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Research Article

COMPATIBLE MAPPINGS OF TYPE (K) IN FUZZY METRIC SPACE

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ABSTRACT
This paper discusses common fixed-point theorem in fuzzy metric space for three self-mappings by using the conditions of compatible mappings of type (K). This theorem generalizes the results of K.B. Manandhar *et al.* (2014), and also the results of Cho (2006).

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Keywords: Common fixed point, compatible mappings, Compatible mappings of type (K), Fuzzy metric space

INTRODUCTION
The study of fixed points is an important area of functional analysis. Common fixed points of mappings that satisfy certain contractive type conditions have been very active field of research. The concept of compatible mappings was proposed by G. Jungck in 1986, as a broader idea than commuting and weakly commuting mappings. The concept of compatible mappings under certain conditions is analogous to the generalization of compatible mappings of type (A) provided by G. Jungck *et al.* in 1993. The concept of compatible type (B) mappings was first introduced by H. K. Pathak and M. S. Khan in 1995. The idea of compatible type (P) mappings was presented by H. K. Pathak *et al.* in 1996. They contrasted it with compatible mappings of type (A) and in a metric space.

The groundwork for fuzzy mathematics was laid in 1965 when L. A. Zadeh introduced fuzzy sets as a way to represent the uncertainty of daily life. By expanding on the idea of M. Frechet metric space (Frechet, 1906), Kramosil and Michalek created the concept of fuzzy metric space in 1975. Fuzzy contractive mappings were first introduced by Heilpern in 1981, and the contraction principle in fuzzy metric spaces was shown by M. Grabiec in 1988. George and Veeramani used the t-norm to modify the notion of fuzzy metric space in 1994.

S. N. Mishra *et al.* introduced the concept of compatible mappings in fuzzy metric space in 1994. Cho *et al.* introduced the concept of compatible mappings of type (B) in fuzzy metric space in 1998. Afterward, many authors introduced various compatible mappings of different types in fuzzy metric space and proved common fixed-point results in that space reader may see the references (Balasubramaniam *et al.*, 2002; Pant & Jha, 2004; Cho, 2006; Cho & Jung, 2006; Mishra *et al.*, 2008; Jain *et al.*, 2009; Jha, 2010; Kumar & Fisher, 2011; Jha, 2013; Manandhar, 2014). In 2014, K. B. Manandhar *et al.* extended compatible mappings of type (E) in fuzzy metric space introduced by M. R. Singh and Y. M. Singh in 2007, in terms of compatible mappings of type (K) in metric space and fuzzy metric space as well and proved a common fixed-point theorem. Later numerous researchers have now achieved fixed point findings in fuzzy metric spaces as a result of their work in this field.

In this paper, we modify common fixed-point theorems obtained in (Manandhar, 2014)] and we characterize the conditions for compatible mappings of type (K) in which two continuous self-mappings of complete fuzzy metric space have a unique common fixed point.

PRELIMINARIES
Definition 2.1. (Klement *et al.*, 2004)
A mapping $*: [0,1] \times [0,1] \to [0,1]$ is called a ***triangular norm*** (shortly t-norm) if it satisfies the following conditions:
For each $a, b, c, d, \in [0,1]$,
(i) $a * 1 = a$;
(ii) $a * b = b * a$;
(iii) $a * (b * c) = (a * b) * c$; and
(iv) $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$.

Example 2.2. (Chaudhary *et al.*, 2021)

$$a * b = \begin{cases} 0 & for\ a = b = 0 \\ a\ or\ b & for\ b = 1\ or\ a = 1 \\ \frac{ab}{a+b} & otherwise \end{cases}.$$

Then $*$ is a triangular norm.

Definition 2.3. (Schweizer & Sklar, 1960)
A triangular norm $*$ is said to be continuous ***t-norm*** if it satisfies the following conditions: For each $a,\ b,\ c,\ d\ \in\ [0,\ 1]$,
(i) $*$ is associative and commutative;

(ii) $*$ is continuous;
(iii) Boundary condition: $a * 1 = a$ and
(iv) Monotonicity: $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$.

Example 2.4: (i) $a * b = ab$
(ii) $a * b = \min\{a, b\}$ for $a, b \in [0, 1]$ are continuous t-norm.

Definition 2.5. (Zadeh, 1965)
If X is a universal set and x is any element of X. Then a ***fuzzy set*** A defined on X is a collection of ordered pairs such that $A = \{(x, \mu A(x)): x \in X, \mu A(x) \in [0,1]\}$.
Where, $\mu A: X \to [0,1]$ is a membership function.

Example 2.6: Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ and A is a fuzzy set of smart students, where smart is a fuzzy term, then

$$A = \{(x_{1,}0.4), (x_{2,}0), (x_{3,}0.5), (x_{4,}0.8), (x_{5,}0.6)\}.$$

Where, $\mu A(x) \in [0,1]$, i.e., the degree of smartness of x_1 is 0.4, and so on.

Definition 2.7. *[George and Veeramani, 1994]* The 3-tuple (X, M, $*$) is called a ***fuzzy metric space*** if X is an arbitrary set, $*$ is a continuous t-norm and M: $X \times X \times (0, \infty) \to [0,1]$ is a fuzzy set satisfying the following conditions: For all x, y, z ∈ X and t, s > 0.
(i) $M(x, y, t) > 0$,
(ii) $M(x, y, t) = 1$ *if and only if* $x = y$,
(iii) $M(x, y, t) = M(y, x, t)$,
(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(v) $M(x, y, \cdot): (0, \infty) \to [0, 1]$ *is continuous.*

Example 2.8. Let (X, d) be a metric space, and let $a * b = ab$. Let
$M(x, y, t) = \frac{t}{t + d(x,y)}$ for all $x, y \in X$ and $t > 0$.
Then $(X, M, *)$ is a fuzzy metric space, and this fuzzy metric M induced by d is called the standard fuzzy metric.

Definition 2.9. (Grabiec, 1988)
A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be ***convergent*** to a point $x \in X$(denoted by $\lim_{n\to\infty} x_n$), if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$.
George and Veeramani show that a sequence $\{x_n\}$ in a fuzzy metric space *(X, M, $*$) converges to a point $x \in X$* if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$.

Definition 2.10. (George & Veeramani, 1994)
A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called a ***Cauchy sequence*** if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $(x_n, x_m, t) > 1 - \epsilon$ for all n, m $\geq n_0$. *A fuzzy metric space* $(X, M, *)$ *in which every Cauchy sequence is convergent is said to be a complete fuzzy metric space.*

Definition 2.11. (Mishra *et al.*, 1994)
The self-mappings S and T of a fuzzy metric space *(X, M, $*$)* are said to be ***compatible*** if $\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some x in X and t > 0.

Definition 2.12. (Cho *et al.*, 1998)
The self-mappings S and T of a fuzzy metric space *(X, M, $*$)* are said to be ***compatible of type (A)*** if $\lim_{n\to\infty} M(STx_n, TTx_n, t) = 1$ and $\lim_{n\to\infty} M(TSx_n, SSx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some t in X and $t > 0$.

Definition 2.13. (Cho *et al.*, 1998)
The self-mappings S and T of a fuzzy metric space$(X, M, *)$ are said to be ***compatible of type (P)*** if $\lim_{n\to\infty} M(TTx_n, SSx_n, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some x in X and t > 0.

Definition 2.14. (Singh & Singh, 2007)
The self-mappings S and $T$of a fuzzy metric space $(X, M, *)$ are said to be ***compatible of type (E)*** iff $\lim_{n\to\infty} M(SSx_n, STx_n, t) = 1$, $\lim_{n\to\infty} M(SSx_n, Tx, t) = 1$, $\lim_{n\to\infty} M(STx_n, Tx, t) = 1$ and $\lim_{n\to\infty} M(TTx_n, TSx_n, t) = 1$, $\lim_{n\to\infty} M(TTx_n, Sx, t) = 1$, $\lim_{n\to\infty} M(TSx_n, Sx, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some x in X and t > 0.

Definition 2.15. (Rohan *et al.*, 2008)
The self-mappings S and $T$of a fuzzy metric space $(X, M, *)$ are said to be ***compatible of type (R)*** if, $\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1$ and $\lim_{n\to\infty} M(SSx_n, TTx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some x in X and t > 0.

Definition 2.16. (Manandhar *et al.*, 2014)
The self-mappings S and T of a fuzzy metric space $(X, M, *)$ are said to be ***compatible of type (K)*** iff $\lim_{n\to\infty} M(SSx_n, Tx, t) = 1$ and $\lim_{n\to\infty} M(TTx_n, Sx, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some x in X and $t > 0$.

Lemma 2.17. (Grabiec, 1988)
Let $(X, M, *)$ be a fuzzy metric space. Then for all $x, y \in X$, $M(x, y, \cdot)$ is ***non-decreasing.***

Lemma 2.18. (Cho, 2006)
Let $(X, M, *)$ be a fuzzy metric space with the condition $\lim_{n\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$. If

there exists $k \in (0,1)$ such that $M(x,y,kt) \geq M(x,y,t)$, then $x = y$.

Lemma 2.19. (Cho, 2008)
Let $\{y_n\}$ be a sequence in a fuzzy metric space $(X, M,*)$ with the condition $\lim_{n\to\infty} M(x,y,t) = 1$ for all $x, y \in X$ and $t > 0$. If there exists $k \in (0,1)$ such that $M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t)$ and $n \in N$, then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2.20. (Cho, 2008)
Let $(X, M,*)$ be a fuzzy metric space and let S and T be continuous self-mappings of X and $[S,T]$ be compatible. Let $\{x_n\}$ be a sequence in X such that $Sx_n \to z$ and $Tx_n \to z$. Then $STx_n \to Tz$.

Lemma 2.21. (Klement *et al.*, 2004)
The only t-norm $*$ satisfying $r * r \geq r$ for all $r \in [0,1]$ is the minimum t-norm, that is, $a * b = min\{a,b\}$ for all $a, b \in [0,1]$.

SOME COMMON FIXED-POINT THEOREMS

In this section, we use the characterization of Cauchy sequences and some lemmas discussed in the previous section to prove new fixed-point theorems for compatible mappings of types (K) on complete fuzzy metric spaces. In 2006, Seong Hoon Cho introduced the following theorem:

Theorem 3.1. Let $(X, M, *)$ be a complete fuzzy metric space, and let $A, B, S, T: X \to X$ be self-mappings of X such that the following conditions are satisfied:

(i) $AX \subset TX$, $BX \subset SX$,
(ii) S and T are continuous,
(iii) the pairs $[A, S]$ and $[B, T]$ are compatible
(iv) there exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,
$M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$.

Then $A, B, S,$ and T have a unique common fixed point in X.

Corollary 3.2. (Singh & Chauhan, 2000)
Let $(X, M, *)$ be a complete fuzzy metric space, and let $A, B, S, T: X \to X$ be self- mappings of X satisfying (i) − (iii) of theorem 3.1 and there exists k ∈ (0, 1) such that
$M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t)$
for every $x, y \in X$ and $t > 0$.
Then $A, B, S,$ and T have a unique fixed point in X.

Corollary 3.3. Let $(X, M, *)$ be a complete fuzzy metric space and let $A, B, S, T: X \to X$ be self-mappings of X satisfying (i) − (iii) of theorem 3.1 and there exists $k \in (0, 1)$ such that
$M(Ax, By, kt) \geq M(Sx, Ty, t)$, for every $x, y \in X$ and $t > 0$.
Then $A, B, S,$ and T have a unique common fixed point in X.

Corollary 3.4. Let $(X, M, *)$ be complete fuzzy metric space and let $A, B, S, T: X \to X$ be self-mappings of X satisfying (i) − (iii) of theorem 3.1 and there exists $k \in (0, 1)$ such that
$M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t)$, for every $x, y \in X$ and $t > 0$.
Then $A, B, S,$ and T have a unique common fixed point in X.

In 2014, K. B. Manandhar *et al.* modified theorem 3.1 for compatible mapping of type (K).

Theorem 3.5. Let $(X, M, *)$ be a complete fuzzy metric space and let $A, B, S, T: X \to X$ be self-mappings of X such that the following conditions are satisfied:

(i) $AX \subset TX, BX \subset SX$,
(ii) $M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$,
For all $x, y \in X, k \in (0,1)$ and $t > 0$,
and
(iii) S and T are continuous.

If (A, S) and (B, T) are compatible with type *(K)*, then $A, B, S,$ and T have a unique common fixed point in X.

MAIN RESULTS

Theorem 4.1: Let $(X, M, *)$ be a complete fuzzy metric space and let $A, S, T: X \to X$ be self-mappings of X such that the following conditions are satisfied:

(i) $AX \subset TX \cap SX$,
(ii) $M(Ax, Ay, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$,
For all $x, y \in X, k \in (0,1)$ and $t > 0$,
and
(iii) S and T are continuous.

If (A, S) and (A, T) are compatible with type *(K)*, then $A, S,$ *and* T have a unique common fixed point in X.

Proof: Since, $AX \subset TX \cap SX$, implies $AX \subset TX$, $AX \subset SX$, so, for any $x_0 \in X$, there exists $x_1 \in X$ such that $Ax_0 = Tx_1$ and for this $x_1 \in X$, there exists $x_2 \in X$ such that
$Ax_1 = Sx_2$.
Inductively, we can define a sequence $\{y_n\}$ in X such that
$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and
$y_{2n} = Sx_{2n} = Ax_{2n-1}$ for $n = 1, 2 \cdots$

From condition (ii),
$M(y_{2n+1}, y_{2n+2}, kt) = M(Ax_{2n}, Ax_{2n+1}, kt)$
$\geq M(Sx_{2n}, Tx_{2n+1}, t) * M(Ax_{2n}, Sx_{2n}, t) * M(Ax_{2n+1}, Tx_{2n+1}, t) * M(Ax_{2n}, Tx_{2n+1}, t)$
$= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+1}, t)$
$\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t)$.

From lemma 2.13 and 2.18, we have
$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t)$ … (4.1.1)

Similarly, we also have
$M(y_{2n+2}, y_{2n+3}, kt) \geq M(y_{2n+1}, y_{2n+2}, t)$ … (4.1.2)

From (4.1.1) and (4.1.2), we have
$M(y_{n+1}, y_{n+2}, kt) \geq M(y_n, y_{n+1}, t)$ ... (4.1.3)

From (4.1.3), we have
$M(y_n, y_{n+1}, t) \geq M(y_n, y_{n-1}, t/k) \geq M(y_{n-2}, y_{n-1}, \frac{t}{k^2}) \geq \cdots \geq M(y_1, y_2, \frac{t}{k^n}) \to 1$ as $n \to \infty$.

So, $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ for any $t > 0$.

For each $\epsilon > 0$ and each $t > 0$, we can choose $n_0 \in N$ such that
$M(y_n, y_{n+1}, t) > 1 - \epsilon$ for all $n > n_0$.

For $m, n \in N$, we assume $m \geq n$. Then we have
$M(y_n, y_m, t) \geq M(y_n, y_{n+1}, t/(m-n)) * M(y_{n+1}, y_{n+2}, t/(m-n)) * \cdots * M(y_{m-1}, y_m, t/(m-n))$
$> (1-\epsilon) * (1-\epsilon) * \cdots * (1-\epsilon)$ *(m-n) times* $\geq 1-\epsilon$
$\Rightarrow M(y_n, y_m, t) \geq 1-\epsilon$
and hence $\{y_n\}$ is a Cauchy sequence in X.
Since $(X, M, *)$ is complete, $y_n\}$ converges to some point $z \in X$, and so
$\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Ax_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z.
Since *(A, S)* and *(A, T)* are compatible of type (K), and from lemma 2.17, we have
$AAx_{2n-2} \to Sz$, $SSx_{2n} \to Az$, $AAx_{2n-1} \to Tz$, *and* $TTx_{2n-1} \to Az$ … (4.1.4)

From *(ii)*, we get
$M(AAx_{2n-2}, AAx_{2n-1}, kt)$
$\geq M(SAx_{2n-2}, TA\, x_{2n-1}, t) * M(AAx_{2n-2}, SAx_{2n-2}, t) * M(AAx_{2n-1}, TA\, x_{2n-1}, t) * M(AAx_{2n-2}, TA\, x_{2n-1}, t)$.

Taking limit as $n \to \infty$, and applying (4.1.4), we get
$M(Sz, Tz, kt) \geq M(Sz, Tz, t) * M(Sz, Sz, t) * M(Tz, Tz, t) * M(Sz, Tz, t)$
$= M(Sz, Tz, t) * 1 * M(Sz, Tz, t)$
$= M(Sz, Tz, t)$.
Implies
$M(Sz, Tz, kt) \geq M(Sz, Tz, t)$, and hence
$Sz = Tz$. … (4.1.5) *(From lemma 2.18)*

Again, from *(ii)*,
$M(Az, AAx_{2n-1}, kt)$
$\geq M(Sz, TAx_{2n-1}, t) * M(Az, Sz, t) * M(AAx_{2n-1}, TAx_{2n-1}, t) * M(Az, TAx_{2n-1}, t)$.

Taking limit as $n \to \infty$, and applying (4.1.4) and (4.1.5), we get
$M(Az, Tz, kt) \geq M(Sz, Tz, t) * M(Az, Sz, t) * M(Tz, Tz, t) * M(Az, Tz, t)$
$= 1 * M(Az, Tz, t) * 1 * M(Az, Tz, t)$
$= M(Az, Tz, t)$.

Implies
$M(Az, Tz, kt) \geq M(Az, Tz, t)$ and hence
$Az = Tz$… (4.1.6)

From (4.1.5) and (4.1.6), we get
$Az = Sz = Tz$ … (4.1.7)
Now, we have to show $Az = z$.
Further, from condition (ii),
$M(Ax_{2n}, Az, kt) \geq M(Sx_{2n}, Tz, t) * M(Ax_{2n}, Sx_{2n}, t) * M(Az, Tz, t) * M(Ax_{2n}, Tz, t)$.
Taking limit as $n \to \infty$, and applying (4.1.7),
$M(z, Az, kt) \geq M(z, Tz, t) * M(z, z, t) * M(Az, Tz, t) * M(z, Tz, t)$
$= M(z, Az, t) * 1 * 1 * M(z, Az, t)$
$= M(z, Az, t)$
Implies
$M(z, Az, kt) \geq M(z, Az, t)$ and hence
Az = z … (4.1.8)

Thus from (4.1.7) and (4.1.8),
z = Az = Sz = Tz and
z is a common fixed point of *A, S* and *T*.

To show uniqueness of a fixed point, let w be other common fixed point of *A, S* and *T*, then
$w = Aw = Sw = Tw$ … (4.1.9)
Using the condition (ii)
$M(z, w, kt) = M(Az, Aw, kt) \geq M(Sz, Tw, t) * M(Az, Sz, t) * M(Aw, Tw, t) * M(Az, Tw, t)$
$= M(z, w, t) * M(z, z, t) * M(w, w, t) * M(z, w, t)$
$= M(z, w, t) * 1 * 1 * M(z, w, t)$
$= M(z, w, t)$
Implies
$z = w$. *(From lemma 2.18)*,
This completes the proof of the theorem.

Corollary 4.2: Let $(X, M, *)$ be a complete fuzzy metric space and let $A, S, T: X \to X$ be self-mappings of X satisfying the following conditions (i), (ii) of theorem 4.1 and if *(A, S)* and *(A, T)* are compatible of type *(K)* there exists k ∈ *(0, 1)* such that for every $x, y \in X$ and $t > 0$,
$M(Ax, Ay, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ay, Sx, 2t) * M(Ax, Ty, t)$.
Then A, S, and T have a unique common fixed point in X.

Proof: We have $M(Ax, Ay, kt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ay, Sx, 2t) * M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Sx, Ty, t) * M(Ty, Ay, t) * M(Ax, Ty, t) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$ and hence, from theorem 4.1, *A, S* and *T* have a unique fixed point in *X*.

Corollary 4.3. Let *(X, M, *)* be a complete fuzzy metric space and let $A, S, T: X \to X$ be self-mappings of X satisfying the following conditions (i), (ii) of theorem 4.1 and if *(A, S)* and *(A, T)* are compatible of type (K)

there exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$, $M(Ax, Ay, kt) \geq M(Sx, Ty, t)$.
Then A, S and T have a unique common fixed point in X.

Proof: We have $M(Ax, Ay, kt) \geq M(Sx, Ty, t) = M(Sx, Ty, t) * 1 \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Sx, 2t) * M(Ay, Ty, t) * M(Ax,Ty, t)$ and hence, from corollary 4.2, A, S and T have a unique fixed point in X.

Corollary 4.4. Let $(X, M, *)$ be a complete fuzzy metric space and let $A, S, T: X \to X$ be self-mappings of X satisfying the following conditions (i), (ii) of theorem 4.1 and if (A, S) and (A, T) are compatible of type (K) there exists $k \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$, $M(Ax, Ay, kt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t)$.
Then A, S and T have a unique common fixed point in X.

Proof: We have $M(Ax, Ay, kt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) =$
$M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) * 1 \geq M(Sx, Ty, t) * M(Sx, Ax, t) *$
$M(Ax, Ty, t) * M(Sx, Ay, 2t) * M(Ay, Ty, t) * M(Ty, Sx, t) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) * M(Sx, Ay, 2t) * M(Ay, Ty, t)$ and hence, from corollary 4.2, A, S and T have a unique fixed point in X.

CONCLUSIONS

In this paper we have established a new theorem in fuzzy metric space by using the notion of compatible mappings of type (K) in three self-mappings. Also, we have constructed and proved four corollaries of the established theorem. This paper generalizes the results of Cho (2006) and Manandhar et al. (2014) as well.

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AUTHOR CONTRIBUTIONS

NPK: Worked in the research and developed the paper, including the concept, research strategy and results; AKC: Contributed to the discussion, conceptualization, text editing and review; R.K. and K.B.M.: Contributed for suggestion and moderation.

CONFLICT OF INTERESTS

The authors declare no conflict of interests.

DATA AVAILABILITY STATEMENT

The data that supports the findings of this study are available from the first author and corresponding author, upon reasonable request.

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