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Research Article

ADOMIAN DECOMPOSITION APPROACH FOR SOLVING THE TWO-DIMENSIONAL FRACTIONAL ADVECTION-DIFFUSION EQUATION

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ABSTRACT

This work focuses on improving the modeling of pollutant dispersion in environment where traditional diffusion approaches are inadequate, such as the Kathmandu valley. Motivated by the need for accurate predictions under stagnant atmospheric conditions, especially during winter, this study addresses the limitations of classical models that fail to capture memory effects and slow pollutant spread. A time-fractional advection-diffusion equation (FADE), incorporating fractional derivatives to represent non-classical diffusion is utilized. The Adomian Decomposition Method (ADM) is applied to derive an approximate analytical solution. The results reveal that lower fractional orders depict slower, sub-diffusive transport, whereas higher orders transition toward classical diffusion behavior. This approach effectively models the anomalous dispersion patterns observed in complex terrains, offering an enhanced framework for air quality assessment in situations where traditional methods are inadequate.

Mathematics Subject Classification (MSC 2020): 35R11, 65M60.

Keywords: Adomian Decomposition, Anomalous Diffusion, Fractional Calculus, Pollutant Dispersion.

INTRODUCTION

Urban air pollution poses a serious environmental threat in topographically complex regions such as Kathmandu, Nepal (Acharya, 2024; Mahapatra *et al.,* 2019). The valley's surrounding hills and mountains significantly limit pollutant dispersion, leading to hazardous accumulation exacerbated during winter by weak winds, temperature inversions, and terrain-induced stagnation (Pariyar et al., 2025). Traditional diffusion models, reliant on parabolic equations, fail to capture the slow, anomalous transport under low-wind conditions, necessitating advanced mathematical frameworks for accurate prediction and mitigation (Gao, 2023; Pariyar & Kafle, 2024).

Fractional calculus addresses these limitations by modeling non-local dynamics and memory effects inherent in complex systems (Ahmed, 2024; Berkowitz, 1998). The Fractional advection-diffusion Equation (FADE), incorporating derivatives like the Caputo operator, which accommodates physical initial conditions, provides a rigorous foundation for anomalous diffusion (Pariyar et al., 2022). Solving such equations efficiently requires robust analytical tools, among which the Adomian Decomposition Method (ADM) excels; it handles nonlinearities without linearization or perturbation, yielding approximate analytical solutions (Acharya, 2024; Liu et al., 2019; Mahapatra, 2019; Pariyar & Kafle, 2024).

The foundation of fractional modeling lies in the concept of Continuous Time Random Walks (CTRW) (Montroll & Weiss, 1965), which was later extended to fractional operators by Berkowitz et al. (1990). Influential contributions by Podlubny (1999) and Metzler (2000) established fractional calculus as a robust framework for modelling memory-dependent transport phenomena. Subsequent studies by Zhang (2017), Odibat (2023), and Liu (2019) demonstrated its effectiveness in representing the dispersion of gaseous pollutants. In recent developments, advanced computational techniques such as wavelet-based solvers (Yadav, 2024), adaptive Grünwald-Letnikov schemes (Ahmed, 2024), and finite element methods (Gao, 2023) have been introduced to solve fractional models more efficiently. Focusing on regional applications (Pariyar et al., 2024) developed a one-dimensional Caputo-based transport model for NH_3, CO, and CO_2, while Pariyar & Kafle (2025) showcased the effectiveness of the Adomian Decomposition Method (ADM) in handling variable-coefficient fractional equations. Despite progress in pollutant dispersion modelling, key challenges persist: one-dimensional models often ignore vertical stratification and terrain heterogeneity, near-stagnant valley dispersion is seldom studied, and analytical solutions for FADEs with complex topography remain limited. This work aims to fill current gaps by developing a Caputo fractional-order model to describe pollution behaviour during low-wind conditions in Kathmandu. Using the Adomian Decomposition Method (ADM), an analytical series solution is obtained to show how the fractional order (α) affects pollutant spread. The main contributions include a terrain-aware fractional advection-dispersion equation (FADE) that captures unusual diffusion and trapping, a straightforward ADM solution method, and new insights into how α controls pollutant persistence under near-stagnant conditions.

MATHEMATICAL MODEL

We consider the two-dimensional time-fractional advection–diffusion equation (FADE) to describe the transport of pollutants and obtain its analytical solution using the Adomian Decomposition Method (ADM). The model is formulated with the Caputo fractional derivative, which enables the representation of memory effects in the temporal evolution of pollutant concentration. The solution is derived through a structured ADM procedure, and its consistency is examined within the proposed mathematical framework.

$$\frac{\partial^\alpha C}{\partial t^\alpha} = D.\nabla^2 C - \vec{u}.\nabla C,\ 0 < \ \leq\ 1;\ 0 \leq x < L, \quad \ldots(1)$$

In this expression, C(x, y, t) denotes the pollutant concentration at spatial coordinates (x, y) and time t. The operator $\frac{\partial^\alpha}{\partial t^\alpha}$ signifies the Caputo fractional derivative of order α, which incorporates memory effects into the temporal evolution of concentration. The parameter D represents the diffusion coefficient, describing the random spreading of pollutant particles, while $\vec{u} = (u_x, u_y)$ is the velocity vector associated with the advective transport caused by airflow or fluid motion.

Where, $\nabla^2 C = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}$ and $\vec{u}.\nabla C = u_x \frac{\partial C}{\partial x} + u_y \frac{\partial C}{\partial y}$, with initial conditions: C(x,y,0) = f(x, y).

Boundary condition: C (0, y, t) = C (L_x, y, t) = 0; C (x, 0, t) = C (x, L_y, t) = 0.

The analytical solution is obtained by applying Equation (1), where the Caputo time-fractional derivative is considered, defined as follows:

$$\frac{\partial^\alpha C}{\partial t^\alpha} = \Gamma(1-\alpha)\int_0^t \frac{\partial C}{\partial \tau}\frac{d\tau}{(t-\tau)^\alpha},\quad 0 < \alpha \leq 1 \qquad \ldots(2)$$

Here, the Riemann-Liouville fractional integral of order

$$J^\alpha[g(t)] = \Gamma(1-\alpha)\int_0^t (t-\tau)^{\alpha-1} g(\tau)\, d\tau \qquad \ldots(3)$$

And the Caputo derivative can be inverted by applying J^α,

$$J^\alpha\left[\frac{\partial^\alpha C}{\partial t^\alpha}\right] = C(x, y, t) - C(x, y, 0),$$

The equation (1) reduces to

$$J^\alpha\left[\frac{\partial^\alpha C}{\partial t^\alpha}\right] = J^\alpha\left[D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)\right] - J^\alpha\left[u_x \frac{\partial C}{\partial x} + u_y \frac{\partial C}{\partial y}\right],$$

$$\text{Or, } C(x, y, t) - C(x, y, 0) = J^\alpha\left[D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)\right] - J^\alpha\left[u_x \frac{\partial C}{\partial x} + u_y \frac{\partial C}{\partial y}\right].$$

$$\text{Or, } C(x, y, t) = f(x, y) + J^\alpha\left[D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)\right] - J^\alpha\left[\left[u_x \frac{\partial C}{\partial x} + u_y \frac{\partial C}{\partial y}\right]\right]$$

The Adomian decomposition method assume a series solution for

$$C(x, y, t) = C_n(x, y, t). \qquad \ldots(4)$$

Equation (4) reduces to,

$$\sum_{n=0}^{\infty} C_n = f(x, y) + J^\alpha\left[D\sum_{n=0}^{\infty}\left(\frac{\partial^2 C_n}{\partial x^2} + \frac{\partial^2 C_n}{\partial y^2}\right)\right] - J^\alpha\left[u_x \sum_{n=0}^{\infty}\frac{\partial C_n}{\partial x} - u_y \frac{\partial C_n}{\partial y}\right].$$

Now, let us construct the recursive terms: $C_0(x, y, t) = f(x, y)$. Recursive terms for $n \geq 0$,

$$\sum_{n=0}^{\infty} C_{n+1} = J^{\alpha}\left[D\sum_{n=0}^{\infty}\left(\frac{\partial^2 C_n}{\partial x^2} + \frac{\partial^2 C_n}{\partial y^2}\right)\right] - J^{\alpha}\left[u_x \sum_{n=0}^{\infty}\frac{\partial C_n}{\partial x} - u_y\frac{\partial C_n}{\partial y}\right].$$

Choosing the initial condition,

$$f(x, y) = \sin\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right). \text{ Then, } C_0(x, y, t) = \sin\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right).$$

Higher-order components C_1, C_2 and $C_3$are obtained iteratively using diffusion and advection operators together with fractional time integrals.

$$\frac{\partial C_0}{\partial x} = \frac{\pi}{L_x}\cos\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right) \text{ and } \frac{\partial C_0}{\partial y} = \frac{\pi}{L_y}\sin\left(\frac{\pi x}{L_x}\right)\cos\left(\frac{\pi y}{L_y}\right),$$

$$\frac{\partial^2 C_0}{\partial x^2} = -\left(\frac{\pi}{L_x}\right)^2\sin\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right) \text{ and } \frac{\partial^2 C_0}{\partial y^2} = -\left(\frac{\pi}{L_y}\right)^2\sin\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right),$$

The R.H.S of equation (1) becomes:

$$D.\nabla^2 C - \vec{u}.\,\nabla C = -D\left(\left(\frac{\pi}{L_x}\right)^2 + \left(\frac{\pi}{L_y}\right)^2\right)\sin\left(\frac{\pi x}{L_x}\right) - u_x\frac{\partial C_0}{\partial x} - u_y\frac{\partial C_0}{\partial y}.$$

$$= -D\left(\left(\frac{\pi}{L_x}\right)^2 + \left(\frac{\pi}{L_y}\right)^2\right)C_0 - u_x\frac{\partial C_0}{\partial x} - u_y\frac{\partial C_0}{\partial y}.$$

$$C_1(x, y, t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)}\left(-D\lambda^2 C_0 - u_x\frac{\partial C_0}{\partial x} - u_y\frac{\partial C_0}{\partial y}\right),$$

Where, $\lambda^2 = \left(\frac{\pi}{L_x}\right)^2 + \left(\frac{\pi}{L_y}\right)^2$; $Q(x, y) = -D\lambda^2 C_0 - u_x\frac{\partial C_0}{\partial x} - u_y\frac{\partial C_0}{\partial y}$.

$$C_1(x, y, t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)}[\,Q(x,y)],$$

$$C_2(x, y, t) = \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}L[\,Q(x,y)] = \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}L\left(-D\lambda^2 C_0 - ux\frac{\partial C_0}{\partial x} - u_y\frac{\partial C_0}{\partial y}\right),$$

$$C_3(x, y, t) = \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}L[\,C_2(x, y, t)].$$

The ADM expansion was terminated at the third correction term C_3 because the solution exhibits fast convergence. Contributions from subsequent terms were observed to be minimal and do not meaningfully alter the concentration profiles, while their inclusion leads to unnecessary algebraic complexity. Limiting the series to C_3 is therefore sufficient to represent the primary transport behavior and the memory effects introduced by the fractional formulation. So each term depend on applying the operator L to the previous C_n, then scaled by the appropriate fractional power of time and gamma function $C(x, y, t) \approx C_0(x, y) + \frac{t^{\alpha}}{\Gamma(1+\alpha)}L(C_0) + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}L^2(C_0) + \ldots$.The obtained analytical solution illustrates the behaviour of pollutant concentration as it changes across time, influenced by both fractional diffusion and advection effects. The order of the fractional derivative α reflects memory dependent dynamics, where lower values of α corresponds to slower values of α corresponds to slower, non-classical diffusion commonly referred to as sub diffusion highlighting the presence of anomalous transport mechanisms.

RESULTS AND DISCUSSION

The Adomian Decomposition Method (ADM) is employed **to** obtain an approximate analytical solution of the two-dimensional time-fractional advection-diffusion equation. This model describes pollutant transport by incorporating diffusion, advection, and Caputo-type fractional time derivatives. The fractional order α ($0 < \alpha \leq 1$) represents memory effects and enables the characterization of anomalous diffusion behavior. The concentration $C(x, y, t)$ is represented as a sum of terms $C_0 + C_1 + C_2 + \cdots$, where each term refines the solution. The first term $C_0(x, y)$ is the initial sinusoidal pollutant distribution centered in the domain. Higher-order terms C_1, C_2… are computed iteratively using diffusion and advection operators combined with fractional integrals weighted by

Gamma functions. In this work, terms up to C_2 approximate the solution at t = 0.01s, capturing

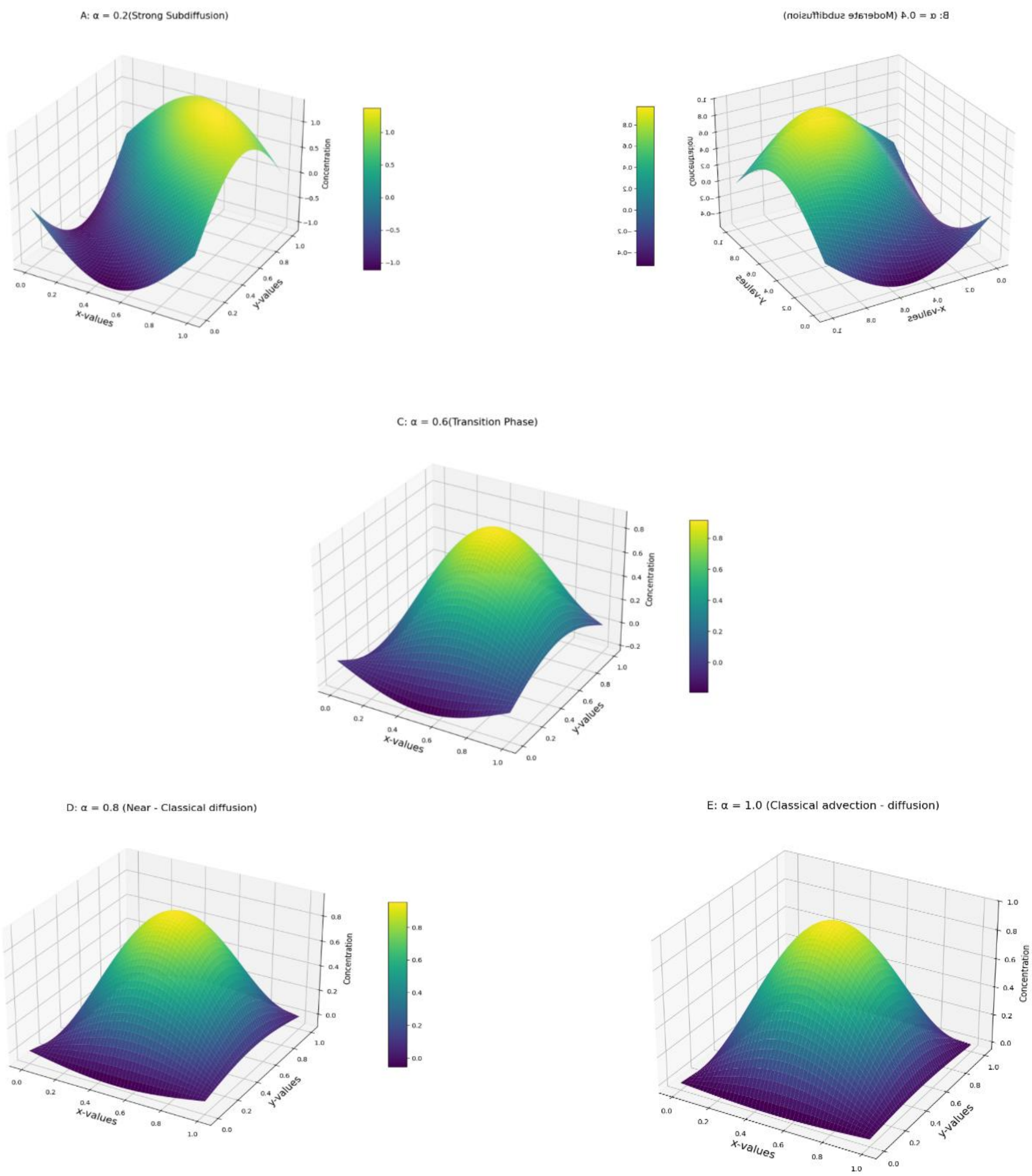

Figure 1: pollutants concentration for fractional orders A: α = 0.2, B: α = 0.4, C: α = 0.6, D: α = 0.8, and E: α = 1.0, with diffusivity D = 0.1 m²/s and constant wind velocities $u_x = u_y = 1.0$ m/s

The ADM breaks down the complex fractional advection-diffusion equation into a sequence of simpler calculations. This model integrates reduced molecular diffusivity D= 0.1 m^2/s consistent advection velocities in both x and y directions ($u_x = u_y = 1.0$ m/s), and varying fractional orders (α = 0.2, 0.4, 0.6, 0.8, 1.0). The resulting concentration surfaces reveal how pollutant spread evolves, demonstrating a shift from slow sub-diffusion at low α values toward classical diffusion as α nears 1.0. This method effectively captures the impact of anomalous diffusion on pollutant dispersion, relevant to the Kathmandu Valley.

Figure 1 depicted pollutant concentration profiles for fractional orders α = 0.2, 0.4, 0.6, 0.8, and 1.0. At A: α = 0.2, the pollutant remains sharply localized due to strong sub-diffusion, with negligible advection influence. As α increases to B: α=0.4 and C:α=0.6, the spread becomes noticeable, with advection gradually skewing the concentration profile. At D: α = 0.8, diffusion dominates, and advection visibly shifts the plume. Finally, at E: α = 1.0, classical diffusion behavior appears, where advection-driven transport results in a broad, uniform dispersion pattern. The fractional order (α) controls how pollutant dispersion shifts from slow, memory-influenced spreading to normal diffusion. This is especially important for pollutants like $PM_{2.5}$, SO_2, CO, and NO_x. In Kathmandu valley, Nepal, fractional models are most useful during the winter months (November to February), when the unique terrain, stagnant air, and urban structures slow down pollutant movement. This approach also helps in understanding pollutant behavior indoors, in soil, and within turbulent atmospheric condition.

CONCLUSIONS

This work effectively utilizes the Adomian Decomposition Method (ADM) to obtain an approximate analytical solution of a two-dimensional time-fractional advection-diffusion equation (FADE), employing Caputo derivatives to represent non-local diffusion behavior. The developed model accurately simulates the anomalous dispersion of pollutants in geographically constrained environments like Kathmandu Valley, where conventional diffusion models inadequately address stagnant atmospheric scenarios. The study's results emphasize the significant influence of the fractional order α on pollutant transport: Lower α values (0.2–0.6) exhibit sub-diffusive transport, where pollutant dispersion is slow and remains highly localized due to strong memory effects and trapping phenomena. Higher α values (0.8–1.0) approach classical diffusion behavior, with advection-driven dispersion patterns aligning with standard Fickian diffusion processes. The ADM approach demonstrated its strength in managing the complexities of memory-dependent transport equations, delivering accurate series solutions without the need for linear approximations. This method offers considerable benefits for modeling long-lived pollutants, such as $PM_{2.5}$ and CO, particularly under low-wind conditions during winter months in valley regions where anomalous diffusion processes are dominant. Moving forward, enhancing the model by integrating realistic topographical features, dynamic boundary conditions, and vertical stratification layers will be vital for improving its predictive capability in urban air quality assessment.

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AUTHOR CONTRIBUTIONS

Conceptualization: SP; Investigation: SP; Methodology: SP; Data curation: JK, TEPP, UDRC; Data analysis: SP; Writing - original draft: SP; Writing - review and editing: JK.

CONFLICT OF INTEREST

There is no conflict of interest among the authors.

DATA AVAILABILITY STATEMENT

Data inquiries should be addressed to the authors.

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