

It means that they are bounded by constant.

Hence, $\| \Delta^m \xi_k \| < \infty$ for all $\xi = (\xi_k)$ so that $\xi = (\xi_k) \in l_\infty(\Delta^m)(\mathbb{C}_2)$.

Therefore, $l_\infty(\Delta^m)(\mathbb{C}_2)$ forms a Banach space. Since $c(\Delta^m)(\mathbb{C}_2)$ and $c_0(\Delta^m)(\mathbb{C}_2)$ are closed subspaces of $l_\infty(\Delta^m)(\mathbb{C}_2)$, they are Banach spaces as well.

Theorem 3. (i) $c_0(\Delta^m)(\mathbb{C}_2) \subset c_0(\Delta^{m+1})(\mathbb{C}_2)$,

(ii) $c(\Delta^m)(\mathbb{C}_2) \subset c(\Delta^{m+1})(\mathbb{C}_2)$,

(iii) $l_\infty(\Delta^m)(\mathbb{C}_2) \subset l_\infty(\Delta^{m+1})(\mathbb{C}_2)$

and the inclusions are strict.

Proof.

(i) Let $\xi = (\xi_k) \in c_0(\Delta^m)(\mathbb{C}_2)$, then $\| \Delta^m \xi_k \| \to 0$ as $k \to \infty$.

Now, $\| \Delta^{m+1} \xi_k \| = \| \Delta^m \xi_k - \Delta^m \xi_{k+1} \| \leq \| \Delta^m \xi_k \| + \| \Delta^m \xi_{k+1} \| \to 0$ as $k \to \infty$.

$\therefore$ $\xi = (\xi_k) \in c_0(\Delta^{m+1})(\mathbb{C}_2)$.

Hence, $c_0(\Delta^m)(\mathbb{C}_2) \subset c_0(\Delta^{m+1})(\mathbb{C}_2)$.

(iii) Let $\xi = (\xi_k) \in l_\infty(\Delta^m)(\mathbb{C}_2)$. Then, $\| \Delta^m \xi_k \| < \infty$.

Now, $\| \Delta^{m+1} \xi_k \| = \| \Delta^m \xi_k - \Delta^m \xi_{k+1} \| \leq \| \Delta^m \xi_k \| + \| \Delta^m \xi_{k+1} \| < \infty$.

$\therefore$ $\| \Delta^{m+1} \xi_k \| < \infty$ for all $\xi = (\xi_k)$. So, $\xi = (\xi_k) \in l_\infty(\Delta^{m+1})(\mathbb{C}_2)$.

Hence $l_\infty(\Delta^m)(\mathbb{C}_2) \subset l_\infty(\Delta^{m+1})(\mathbb{C}_2)$.

The inclusion relation is strict. It can be illustrated by the following example.

Consider the sequence $\xi = (k^m j)$. Then, $\xi \in c_0(\Delta^{m+1})(\mathbb{C}_2)$, but it doesn't belong to $c_0(\Delta^m)(\mathbb{C}_2)$.

Theorem 4. The sequence space $l_\infty(\Delta^m)(\mathbb{C}_2)$ possesses the property of convexity.

Proof.

Suppose, $\xi = (\xi_k)$, $\eta = (\eta_k) \in l_\infty(\Delta^m)(\mathbb{C}_2)$ and α is any real number satisfying

$0 \leq \alpha \leq 1$. Then, $\sup_k \|\Delta^m \xi_k\| < \infty$ and $\sup_k \|\Delta^m \eta_k\| < \infty$.

Now, $\sup_k \|\Delta^m \{\alpha\, \xi_k + (1-\alpha)\, \eta_k\}\| \leq \sup_k \|\Delta^m(\alpha\, \xi_k)\| + \sup_k \|\Delta^m \{(1-\alpha)\, \eta_k)\}\|$

$$\leq |\alpha| \sup_k \|\Delta^m \xi_k\| + |(1-\alpha)| \sup_k \|\Delta^m \eta_k\|$$

$$< \infty.$$

$\therefore \sup_k \|\Delta^m \{\alpha\, \xi_k + (1-\alpha)\, \eta_k\}\| < \infty$.

Hence, $\alpha\, \xi_k + (1-\alpha)\, \eta_k \in l_\infty(\Delta^m)(\mathbb{C}_2)$.

Therefore, $l_\infty(\Delta^m)(\mathbb{C}_2)$ is convex.

CONCLUSION

In this article, we established linearity, completeness and convexity property of generalized difference sequence spaces of bi-complex numbers. We also discussed some inclusion relations. The extension of algebraic, topological and geometric properties of generalized difference double sequence spaces of bi-complex numbers presents a promising direction for further research.

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AUTHOR CONTRIBUTIONS

Conceptualization: MPJ, NPP; Investigation: PP; Methodology: NPP; Data curation: MPJ; Data analysis: PP; Writing - original draft: MPJ; Writing - review and editing: MPJ, NPP, PP.

CONFLICT OF INTEREST

The authors confirm that they have no competing interests in relation to the publication of this article.

REFERENCES

Alpay, D., Luna Elizarrarás, M.E., Shapiro, M., & Struppa, D. C. (2014). Basics of functional analysis with bi-complex scalars and bi-complex Schur analysis. *Springer Science and Business Media.*

Bera, S., & Tripathy, B.C. (2023). Statistical convergence in bi-complex valued metric space. *Ural Mathematical Journal*, *9*(1), 49-63.

Bera, S., & Tripathy, B.C. (2023). Ceśaro convergence of sequences of bi-complex numbers using BC- Orlicz function. *Filomat, 37*(28), 9769-9775.

Degirmen, N., & Sagir, B. (2022). Some new notes on bi-complex sequence space $l_p(\mathbb{C}_2)$. *Journal of Fractional Calculus and Applications*, *13*(2), 66-76.

Degirmen, N., & Sagir, B. (2022). Some geometric properties of bi-complex sequence spaces. *Konuralp Journal of Mathematics*, *10*(1), 44-49.

Et, M. (1993). On some difference sequence spaces. *Turkish Journal of Mathematics, 17,* 18-24.

Et, M., & Colak, R. (1995). On some generalized difference sequence spaces. *Soochow Journal of Mathematics, 21*(4), 377-386.

Ghimire, J. L., & Pahari, N.P. (2022). On certain type of difference sequence spaces defined by φ − function. *Journal of Nepal Mathematical Society*, *5*(2), 11-17.

Herawati, E., & Gulton, S. N. R. (2019). On a certain type of sequence spaces defined by φ − function. *Journal of Research in Mathematics Trends and Technology*, *1*, 61-67.

Kizmaz, H. (1981). On certain sequence spaces. *Canadian Mathematical Bulletin, 24*, 169-176.

Kumar, S., & Tripathy, B. C. (2024). Double sequences of bi-complex numbers. *Proceedings of National Academy of Sciences, India Section A: Physical Sciences*, http:// doi. org/10.1007/s 40010-024-00895-7.

Kumar, S., & Tripathy, B. C. (2024). Difference double sequences of bi-complex numbers. *Annals of the Academy of Romanian Scientists: Series on Mathematics and its Applications*, *16*(2), 135-149.

Kumar, S., & Tripathy, B. C. (2024). Almost convergence double sequences of bi-complex numbers. *Filomat, 38*(11), 3957-3970.

Parajuli, P., Pahari, N. P., Ghimire, J. L., & Jaiswal, M. P. (2025). On some sequence spaces of bi-complex numbers. *Nepal Journal of Mathematical Sciences, 6*(1), 35-44.

Parajuli, P., Pahari, N. P., Ghimire, J. L., & Jaiswal, M. P. (2025). On some difference sequence spaces of bi-complex numbers. *Journal of Nepal Mathematical Society, 8*(1), 76-82.

Price, G. B. (1991). An introduction to multi-complex spaces and functions. Marcel Dekker Inc.

Rochon, D., & Shapiro, M. (2004). On algebraic properties of bi- complex and hyperbolic numbers. *Annals of Oradea University: Mathematics Fascicola, 11*, 1-28.

Sager, N., & Sagir, B. (2020). On completeness of some bi- complex sequence spaces. *Palestine Journal of Mathematics, 9*(2), 891-902.

Segre, C. (1892). Le rappresenttazioni reali delle forme complessee gli enti iperal gebrici. *Annals of Mathematics*, *40*(3), 413-467.

Tripathy, B. C., & Esi, A. (2006). A new type of difference sequence spaces. *International Journal of Science and Technology, 1*(1), 11-14.

Wagh, M. A. (2014). On certain spaces of bi-complex sequences. *International Journal of Physics Chemistry and Mathematics,* 7(1), 1-6.