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Research Article

HALF-CAUCHY CHEN DISTRIBUTION WITH THEORIES AND APPLICATIONS

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ABSTRACT

In this article, we have suggested the three-parameter half-Cauchy Chen distribution which has been derived by compounding a continuous Chen distribution with the half-Cauchy family of distribution. The hazard rate function (HRF), quantile function, reversed hazard rate function, survival function, cumulative distribution function (CDF), probability density function (PDF), kurtosis, and skewness of the suggested distribution are some of its statistical properties and characteristics that are explored. Utilizing the methods of LSE, CVM, and MLE, the parameters of the new distribution are computed. All the calculations are performed with the aid of R programming software. To assess the application of the new distribution, two real data sets are analyzed and performed the goodness-of-fit. It is found that the half-Cauchy Chen distribution outperformed a few other existing distributions. We hope that this distribution will contribute to the field of real data analysis.

Keywords: Chen distribution, half-Cauchy distribution, hazard function, MLE

INTRODUCTION

Last few decades several classical probability models have been used to model the real datasets related to finance, engineering, medicine, geology, climatology, biology, hydrology, reliability, ecology, risk analysis, and life testing, etc. do not provide a good fit. Therefore, it would seem necessary to expand the existing distributions in order to address the problems in these areas. By performing some change or inserting one or more parameters to the baseline model we can develop more flexible distributions which can provide a better fit as compared to existing classical models.

In this study we have considered the Chen distribution as parent distribution which was defined by (Chen, 2000) having increasing failure rate (IFR) function or bathtub shaped. The Chen distribution's cumulative distribution function (CDF) is

$$G(x;\beta,\lambda)=1-\exp\left\{\lambda(1-e^{x^{\beta}})\right\};\ x>0,\ \beta,\lambda>0 \quad (1)$$

And the corresponding probability density function (PDF) is

$$g(x;\beta,\lambda)=\beta\lambda x^{\beta-1}e^{x^{\beta}}\ \exp\left\{\lambda(1-e^{x^{\beta}})\right\} \quad (2)$$

Introducing a more adaptable model that can show the many varied shapes of the density and hazard functions inspired the modification of the Chen distribution. The Bayesian analysis of the Chen model was introduced by (Srivastava & Kumar, 2011). Bhatti *et al.* (2019) have defined the extended Chen (EC) distribution as derived from the nexus between the gamma and exponential variable. The Weibull–Chen (WC) distribution has been defined by Tarvirdizade and Ahmadpour (2019) having increasing, decreasing or bathtub-shaped hazard rate function. Joshi and Kumar (2020) have developed the Lindley-Chen distribution. Also, another extension of Chen distribution was introduced by Joshi and Kumar (2021a) called Logistic Chen distribution. The Poisson Chen distribution has been defined by (Joshi & Kumar, 2021b). In this study, we have taken into consideration the half-Cauchy distribution that results from the Cauchy distribution by folding the curve on the origin so that only positive values may be seen. As an alternative to modeling spreading distances, Shaw (1995) employed the half-Cauchy distribution with a strong tail since it can predict more frequent long-distance scattering events. In addition, the half-Cauchy model is also used by (Paradis *et al.*, 2002) to model ringing data on tits having two species in Ireland and Britain. If a non-negative random variable X follows the half-Cauchy distribution, then its cumulative distribution function can be written as

$$G\left(x;\theta\right)=\frac{2}{\pi}\tan^{-1}\left(\frac{x}{\theta}\right),\quad x>0,\theta>0. \quad (3)$$

and the probability density function (PDF) corresponding to (3) is, $g\left(x;\theta\right)=\frac{2}{\pi}\left(\frac{\theta}{\theta^{2}+x^{2}}\right),\quad x>0,\theta>0. \quad (4)$

Last some decades many researchers have been used the half-Cauchy distribution as a parent distribution. The modification of the half-Cauchy distribution was introduced by (Cordeiro & Lemonte, 2011) called the beta-half-Cauchy distribution, Jacob and Jayakumar (2012) has presented the modification of half-Cauchy distribution applying Marshall-Olkin transformation and studied the autoregressive process of first order and Polson and Scott (2012) have been used the half-Cauchy distribution as prior for a universal scale parameter for Bayesian analysis. The Kumaraswamy-half-Cauchy distribution is a further development of the half-Cauchy model that was introduced by (Ghosh, 2014).The gamma half-Cauchy model has introduced by (Alzaatreh *et al.*, 2016). Cordeiro *et al.* (2017) has developed the family of distribution using half-Cauchy distribution as generalized odd half-Cauchy family of distribution. Chaudhary and Kumar (2022) have developed three parameter half-Cauchy modified exponential distribution which has been derived by combing modified exponential distribution with half -Cauchy family of distribution. Chaudhary *et al.*, (2022) have also presented half-Cauchy extended exponential distribution which has been derived by compounding extended exponential distribution with half -Cauchy family of distribution.

Therefore, utilizing the half-Cauchy family of distribution, we are interested in creating new distributions. Zografos and Balakrishnan (2009) created the generating family of distribution, and its CDF can be derived as

$$F(x) = \int_0^{-\ln[1-G(x)]} r(t)\, dt, \quad (5)$$

Here, $G(x)$ stands for the CDF of any baseline distribution and $r(t)$ stands for the PDF of any distribution. Using $r(t)$ as the PDF of the half-Cauchy distribution defined in formula (4), the family of half-Cauchy distribution whose CDF may be defined as

$$\begin{aligned} F(x) &= \int_0^{-\ln[1-G(x)]} \frac{2}{\pi}\frac{\theta}{\theta^2+t^2}\, dt \\ &= \frac{2}{\pi}\arctan\left(-\frac{1}{\theta}\ln\left[1-G(x)\right]\right);\ x>0, \theta>0 \end{aligned} \quad (6)$$

The PDF corresponding to (6) can be expressed as

$$f(x) = \frac{2}{\pi\theta}\frac{g(x)}{1-G(x)}\left[1+\left\{-\frac{1}{\theta}\log\left[1-G(x)\right]\right\}^2\right]^{-1} \quad (7)$$

The remaining section of this article is arranged as follows. We define half-Cauchy Chen distribution and also, we derive the statistical and mathematical properties of the suggested model such as revised hazard rate function, hazard function, survival function, probability density function, cumulative distribution function, cumulative hazard function, quantiles, the measures of skewness based on quartiles and kurtosis based on octiles. The estimation of the parameters of the proposed model is carried out using the three widely used estimation techniques namely least-square (LSE), maximum likelihood estimators (MLE) and Cramer-Von-Mises (CVM) methods. The application of the proposed model is presented using two real –life data sets. Finally, some explanatory remarks are made.

Half-Cauchy Chen (HCC) distribution

In this section we have introduced a new distribution called half-Cauchy Chen (HCC) distribution having three parameters (α, λ, θ) which is the extension of Chen distribution. If a positive random variable X follows HCC distribution, then its CDF can be obtained by substituting equation (1) in (6) as

$$F(x) = \frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x^\beta})\right\};\ x>0,\ \beta,\lambda,\theta>0 \quad (8)$$

The PDF of HCC (α, λ, θ) corresponding to (8) can be written as

$$f(x) = \frac{2}{\pi}\beta\lambda\theta x^{\beta-1}e^{x^\beta}\left[\theta^2+\left\{-\lambda(1-e^{x^\beta})\right\}^2\right]^{-1} \quad (9)$$

Reliability function

The reliability function of HCC (α, λ, θ) distribution is

$$\begin{aligned} R(x) &= 1-F(x) \\ &= 1-\frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x^\beta})\right\};\ x>0,\ \beta,\lambda,\theta>0 \end{aligned} \quad (10)$$

Hazard rate function (HRF)

The HRF of HCC (α, λ, θ) can be obtained as

$$\begin{aligned} h(x) &= \frac{f(x)}{R(x)} \\ &= 2\beta\lambda\theta x^{\beta-1}e^{x^\beta}\left[\pi-2\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x^\beta})\right\}\right]^{-1} A \\ &\text{Where } A = \left[\theta^2+\left\{-\lambda(1-e^{x^\beta})\right\}^2\right]^{-1} \end{aligned} \quad (11)$$

The reverse hazard function (RHR)

The reverse failure rate function is,

$$\text{RHR} = \frac{f(x)}{F(x)} = 2\beta\lambda\theta x^{\beta-1}e^{x^\beta}\left[\frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x^\beta})\right\}\right]^{-1} T, \quad \text{Where } T = \left[\theta^2+\left\{-\lambda(1-e^{x^\beta})\right\}^2\right]^{-1} \quad (12)$$

Cumulative hazard function (CHF)

The Cumulative hazard function of the proposed model is defined as

$$\begin{aligned} H(x) &= \int_{-\infty}^{x} h(y)dy \\ &= -\log\left[1-F(x)\right] \\ &= -\log\left[1-\frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x^\beta})\right\}\right] \end{aligned} \quad (13)$$

Quantile function (QF) of the HCC distribution

The QF can be obtained by taking the inverse function of (8) as

$$Q(u) = F^{-1}(u)$$

Hence QF is obtained as,

$$Q(u) = \left[\ln\left[1 + \frac{\theta}{\lambda}\tan\left\{\frac{\pi u}{2}\right\}\right]\right]^{1/\beta} ;0 < u < 1 \qquad (14)$$

where u stands for the uniform random variable of U (0,1).

For the HCC distribution, the random deviates can be produced using (14) as

$$x = \left[\ln\left[1 + \frac{\theta}{\lambda}\tan\left\{\frac{\pi v}{2}\right\}\right]\right]^{1/\beta} ;0 < v < 1.$$

The median of HCC distribution can be calculated using relation

$$median = \left[\ln\left\{1 + \frac{\theta}{\lambda}\right\}\right]^{1/\beta}.$$

Skewness of HCC distribution

The Bowley's coefficient of skewness based on quantiles can be obtained as

$$S = \frac{Q(0.75) - 2Q(0.5) + Q(0.25)}{Q(3/4) - Q(1/4)}.$$

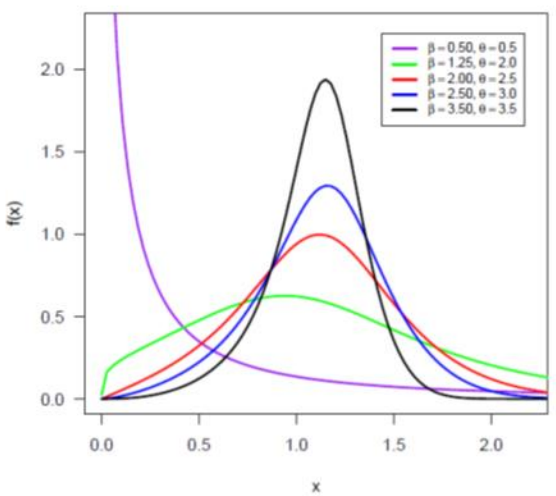
Kurtosis of HCC distribution

Moors (1988) defined the coefficient of kurtosis using octiles as

$$K_u(M) = \frac{Q(0.875) + Q(0.375) - Q(0.625) - Q(0.125)}{Q(3/4) - Q(1/4)}$$

For various parameter values of the HCC distribution, we have presented the PDF and HRF graphs in figure 1 keeping lambda=1 as constant.

The shapes of PDF can have decreasing, right skewed or symmetrical and HRF is increasing, decreasing or uni-modal hazard rate.



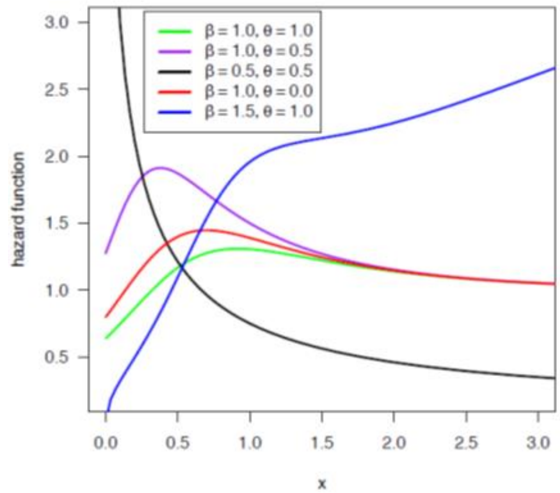


Figure 1. The PDF's graph (upper section) and HRF's (lower section) for various values of β and θ keeping λ=1 as constant.

Parameter estimation

Maximum Likelihood Estimation (MLE)

Here, the MLE method is applied to evaluate the ML estimators (MLE's) of the HCC distribution. Let a random sample $\underline{x} = (x_1, \ldots, x_n)$ of size 'n' be drawn from $HCC(\beta, \lambda, \theta)$ then the log likelihood function can be written as,

$$\ell(\beta,\lambda,\theta \mid \underline{x}) = n\ln(2/\pi) + n\ln(\beta\lambda\theta) + (\beta-1)\sum_{i=1}^{n}\ln x_i + T_1 \qquad (15)$$

Where $T_1 = \sum_{i=1}^{n} x_i^{\beta} - \sum_{i=1}^{n}\ln\left\{\theta^2 + \left\{-\lambda(1-e^{x_i^{\beta}})\right\}^2\right\}$

After differentiating (15) with respect to β, λ and θ, we get

$$\frac{\partial\ell}{\partial\lambda} = \frac{n}{\lambda} - 2\lambda\sum_{i=1}^{n}\left[(1-e^{x_i^{\beta}})\left[\theta^2 + \left\{-\lambda(1-e^{x_i^{\beta}})\right\}^2\right]^{-1}\right]$$

$$\frac{\partial\ell}{\partial\theta} = \frac{n}{\theta} - 2\theta\sum_{i=1}^{n}\left[\theta^2 + \left\{-\lambda(1-e^{x_i^{\beta}})\right\}^2\right]^{-1}$$

Equating $\frac{\partial\ell}{\partial\beta} = \frac{\partial\ell}{\partial\lambda} = \frac{\partial\ell}{\partial\theta} = 0$ and solving simultaneously for the β, λ and θ we obtain the corresponding ML estimators of the $HCC(\beta,\lambda,\theta)$ model. But normally, it is not possible to solve non-linear equations above so one can solve them easily with the aid of suitable computer software. If $\Theta = (\beta,\lambda,\theta)$ represent the parameter vector of $HCC(\beta,\lambda,\theta)$ and the corresponding MLE of Θ as $\underline{\hat{\Theta}} = (\hat{\beta},\hat{\lambda},\hat{\theta})$, then the asymptotic normality results in, $(\underline{\hat{\Theta}} - \underline{\Theta}) \rightarrow N_3\left[0,\left(I(\underline{\Theta})\right)^{-1}\right]$ where $I(\underline{\Theta})$ stands for the Fisher's information matrix which is defined by,

$$I\left(\underline{\Theta}\right)=-\begin{pmatrix} E\left(\frac{\partial^2 l}{\partial\beta^2}\right) & E\left(\frac{\partial^2 l}{\partial\beta\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\beta\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\beta\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) \\ E\left(\frac{\partial^2 l}{\partial\beta\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\theta}\right) & E\left(\frac{\partial^2 l}{\partial\theta^2}\right) \end{pmatrix}$$

The MLE's asymptotic variance $\left(I\left(\underline{\Theta}\right)\right)^{-1}$ is meaningless since we don't know *Θ*. As a result, we use the estimated parameter values to try and approximate the asymptotic variance. An estimation of the information matrix $I\left(\underline{\Theta}\right)$ provided by the observed fisher information matrix $O(\underline{\Theta})$ is employed. The observed fisher information matrix $O(\underline{\Theta})$ is given by

$$O(\underline{\Theta})=-\begin{pmatrix} \frac{\partial^2 l}{\partial\hat{\beta}^2} & \frac{\partial^2 l}{\partial\hat{\beta}\partial\hat{\lambda}} & \frac{\partial^2 l}{\partial\hat{\beta}\partial\hat{\theta}} \\ \frac{\partial^2 l}{\partial\hat{\beta}\partial\hat{\lambda}} & \frac{\partial^2 l}{\partial\hat{\lambda}^2} & \frac{\partial^2 l}{\partial\hat{\theta}\partial\hat{\lambda}} \\ \frac{\partial^2 l}{\partial\hat{\beta}\partial\hat{\theta}} & \frac{\partial^2 l}{\partial\hat{\theta}\partial\hat{\lambda}} & \frac{\partial^2 l}{\partial\hat{\theta}^2} \end{pmatrix}_{\left(\hat{\beta},\hat{\lambda},\hat{\theta}\right)} = -H\left(\underline{\Theta}\right)\Big|_{\left(\underline{\Theta}=\hat{\underline{\Theta}}\right)} \quad (16)$$

where H stands for the Hessian matrix.

The observed information matrix is generated by the Newton-Raphson method with the goal of maximizing likelihood. Consequently, the variance-covariance matrix is represented by

$$\left[-H\left(\underline{\Theta}\right)\Big|_{\left(\underline{\Theta}=\hat{\underline{\Theta}}\right)}\right]^{-1}=\begin{pmatrix} \text{var}(\hat{\beta}) & \text{cov}(\hat{\beta},\hat{\lambda}) & \text{cov}(\hat{\beta},\hat{\theta}) \\ \text{cov}(\hat{\beta},\hat{\lambda}) & \text{var}(\hat{\lambda}) & \text{cov}(\hat{\lambda},\hat{\theta}) \\ \text{cov}(\hat{\beta},\hat{\theta}) & \text{cov}(\hat{\lambda},\hat{\theta}) & \text{var}(\hat{\theta}) \end{pmatrix} \quad (17)$$

Hence, approximate *100(1-α) %* confidence intervals for estimating β, λ and θ of $HCC(\beta,\lambda,\theta)$ from the asymptotic normality of MLEs can be constructed as,

$\hat{\alpha}\pm Z_{\alpha/2}\sqrt{\text{var}(\hat{\alpha})}$, $\hat{\lambda}\pm Z_{\alpha/2}\sqrt{\text{var}(\hat{\lambda})}$ and

$\hat{\theta}\pm Z_{\alpha/2}\sqrt{\text{var}(\hat{\theta})}$.

where $Z_{\alpha/2}$ stands for the upper percentile of standard normal variate.

Method of Least-Square Estimation (LSE)

In order to estimate the unknown parameters β, λ and θ of the HCC distribution, we also used the least-square estimation approach, which can be derived by minimizing (18) with respect to the unknown parameters β, λ and θ.

$$B\left(X;\beta,\lambda,\theta\right)=\sum_{i=1}^{n}\left[F(X_i)-\frac{i}{n+1}\right]^2 \quad (18)$$

Assume that a random sample $\{X_1, X_2, \ldots, X_n\}$ is drawn size n from a distribution function F, where $F(X_i)$ denote the CDF of the ordered random variables $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$, then least-square estimators of β, λ and θ, denoted respectively as $\hat{\beta},\hat{\lambda}$ *and* $\hat{\theta}$, can be produced by minimizing (19) with respect β, λ and θ.

$$B\left(X;\beta,\lambda,\theta\right)=\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x_i^{\beta}})\right\}-\frac{i}{n+1}\right]^2 \quad (19)$$

Differentiating (19) with respect to λ ,β and θ, we get

$$\frac{\partial B}{\partial\beta}=\frac{4\lambda}{\pi\theta}\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{U\left(x_i\right)\right\}-\frac{i}{n+1}\right]e^{x_i^{\beta}}x_i^{\beta}\log(x_i)\left[1+\left\{U\left(x_i\right)\right\}^2\right]^{-1}$$

$$\frac{\partial B}{\partial\lambda}=\frac{-4}{\pi\theta}\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{U\left(x_i\right)\right\}-\frac{i}{n+1}\right](1-e^{x_i^{\beta}})\left[1+\left\{U\left(x_i\right)\right\}^2\right]^{-1}$$

$$\frac{\partial B}{\partial\theta}=\frac{4\lambda}{\pi\theta^2}\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{U\left(x_i\right)\right\}-\frac{i}{n+1}\right](1-e^{x_i^{\beta}})\left[1+\left\{U\left(x_i\right)\right\}^2\right]^{-1}$$

where $U\left(x_i\right)=-\frac{\lambda}{\theta}(1-e^{x_i^{\beta}})$

The weighted least square estimators are calculated similarly by minimizing (20) with respect to β, θ and λ.

$$B\left(X;\beta,\lambda,\theta\right)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right] \quad (20)$$

Here, $w_i = weights = \frac{1}{Var(X_{(i)})}=\frac{\left(n+1\right)^2\left(n+2\right)}{i\left(n-i+1\right)}$

As a result, by minimizing (21) with regard to β, λ and θ respectively, we can obtain the weighted least square estimators of β, λ and θ.

$$B\left(X;\beta,\lambda,\theta\right)=\sum_{i=1}^{n}\frac{\left(n+1\right)^2\left(n+2\right)}{i\left(n-i+1\right)}\left[\frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x_i^{\beta}})\right\}-\frac{i}{n+1}\right]^2 \quad (21)$$

Method of Cramer-Von-Mises estimation (CVME)

By minimizing the function (22) with respect to β, λ and θ, the Cramer-Von-Mises estimators for each of these parameters are respectively derived.

$$J\left(X;\beta,\lambda,\theta\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}\mid\beta,\lambda,\theta\right)-\frac{2i-1}{2n}\right]^2$$

$$=\frac{1}{12n}+\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{-\frac{\lambda}{\theta}(1-e^{x_i^{\beta}})\right\}-\frac{2i-1}{2n}\right]^2 \quad (22)$$

Differentiating (22) with respect to β, λ and θ we get,

$$\frac{\partial J}{\partial\beta}=\frac{4\lambda}{\pi\theta}\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{U\left(x_i\right)\right\}-\frac{2i-1}{2n}\right]e^{x_i^{\beta}}x_i^{\beta}\log(x_i)\left[1+\left\{U\left(x_i\right)\right\}^2\right]^{-1}$$

$$\frac{\partial J}{\partial\lambda}=\frac{-4}{\pi\theta}\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{U\left(x_i\right)\right\}-\frac{2i-1}{2n}\right](1-e^{x_i^{\beta}})\left[1+\left\{U\left(x_i\right)\right\}^2\right]^{-1}$$

$$\frac{\partial J}{\partial \theta}=\frac{4\lambda}{\pi\theta^2}\sum_{i=1}^{n}\left[\frac{2}{\pi}\arctan\left\{U\left(x_i\right)\right\}-\frac{2i-1}{2n}\right](1-e^{x_i^{\beta}})\left[1+\left\{U\left(x_i\right)\right\}^2\right]^{-1}$$

where $U\left(x_i\right)=-\frac{\lambda}{\theta}(1-e^{x_i^{\beta}})$

We will get the CVM estimators after solving non-linear equations $\frac{\partial J}{\partial \beta}=0, \frac{\partial J}{\partial \lambda}=0$ *and* $\frac{\partial J}{\partial \theta}=0$ at the same time.

Applications to real dataset

In this part, we have used two real datasets from previous works to show the applicability and suitability of the HCC distribution. We have taken into account the four distributions namely Generalized exponential distribution (Gupta & Kundu, 1999), Chen distribution (Chen, 2000), Weibull extension (WE) distribution (Tang *et al.*, 2003) and Gompertz distribution (Murthy *et al.*, 2003) to compare the potential of the suggested model.

Data set I: BP data

The third data set that represent the fibre and bundle strength in hybrid composites (Bader & Priest, 1982).

1.312, 1.314, 1.479, 1.552, 1.700, 1.803, 1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.14, 2.179, 2.224, 2.240, 2.253, 2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426,2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.57, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726, 2.770, 2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.88, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585.

By maximizing the likelihood function (15), we have estimated the MLEs of the HCC distribution using the R software's optim() function (R Core Team, 2020). The log likelihood value that we have determined is l = -50.1663. For β, λ and θ, we have displayed the MLEs and associated standard errors (SE) in Table 1.

Table 1. MLE and SE for β, λ and θ

Parameter	MLE	SE
beta	1.6394	0.07402
lambda	0.0287	0.09691
theta	2.2594	7.58837

The profile log-likelihood function for the three parameters β, λ and θ is depicted graphically in figure 2. It is possible to compute the ML estimates uniquely, as we have found.

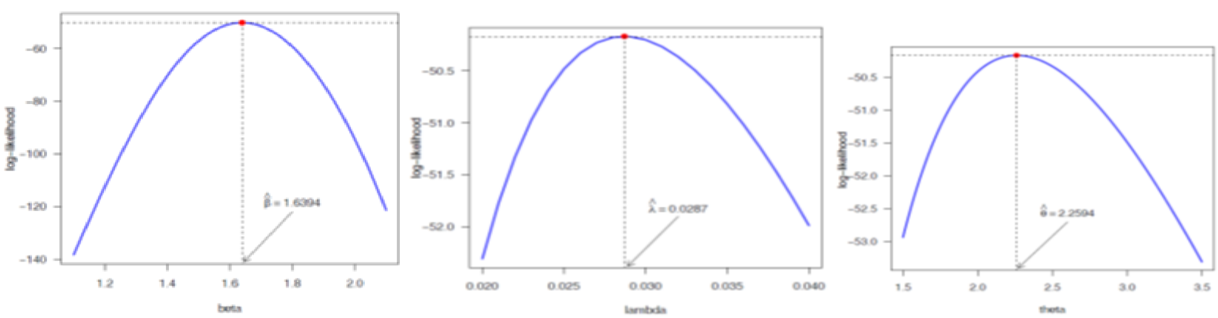


Figure 2. Profile log-likelihood function of the parameters β, λ and θ.

The HCC distribution successfully fits the data, as shown by figure 3's graphs for the P-P plot and Q-Q plot.

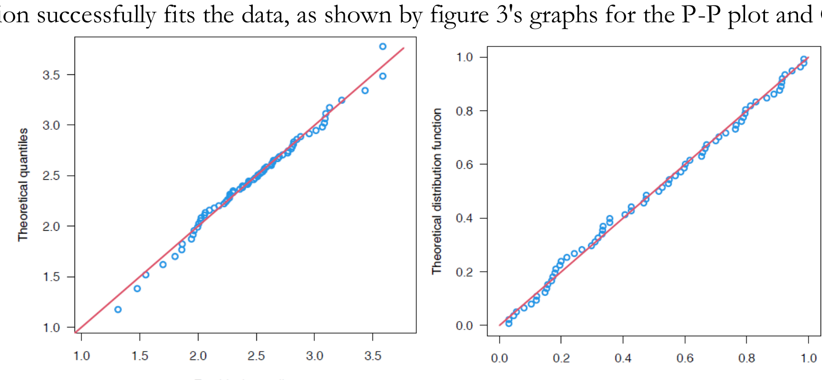


Figure 3. The P-P plot (left panel) and Q-Q plot (right panel) of the HCC distribution.

In Table 2, we have shown the HCC distribution's estimated parameters values along with their accompanying negative log-likelihoods and AIC criteria. We have used the MLE, LSE and CVE methods to arrive at these results.

Table 2. Estimated parameters values, AIC and log-likelihood

Estimation methods	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$	LL	AIC
MLE	1.6394	0.0287	2.2594	-50.1663	106.3325
LSE	1.5904	0.0286	1.8603	-50.3817	106.7633
CVE	1.6060	0.0319	2.2001	-50.2668	106.5335

The A^2, KS, and W statistics, as well as the related p-value, are shown in Table 3 for the MLE, LSE, and CVE estimations.

Table 3. The KS, A^2, and W statistics with p-value

Estimation methods	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
MLE	0.0489(0.9965)	0.0220(0.9951)	0.1855(0.9939)
LSE	0.0449(0.9991)	0.0187(0.9982)	0.1990(0.9907)
CVE	0.0425(0.9996)	0.0178(0.9987)	0.1800(0.9950)

In figure 4, we have shown the graphs of the Q-Q plot, the histogram, and the density function of fitted distributions of the HCC model using estimation methods LSE, MLE, and CVM. It turns out that the HCC model is good fitted to the BP data set.

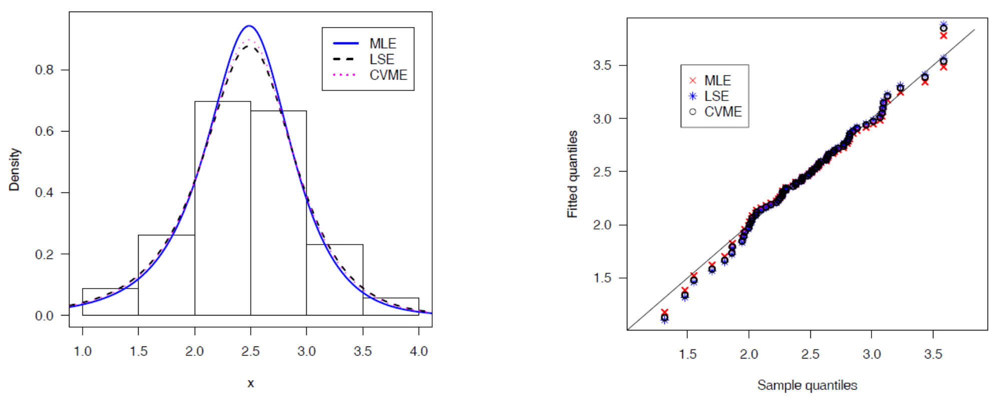


Figure 4. The fitted distributions' histogram and density function (left panel) and Q-Q plot (right panel)of the HCC distribution LSE, MLE, and CVM estimation methods

The Hannan-Quinn information criterion (HQIC), Bayesian information criterion (BIC), Corrected Akaike information criterion (CAIC), Akaike information criterion (AIC), and all of which are shown in Table 4, have been demonstrated for the assessment of the applicability and adaptability of the HCC distribution.

Table 4 Log-likelihood (LL), AIC, BIC, CAIC and HQIC

Model	LL	AIC	BIC	CAIC	HQIC
HCC	-50.1663	106.3325	113.0349	106.7018	108.9916
WE	-50.7239	107.4479	114.1502	107.8171	110.1069
GZ	-53.6249	111.2497	115.7179	111.4315	113.0224
GE	-54.6205	113.2409	117.7091	113.4227	115.0136
Chen	-55.0534	114.1069	118.5751	114.2887	115.8796

Figure 5 shows the graph of the HCC distribution's goodness-of-fit as well as a few other chosen distributions.

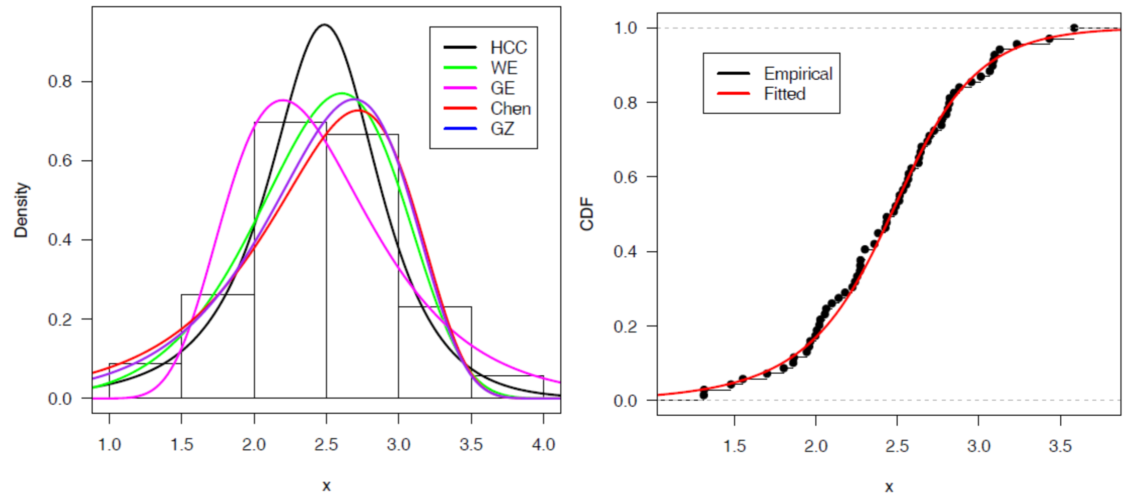


Figure 5. The empirical distribution function with estimated distribution function (right panel) and HCC distribution's histogram, density function for fitted distributions (left panel).

We have also shown the Cramer-Von Mises (CVM), Anderson-Darling (AD) and Kolmogorov-Simnorov (KS) statistics values in Table 5 to evaluate the HCC distribution's goodness-of-fit with other rival models. With a lower test statistic value and a higher p-value than the other distributions used as comparisons, we may infer that the HCC distribution has a good fit to the BP data set and consistently produces more reliable results.

Table 5. Statistics for goodness-of-fit and the accompanying p-value

Model	*KS(p-value)*	*AD(p-value)*	*CVM(p-value)*
HCC	0.0489(0.9965)	0.0220(0.9951)	0.1855(0.9939)
WE	0.0647(0.9348)	0.0568(0.8357)	0.4431(0.8046)
GZ	0.0847(0.7048)	0.1268(0.4696)	0.9280(0.3970)
GE	0.0949(0.5629)	0.1603(0.3603)	1.1235(0.2983)
Chen	0.0956(0.5537)	0.164(0.3503)	1.2152(0.2617)

Data set II: LG data

59 conductors underwent an accelerated life test, and the results are shown below (Nelson & Doganaksoy, 1995). Microcircuit failures can be caused by electro-migration, which is the movement of atoms within the conductors of the circuit. The failure times are given in hours, and the observations are uncensored.

9.289, 6.545, 6.956, 7.543, 5.459, 6.492, 4.706, 8.120, 2.997, 8.687, 6.129, 8.591, 5.381, 11.038, 4.288, 6.958, 4.137, 6.522, 7.495, 7.459, 6.538, 6.573, 6.087, 5.589, 6.725, 5.807, 8.532, 6.369, 9.663, 7.024, 9.218, 8.336, 7.945, 6.869, 4.700, 6.352, 9.254, 6.948, 7.489, 5.009, 6.033, 7.398, 7.496, 10.092, 7.974, 4.531, 7.683, 8.799, 7.365, 7.224, 5.640, 6.923, 5.434, 7.937, 6.476, 6.515, 6.071, 5.923, 10.491.

The log likelihood value that we have determined is $l = -111.7319$. For β, λ and θ, we have shown the MLEs in Table 6 along with their standard errors (SE).

Table 6. MLE and SE for β, λ and θ

Parameter	MLE	SE
beta	0.9753	0.04004
lambda	0.0398	0.02643
theta	29.0272	11.11081

In figure 6, we have plotted below the graphs of the profile log-likelihood function for β, λ and θ in figure 6 and it is obtained that the ML estimates have unique values.

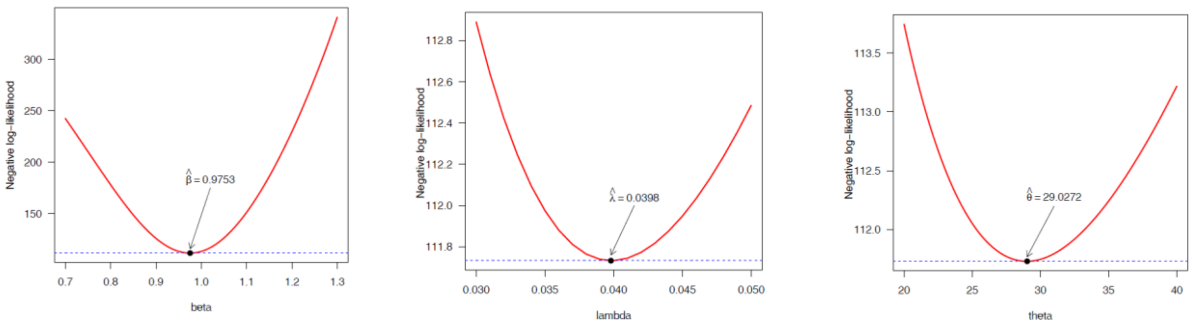


Figure 6. Profile log-likelihood function of β, λ and θ

The graphs of the P-P plot and Q-Q plot are shown in Figure 7.We have found that the data is well fitted by the HCC model.

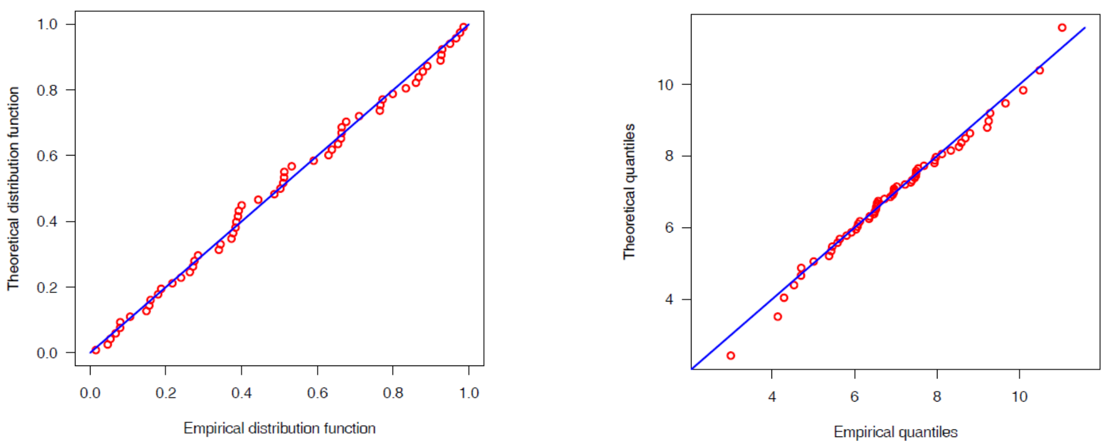


Figure 7. The Q-Q plot (right panel) and P-P plot (left panel) of the HCC model

The estimated values of the HCC model's parameters, together with their related negative log-likelihoods and the AIC criterion, are shown in Table 7 using the MLE, LSE, and CVE methods.

Table 7. Estimated parameters, log-likelihood, and AIC

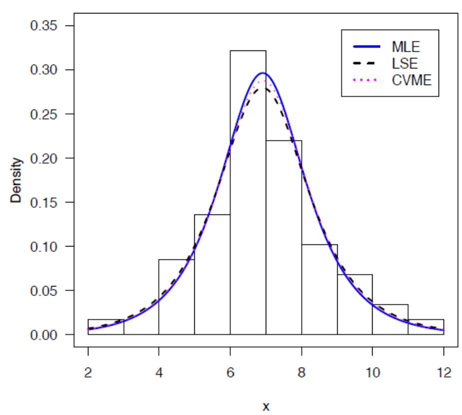
Estimation methods	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$	LL	AIC
MLE	0.9753	0.0398	29.0272	-111.7319	229.4638
LSE	0.9556	0.0090	5.1356	-111.8516	229.7032
CVE	0.9654	0.0079	5.1424	-111.7644	229.5288

Table 8 shows the KS, W, and A^2 statistics along with the related p-value for the MLE, LSE, and CVE estimates.

Table 8. The A^2, KS and W statistics with a p-value

Estimation methods	*KS(p-value)*	*W(p-value)*	*A^2(p-value)*
MLE	0.0577(0.9830)	0.0238(0.9924)	0.1705(0.9965)
LSE	0.0527(0.9938)	0.0238(0.9924)	0.1695(0.9966)
CVE	0.0584(0.9808)	0.0231(0.9936)	0.1583(0.9979)

We have displayed the graphs of the Histogram and the density function of fitted distributions of HCC distribution and Q-Q plot using LSE, MLE and CVM estimation methods in figure 8 and it is found that the LG data set is good fitted by the HCC distribution.



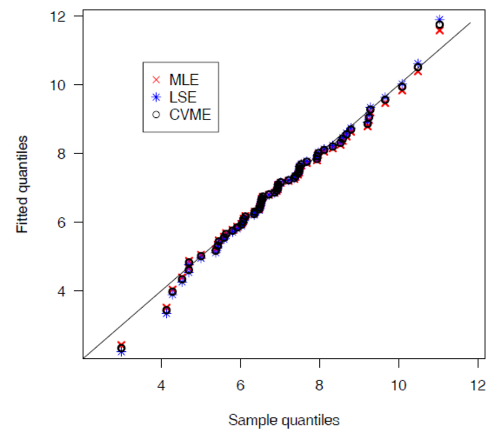


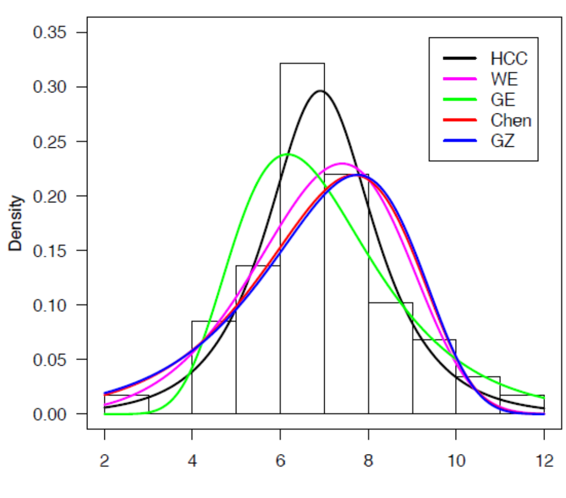
Figure 8. The fitted distributions' Histogram and the density function (left panel) of MLE, LSE and CVM estimation methods of HCC distribution and Q-Q plot (right panel).

We have provided examples of AIC, BIC, CAIC, and HQIC in Table 9 for the appraisal of the applicability and suitability of the HCC model.

Table 9. AIC, Log-likelihood (LL), BIC, HQIC and CAIC

Model	LL	AIC	BIC	CAIC	HQIC
HCC	-111.7319	229.4638	235.6964	229.9002	231.8968
WE	-113.6745	233.3491	239.5817	233.7855	235.7821
GE	-114.9473	233.8946	238.0497	234.1089	235.5166
Chen	-116.3874	236.7748	240.9299	236.9891	238.3968
GZ	-117.1740	238.3480	242.5031	238.5623	239.9700

In figure 9, we have displayed the graphs of goodness-of-fit of HCC model and some chosen competing models.



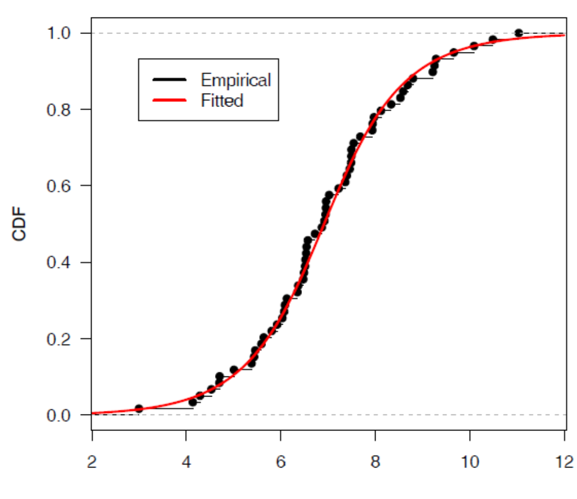


Figure 9. The fitted distributions' histogram and the density function (left panel) and Empirical distribution function with estimated distribution function (right panel) of HCC distribution.

The KS-value, AD-value, and CVM statistic and their related p-values are also shown in Table 10 to allow comparisons of the HCC model's goodness-of-fit with those of rival models. It can be concluded that the HCC distribution has a much better fit to the data and more consistent and trustworthy findings from other distributions used as a comparison because it has the highest p-value and lowest test statistic value.

Table 10. The goodness-of-fit statistics and their corresponding p-value

Model	*KS(p-value)*	*AD(p-value)*	*CVM(p-value)*
HCC	0.0577(0.9830)	0.0238(0.9924)	0.1705(0.9965)
WE	0.1067(0.4796)	0.1154(0.5160)	0.6800(0.5751)
GE	0.1042(0.5103)	0.1173(0.5079)	0.7368(0.5282)
Chen	0.1238(0.3006)	0.1913(0.2855)	1.1741(0.2774)
GZ	0.1306(0.2464)	0.2160(0.2387)	1.3143(0.2277)

CONCLUSIONS

In this article, we have presented three parameter half-Cauchy Chen distribution which has been derived by combing a continuous Chen distribution with the half - Cauchy family of distribution .A thorough analysis of some of the new distribution's statistical properties, including the precise formulations for its quantile function, skewness, kurtosis, hazard rate function, survival function, cumulative hazard function and reversed hazard rate function, has been offered. Three popular estimation techniques namely, MLE, CVME, and LSE are employed to estimate the parameters of the suggested model for two real data sets. We have discovered that MLEs perform relatively better than CVM and LSE techniques. The PDF of the suggested model's curves have demonstrated that it is versatile for modeling real-life data and may take on a variety of shapes, including increasing-decreasing and right-skewed. According to the values of the model parameters, the hazard function graph is also reverse j-shaped, constant, or monotonically increasing. Two real-life datasets are used to assess the applicability and adaptability of the suggested model, and the results showed that it is significantly more flexible than some other fitted distributions.

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AUTHOR CONTRIBUTIONS

AKC: formulated the concept, conducted the research, manuscript writing; RSY: data analysis, manuscript writing; VK: developed the distribution, manuscript writing.

CONFLICT OF INTEREST

There is no conflict of interest between the authors in this publication.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author, upon reasonable request.

REFERENCES

Alzaatreh, A., Mansoor, M., Tahir, M.H., Zubair, M., & Ghazali, S.A. (2016). The gamma half-Cauchy distribution: Properties and applications. *Hacettepe Journal of Mathematics and Statistics*, *45*(4), 1143-1159.

Bader, M.G., & Priest, A.M. (1982). Statistical aspects of fibre and bundle strength in hybrid composites. In Hayashi, T., Kawata, S., & Umekawa, S. (Eds.) *Progress in science and engineering of composites* (pp. 1129-1136), ICCM-IV, Tokyo.

Bhatti, F.A., Hamedani, G.G., Najibi, S.M., & Ahmad, M. (2019). On the extended Chen distribution: development, properties, characterizations and applications. *Annals of Data Science*, *8*, 159-180.

Chaudhary, A.K., & Kumar, V. (2022). Half Cauchy-modified exponential distribution: properties and applications. *Nepal Journal of Mathematical Sciences*, *3*(1), 47-58.

Chaudhary, A.K., Sapkota, L.P., & Kumar, V. (2022). Some properties and applications of Half Cauchy extended exponential distribution. *International Journal of Statistics and Applied Mathematics, 7*(4), 226-235.

Chen, Z. (2000). A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function. *Statistics & Probability Letters, 49*(2), 155–161.

Cordeiro, G.M., & Lemonte, A.J. (2011). The beta-half-Cauchy distribution. *Journal of Probability and Statistics*, 904705. https://doi.org/10.1155/2011/904705.

Cordeiro, G.M., Alizadeh, M., Ramires, T.G., & Ortega, E.M. (2017). The generalized odd half-Cauchy family of distributions: properties and applications.

Communications in Statistics-Theory and Methods, *46*(11), 5685-5705.

Ghosh, I. (2014). The Kumaraswamy-half-Cauchy distribution: properties and applications. *Journal of Statistical Theory and Applications*, *13*(2), 122-134.

Gupta, R.D., & Kundu, D. (1999). Generalized exponential distributions. *Australian and New Zealand Journal of Statistics, 41*(2), 173-188.

Jacob, E., & Jayakumar, K. (2012). On half-Cauchy distribution and process. *International Journal of Statistika and Mathematika, 3*(2), 77 - 81.

Joshi, R.K., & Kumar, V. (2020). Lindley-Chen distribution with applications. *International Journals of Engineering, Science & Mathematics*, *9*(10), 12-22.

Joshi, R.K., & Kumar, V. (2021a). Logistic Chen distribution with properties and applications, *International Journal of Mathematics Trends and Technology, 67*(1), 141-151.

Joshi, R.K., & Kumar, V. (2021b). Poisson Chen distribution: properties and application. *International Journal of Latest Trends in Engineering and Technology*, *18*(4), 001-012.

Kumar, V., & Ligges, U. (2011). reliaR : A package for some probability distributions. http://cran.r-project.org/web/packages/reliaR/index.html.

Kundu, D., & Raqab, M.Z. (2005). Generalized Rayleigh distribution: different methods of estimation, *Computational Statistics and Data Analysis*, *49*, 187-200.

Moors, J.J.A. (1988). A quantile alternative for kurtosis. *Journal of the Royal Statistical Society: Series D (The Statistician)*, *37*(1), 25-32.

Murthy, D.N.P., Xie, M., & Jiang, R. (2003). *Weibull models*, Wiley, New York.

Nelson, W., & Doganaksoy, N. (1995). Statistical analysis of life or strength data from specimens of various sizes using the power-(log) normal model. InBalakrishnan, N. (Ed.) *Recent advances in life-testing and reliability* (pp. 377-408), Taylor & Francis.

Paradis, E., Baillie, S.R, & Sutherland, W.J. (2002). Modeling large-scale dispersal distances. *Ecological Modelling, 151*, 279–292.

Polson, N.G., & Scott, J.G. (2012). On the half-Cauchy prior for a global scale parameter. *Bayesian Analysis*, *7*(4), 887-902.

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. https://www.R-project.org/.

Shaw, M.W. (1995). Simulation of population expansion and spatial pattern when individual dispersal distributions do not decline exponentially with distance. *Proceedings of the Royal Society B: Biological Sciences*, *259*, 243–248.

Srivastava, A.K., & Kumar, V. (2011). Markov Chain Monte Carlo methods for Bayesian inference of the Chen model. *International Journal of Computer Information Systems*, *2*(2), 07-14.

Tang, Y., Xie, M., & Goh, T.N. (2003). Statistical analysis of a Weibull extension model. *Communications in Statistics-Theory and Methods*, *32*(5), 913-928.

Tarvirdizade, B., & Ahmadpour, M. (2019). A new extension of Chen distribution with applications to lifetime data. *Communications in Mathematics and Statistics*, *9*, 23-38.

Zografos, K., & Balakrishnan, N. (2009). On families of beta-and generalized gamma-generated distributions and associated inference. *Statistical Methodology*, *6*(4), 344-362.