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Research Article

KUMARASWAMY UNIFORM DISTRIBUTION: MODEL, PROPERTIES, AND APPLICATIONS

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ABSTRACT

Three parameters Kumaraswamy Uniform distribution has been derived from the Kumaraswamy family of distribution with uniform distribution, where θ is scale parameter, and $a \text{ and } b$ are the shape parameters. The proposed model is unimodal and negatively skewed, whereas the hazard rate function is bathtub and inverted bathtub shaped. The statistical properties like as, the reliability/survival function, the hazard rate function, the quantile function, the median, and the mode have been derived from the proposed model. The parameters are obtained from the maximum loglikelihood function which is equivalent to the maximum likelihood function. Using real-data analysis, the proposed model is unimodal, and the negatively skewed distribution is well-fitted distribution observed by the P-P plot, estimated CDF with empirical distribution, and KS test value. Finally, the proposed model is compared to various competitive models available in the literature, and the results revealed that the proposed model performs better than other models in terms of finding the least value of AIC, BIC, CIAC, and HQIC. Hence, the proposed model is an alternative model of lifetime data.

Keywords: Kumaraswamy uniform distribution, maximum likelihood estimation, parameters, quantile function, total time on test (TTT) plot

INTRODUCTION

Recently, some efforts have been made to define new families of distributions in order to extend well-known models while providing great flexibility when modelling data in practice. A variety of techniques have been used to add extra parameters to an existing distribution, resulting in the formation of a new family of distributions or probability distributions. The distribution that Kumaraswamy (1980) first proposed is quite flexible, but it has not been explored much in the literature but, Jones (2009) introduced a new family of distribution is Kumaraswamy- G family having a cumulative distribution function (c.d.f.) with a simple form $F(x,\Theta) = 1 - [1-(G(x,\phi))^a]^b$ where a>0 and b>0 are the two shape parameters to introduce skewness and to vary the tail weights. The KW-G distribution can be used very successfully even with censored data due to its tractable distribution function. The corresponding density function is defined by $f(x,\Theta) = abg(x,\phi)(G(x,\phi))^{a-1}[1-(G(x,\phi))^a]^{b-1}$;$0 < x < 1$ and $\Theta = (a,b,\phi^T)^T$ is the parameter space of the family, which can be unimodal, and increasing, decreasing or constant, depending on the parameter values. Jones (2009) advocated the KW distribution as a generator since its quantile function takes a simple form. In his paper, mentioned several advantages over beta distribution: such as the simple normalizing constant, normal explicit formula for the distribution and quantile functions. It does not involve any distinct functions for quantile function and random variate generation.

The existing one- or two-parameter models have been modified to create new classes of models. The distribution becomes richer and more adaptable for modelling data when one or more parameters are added. There are various approaches to adding a parameter or parameters to a distribution. The distribution that results from such parameter additions is richer and more adaptable for modelling data. So, the KW-G distribution is acquired by incorporating two shape parameters a and b to the G distribution. It includes distributions with bathtub-shaped hazard rate functions, and unimodal. Likewise, Marshall-Olkin extended inverted Kumaraswamy (MOEIK) distribution is discovered after the three parametric distributions with parameters $\alpha > 0, \beta > 0, \lambda > 0$. This generalization includes some well-known sub-models, including the Lomax, Beta type II, and the log-logistic (Fisk) distribution (Usman & ul-Haq, 2018). Similarly, Almalki et al. (2021) developed a new distribution known as the partially constant-stress accelerated life tests (PALTs) model from the Kumaraswamy distribution with adaptive Type-II progressive censoring. To solve the problem of statistical inference, based on censored data has been used in this model. The Boots trap, MLEs, Bayes estimates of the unknown parameters and the acceleration factor are used to estimate the population parameters. A new distribution with two shape parameters α>0, β>0 developed by El-Sayed & Ahmed (2014) known as the Kumaraswamy –Kumaraswamy (KW-KW) distribution, a special model from the class of Kumaraswamy Generalized (KW-G) distributions. Furthermore, Lemonte et al. (2013) introduced the exponentiated Kumaraswamy distribution by generalization of the Kumaraswamy distribution with three parameters. For better fit than some known models which are available in the literature, the authors used lifetime data quite effectively in the analysis. They also proposed a related distribution, referred to as the log-exponentiated Kumaraswamy (log-EK) distribution and, which extends the generalized exponential and double generalized exponential distributions. The inverted Kumaraswamy distribution is derived by Hameed et al. (2020). The estimation of stress strength (S-S) reliability for two shape parameters α and β using this distribution. Eldin et al. (2014) conducted research on parameter estimation for the Kumaraswamy distribution using general progressive type II censoring. Alduais et al. (2022) proposed Bayesian estimators of the Kumaraswamy distribution (KD) to estimate the parameters using type-II censoring data. Bayesian estimation approaches have been used to examine the effectiveness of Bayesian estimators for the shape parameter of the KD.

Moreover, for better suitability, as compared to competitive distributions a new probability distribution named Transmuted Inverted Kumaraswamy (TIK) distribution based on a quadratic rank transmutation map has been proposed by Sherwani et al. (2021). It is an extension of the inverted Kumaraswamy distribution. Ahmed (2020) introduced a distribution, the so-called alpha power Kumaraswamy (AK) distribution by applying alpha power transformation (APT) to the Kumaraswamy distribution. Similarly, new distribution known as type II half logistic Kumaraswamy (TIIHLKw) distribution, a simple and more flexible with a unit interval has been proposed by including an extra parameter in the existing model to improve its ability to fit complex data sets (Zein Eldin et al., 2020). A newly generated class (new G class) of models, that is the extended generalized inverted Kumaraswamy generated (EGIKw-G) family of distributions has been derived. Furthermore, another special model, the extended generalized inverted Kumaraswamy Burr XII (EGIKw-Burr XII) model with a four-parameter was also generated. The EGIKw-Frechet, EGIKw-Burr XII, EGIKw-Uniform, and EGIKw-Normal distributions have been explained as sub models of the proposed class (Ramzan et al., 2022).

Likewise, the other distribution, a new Exponentiated Odd Lomax Exponential (EOLE) to four-parameter by making the distribution of Lomax distribution as a generator with an exponentiated odd function (Dhungana & Kumar, 2022). The three parameters, half logistic inverted Weibull distribution is developed by type I half logistic-G family with inverted Weibull distribution (Dhungana & Kumar, 2022). Furthermore, a new Modified Half Logistic Weibull (MHLW) distribution is developed in the type-I half logistic-G family of distributions (Dhungana & Kumar, 2022). Authors combined the Rayleigh distribution with exponentiated -G Poisson family formed exponentiated Rayleigh Poisson distribution (Joshi & Dhungana, 2020). Another new Poisson Inverted Exponential distribution having two parameters has developed from the Poisson family of distribution (Dhungana, 2020), and Tharu et al. (2021) proposed the new univariate continuous Exponentiated Marshall –Olkin Exponential distribution by compounding exponential distribution with Marshall-Olkin family of distribution. The motivation for developing this model lies in its applicability to enhance reliability testing in industrial data. It aids in the measurement of risk tolerance, prediction, and forecasting of complex data modeling in the future. Hence, the aim of the study is to develop the sophisticated model Kumaraswamy Uniform "KwU" distribution which will be applied in different areas including engineering, medicine, environmental science, biology, demography, etc. in data modeling.

MATERIALS AND METHODS

Jones (2009) introduced a new family of distribution known as the Kumaraswamy- G family having a cumulative distribution function (c.d.f.) which has a simple form as

$$F(x,\Theta) = 1 - [1 - (G(x,\phi))^a]^b \tag{1}$$

where b >0 and a>0.The corresponding density function is given by

$$f(x,\Theta) = abg(x,\phi)(G(x,\phi))^{a-1}[1-(G(x,\phi))^a]^{b-1}; 0 < x < 1 \tag{2}$$

The uniform probability distribution is a continuous probability distribution that deals with events that are equally likely to occur. When solving problems with a uniform distribution, keep in mind whether the data is inclusive or exclusive of endpoints. The CDF of Uniform distribution is defined as,

$$G(x) = \frac{x}{\theta} \tag{3}$$

The corresponding probability density function (pdf) of uniform distribution is

$$g(x) = \frac{1}{\theta}, 0 < x < \theta \tag{4}$$

Now, the equation (3) and (4) are used in equation (2), we have to explore the new Kumaraswamy Uniform "KwU" distribution having the three parameters, the corresponding cdf of the proposed model is

$$F(x,\Theta) = 1 - \left[1 - \left(\frac{x}{\theta}\right)^a\right]^b \tag{5}$$

Having the corresponding PDF of the proposed model is

$$f(x,\theta)=\frac{ab}{\theta}\left(\frac{x}{\theta}\right)^{a-1}\left[1-\left(\frac{x}{\theta}\right)^{a}\right]^{b-1};a>0;b>0;0<x<\theta<1 \qquad (6)$$

The proposed model of pdf curve explorer has different characteristics such as unimodal, shifted to low peak as the value of each parameter is increased but there is no change in flatness. The cumulative density function curve has provided a good approximation of cdf nature.

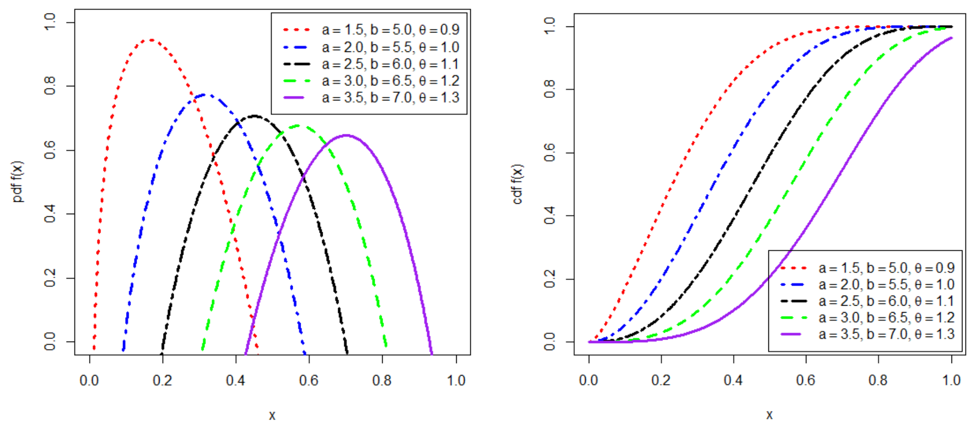

Figure 1. Plot of probability density function (left panel) and cumulative density function (right panel)

Similarly, the reliability function and hazard rate function of the proposed model is defined as

$$R(x)=1-F(x,\Theta)=\left[1-\left(\frac{x}{\theta}\right)^{a}\right]^{b} \qquad (7)$$

And hazard function is the defined as

$$h(x)=\frac{f(x)}{1-F(x)}=\frac{ab}{\theta}\left(\frac{x}{\theta}\right)^{a-1}\left[1-\left(\frac{x}{\theta}\right)^{a}\right]^{-1} \qquad (8)$$

The proposed model of survival curve explores the different characteristics as well as its hazard rate function. The hazard rate function is inverted bath tube shaped which exhibit the good characteristics of proposed distribution because the inverted bathtub shape of hazard rate function is a special characteristic.

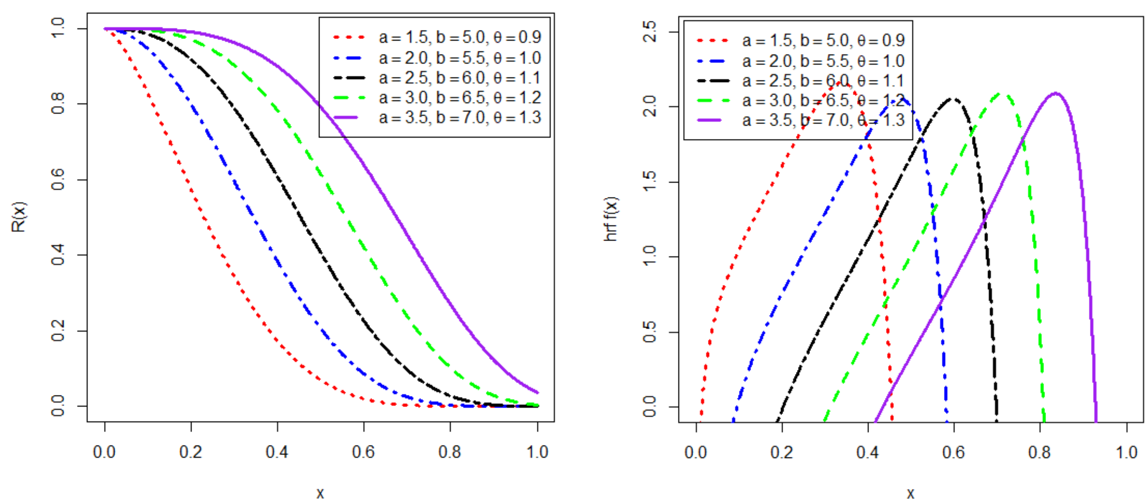

Figure 2. Plot of survival function (left panel) and hazard rate function (right panel)

Statistical Properties

Some characteristics of the KuU distribution have been derived in this section. To derive distribution from the generalized binomial and exponential series. For, $|\varsigma| < 1, n > 0$; we have,

$(1+\varsigma)^{-n} = \sum_{i=0}^{\infty}(-1)^i \binom{n+i-1}{i} \varsigma^i$; and $(1-\varsigma)^n = \sum_{j=0}^{\infty}(-1)^j \binom{n}{j} \varsigma^j$;

The PDF of proposed distribution (6) derived by using the generalized binomial series as,

$$f(x) = \frac{ab}{\theta}\left(\frac{x}{\theta}\right)^{a-1} \sum_{j=0}^{\infty}(-1)^j \binom{b-1}{j} \left(\frac{x}{\theta}\right)^{aj}; \qquad (9)$$

Similarly, the CDF of proposed distribution (5) derived by using the generalized binomial series as,

$$F(x) = 1 - \sum_{j=0}^{\infty}(-1)^j \binom{b}{j} \left(\frac{x}{\theta}\right)^{aj} \qquad (10)$$

Quantile Function and Median

In theoretical aspects of probability theory, quantile functions are used. It is an alternative to CDF and PDF is used to get statistical measure like as median, kurtosis, and skewness. The quantile function defined as $Q(u) = F^{-1}(x)$. Consequently, the corresponding quantile function of proposed distribution becomes:

$$Q(u) = \theta\left[1-(1-u)^{\frac{1}{b}}\right]^{\frac{1}{a}};0<u<1. \qquad (11)$$

Where, $u \sim U(0,1)$. In particular, the median is derived by placing $u = \frac{1}{2}$ in equation (11), then we obtain:

$$Median = \theta\left[1-\left(\frac{1}{2}\right)^{\frac{1}{b}}\right]^{\frac{1}{a}}$$

Mode

The mode is maximum recurring value of proposed distribution. To calculate the mode, we have to differentiate with respect to x in equation (6) or which is equivalent to log of equation (6), which is:

$$ln\, f(x,\theta) = ln\left(\frac{ab}{\theta}\right) + (a-1)\, ln\left(\frac{x}{\theta}\right) + (b-1)\, ln\left[1-\left(\frac{x}{\theta}\right)^a\right] \qquad (12)$$

The equation (12) is differentiated with respect to x and apply the condition $f(x,\theta) \neq 0$ and $f'(x,\theta) = 0$, the mode of proposed distribution is

$$(a-1)\left(\frac{1}{x}\right) - (b-1)\left(\frac{a}{x}\right)\left(\frac{x}{\theta}\right)\left[1-\left(\frac{x}{\theta}\right)^a\right]^{-1} = 0$$

The above equation is a nonlinear equation which is solved by analytical methods.

Maximum Likelihood Estimation

We have to estimate unknown parameters of the proposed model using maximum likelihood estimation. Let, $x_1, x_2, \ldots, x_n$ are random sample drawn from KwU distribution with parameters (a, b, and θ), then likelihood function of proposed distribution is product of n^{th} time of sample of proposed distribution. Mathematically, $\ell(x;\zeta) = \prod_{i=1}^{n} f(x_i;\zeta)$ where, ζ is the parameter space belong to (a, b, and θ). The likelihood function is equivalent to the log likelihood function. Therefore, log likelihood function of proposed distribution becomes:

$$\ell(x;\zeta) = ln\left(\frac{ab}{\theta}\right) + (a-1)\, ln \sum_{l=1}^{n}\left(\frac{x_i}{\theta}\right) + (b-1)\, ln \sum_{i=1}^{n}\left[1-\left(\frac{x_i}{\theta}\right)^a\right] \qquad (13)$$

The parameters are obtained by differentiating (13) partially with respect to (a, b, and θ). We have

$$\frac{\partial\ell(x;\zeta)}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n} ln\left(\frac{x_i}{\theta}\right) - (b-1)\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^a ln\left(\frac{x_i}{\theta}\right)\left[1-\left(\frac{x_i}{\theta}\right)^a\right]^{-1} \qquad (14)$$

$$\frac{\partial\ell(x;\zeta)}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n} ln\left(1-\left(\frac{x_i}{\theta}\right)^a\right) \qquad (15)$$

$$\frac{\partial\ell(x;\zeta)}{\partial \theta} = -\frac{an}{\theta} + \frac{a(b-1)}{\theta}\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^a\left[1-\left(\frac{x_i}{\theta}\right)^a\right]^{-1} \qquad (16)$$

Finally, solve non-linear equations $\frac{\partial\ell(x;\zeta)}{\partial a} = 0$, $\frac{\partial\ell(x;\zeta)}{\partial b} = 0, \frac{\partial\ell(x;\zeta)}{\partial \theta} = 0$ and estimate ($\hat{a}$, $\hat{b}$,and $\hat{\theta}$) for parameters (a, b, and θ). Additionally, the asymptotic normality of MLEs, and approximate $100(1-\gamma)\%$ confidence intervals of (a, b, and θ) can be formed as: $\hat{a} \pm z_{\gamma/2}SE(\hat{a})$, $\hat{b} \pm z_{\gamma/2}SE(\hat{b})$,and $\hat{\theta} \pm z_{\gamma/2}SE(\hat{\theta})$ and; $z_{\gamma/2}$is the upper percentile of standard normal variate.

DATA ANALYSIS

Data analysis is a technique for arriving at a sound conclusion based on facts or information. We used real data analysis to determine the proposed model's suitability for the given data set. The data set has been used in several studies, including Dasgupta (2011), the distribution of burr with applications and Bakouch (2020), family of extended half-distributions. The data set contains 50 observations on Burr (in millimeters) with hole diameters of 12 mm and sheet thicknesses of 3.15 mm.

0.24, 0.06, 0.06, 0.14 ,0.22, 0.08, 0.04, 0.26, 0.02, 0.14 ,0.08, 0.26, 0.04, 0.14, 0.12, 0.16, 0.16, 0.04, 0.02, 0.16, 0.28, 0.12, 0.26, 0.32, 0.18, 0.14, 0.24, 0.24, 0.18, 0.22, 0.18, 0.32, 0.24, 0.12, 0.24, 0.18, 0.16, 0.14, 0.08, 0.16, 0.22, 0.32, 0.22 ,0.06, 0.16, 0.08, 0.14, 0.12,0.24, 0.16

The summary of the finding of the given data set is follows:

Table 1. Descriptive statistic of proposed data set.

Sample(n)	Min.	Q_1	Median	Mean	Q_3	Max.	Skewness	Kurtosis	Standard deviation
50	0.020	0.120	0.160	0.163	0.235	0.320	0.07233	2.216649	0.08105

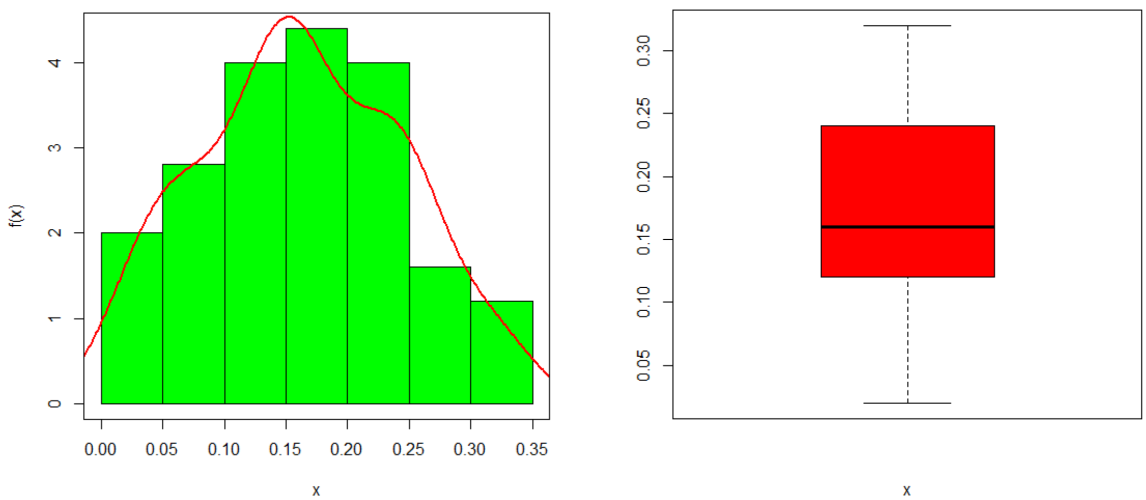

Figure 3. Histogram and density plot (left panel) and boxplot (right panel).

Similarly, we have to present the graphical representation of data set as follows. The data set has the symmetrical with there is no outliers. The kernel density plot is provided of the symmetrical pattern in all over the data set.

We have estimated the value of parameters with standard error by using the method of maximum likelihood estimation which maximizes the log-likelihood function (13) directly using *maxlik ()* function in "BFGS" method from R software (R core team, 2022). Likewise, the estimated parameters with standard error (SE) are presented in table (Table 2).

Table 2. Estimated value of MLE with SE of given data set.

Parameters	MLE	SE	t-value	p-value
$\hat{a}$	1.53821	0.33721	4.562	<0.001
$\hat{b}$	1.72213	0.80541	2.138	0.0325
$\hat{\theta}$	0.33657	0.02616	12.863	<0.001

where the variance covariance matrix as follows

$$\begin{array}{cc} & \begin{array}{ccc} a & b & \theta \end{array} \\ \begin{array}{c} a \\ b \\ \theta \end{array} & \begin{pmatrix} 0.11371 & 0.22826 & 0.00597 \\ 0.22826 & 0.64867 & 0.01896 \\ 0.00597 & 0.01896 & 0.00068 \end{pmatrix} \end{array}$$

The total time on test (TTT) plot is used to determine the behavior of the HRF in the data. We know the data has a constant HRF when we get a diagonal line. A concave TTT plot shows that the data's HRF is increasing, while a convex TTT plot shows the data's HRF is decreasing. The increasing HRF of plotted data indicates that the proposed distribution is appropriate for modeling. Similarly, the empirical distribution is closed with the theoretical distribution. We have tested the goodness of fit by Kolmogorov-Smirnov test (D = 0.078852, p-value = 0.7798). The finding revel that proposed distribution is valid by presenting the empirical cumulative distribution function versus theoretical cumulative distribution function (Fig. 4).

We have to plot the P-P plot of proposed models; it provides the good fit of proposed distribution. Likewise, after estimation of the parameter value of proposed distribution, the predicated hazard rate function of proposed model is bathtub shaped which indicates the model has satisfied every characteristic of data handling (Fig. 5).

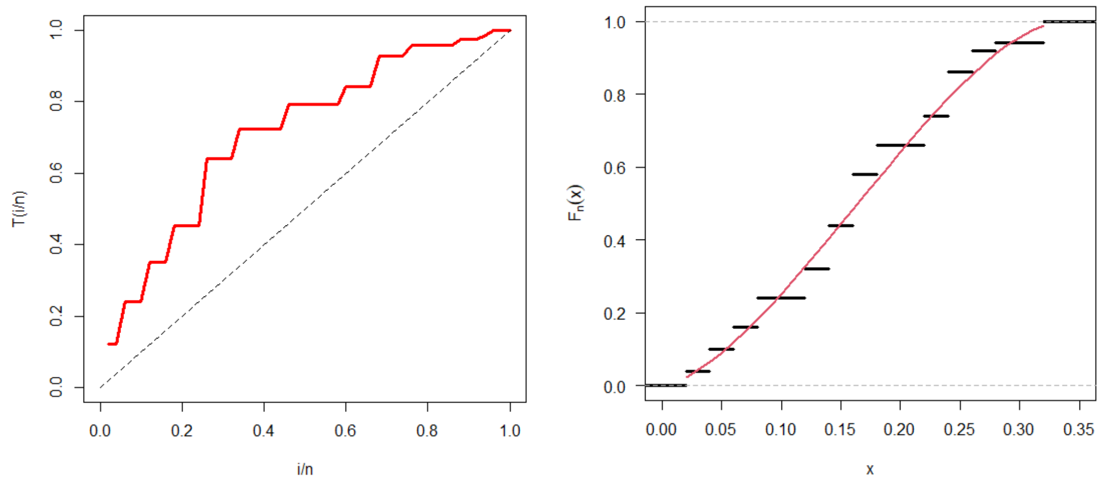

Figure 4. TTT plot (left panel) and empirical distribution verses theoretical distribution (right panel)

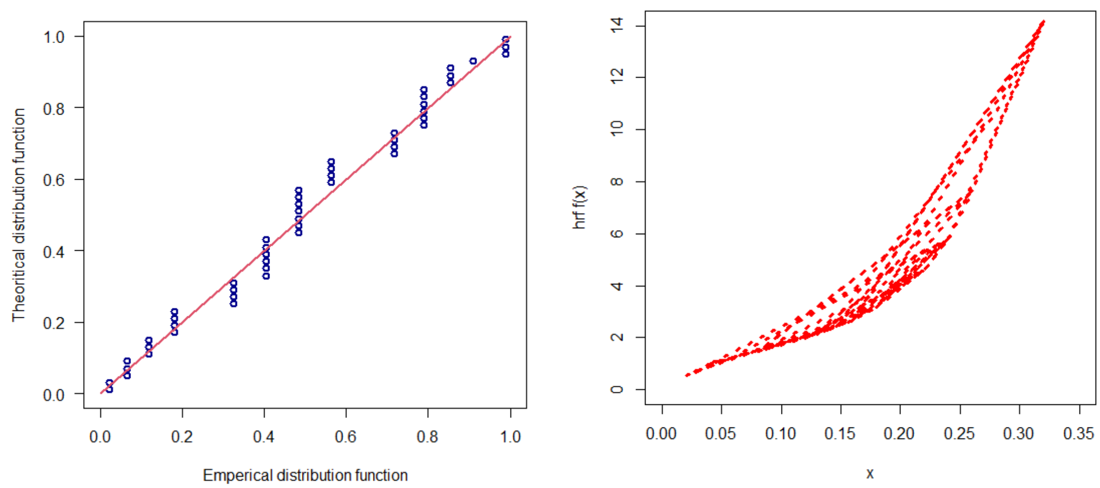

Figure 5. P-P plot (left panel) and fitted hazard rate function (right panel)

Descriptive statistics of proposed Model

After estimating the parameters, we have to compute the descriptive characteristic of the proposed distribution, which reveals that it is negatively skewed and leptokurtic distribution. The finding shows that mean<median<mode (Table 3).

Table 3. Descriptive statistics of proposed model

Mean	Median	Mode	Standard deviation	Skewness	Kurtosis
1.161	1.267	1.386	0.324	-2.022	6.354

DISCUSSION

We have to compare the proposed model with the following competitive models which are available in literature. A new extension of the exponentiated Weibull model formed Lomax exponentiated Weibull distribution having three parameters (Ansari & Nofal, 2021).

$$f_{LEW}(x) = \frac{\alpha\beta\vartheta x^{\beta-1}e^{-x^{\beta}}\left(1-e^{-x^{\beta}}\right)^{\alpha-1}}{\left[1-\left(1-e^{-x^{\beta}}\right)^{\alpha}\right]^{2}} \left\{1+\frac{\left(1-e^{-x^{\beta}}\right)^{\alpha}}{1-\left(1-e^{-x^{\beta}}\right)^{\alpha}}\right\}^{-(\vartheta+1)} \quad ; x \geq 0, \alpha \geq 0, \beta \geq 0, \vartheta \geq 0$$

The PDF of the LxEW distribution has important shapes, such as right skewed, symmetric, left skewed, and bimodal. Similarly, the hazard rates of the LxEW distribution is constant bathtub, increasing, decreasing, unimodal. It demonstrates that LxEW distribution is empirically important in the modeling of lifetime data. Likewise, Badr (2019) proposed the compound Rayleigh model. Initially, Wu and Kus (2009) introduced the compound Rayleigh distribution (CRD) based on a new life test plan termed as a progressive first failure-censored plan. This was accomplished by employing the conjugate prior for the shape parameter, and the discrete prior for the scale parameter. The symmetric and asymmetric Bayes estimators have been acquired in closed forms of CR distribution having the pdf

$$f_{CR}(x) = 2\alpha\beta^{\alpha}x(\beta + x^2)^{-(\alpha+1)}; x > 0, \alpha > 0, \beta > 0.$$

Furthermore, Exponentiated Rayleigh Poisson distribution (ERP) derived by Joshi and Dhungana (2020) which is developed from the Exponentiated-G Poisson family of distribution with the Rayleigh distribution. The hazard function shows the upside curve (concave) shape of this distribution. The flexibility and significance of the new distribution formed which is illustrated by various field for modeling having the pdf:

$$f(x) = \frac{\alpha\beta\lambda^2 x \ e^{-(\lambda x)^2}\left[1 - e^{-(\lambda x)^2}\right]^{\alpha-1} e^{-\beta\left[1-e^{-(\lambda x)^2}\right]^{\alpha}}}{1 - e^{-\beta}}; x > 0, \alpha > 0, \beta > 0, \lambda > 0$$

Finally, unimodal and increasing hazard function distribution called Exponentiated Chen distribution derived by Dey et al. (2017). The proposed model has the good qualities of the method such as asymptotic efficiency, normality, consistency, and invariance. Hence, EC distribution is recommended for all practical purposes having the pdf

$$f(x) = \alpha\beta\lambda x^{\beta-1} e^{x^{\beta}} e^{\lambda\left(1-e^{x^{\beta}}\right)}\left[1 - e^{\lambda\left(1-e^{x^{\beta}}\right)}\right]^{\alpha-1}; x > 0, \alpha > 0, \beta > 0, \lambda > 0$$

Firstly, we have estimated parameters of proposed model and competitive models. Each models' parameters are estimated by maximum likelihood estimation technique by maximizing the log-likelihood function by using R (R core team, 2022) and Henningsen et al. (2011). The estimated value of each parameter is presented in the following table (Table 4).

Table 4. Estimated value of competitive models, along with proposed model.

Models	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{a}$	$\hat{b}$	$\hat{\theta}$
KwU	-	-	-	1.53821 (0.33721)	1.72213 (0.80541)	0.33657 (0.02616)
LEW	0.0040 (-)	2.4802 (-)			-	1.6220 (-)
CR	41.262 (-)	1.336 (-)		-	-	-
ERP	4.8385 (1.4303)	3.6490 (0.944)	11.8152 (0.977)	-	-	-
EC	0.5347 (0.0994)	3.1131 (0.1404)	98.8044 (8.8110)	-	-	-

Now, we have compared the proposed model with all other competitive models by various goodness of fit criteria's like; (i) Akaike's information criterion, (ii) Value of log likelihood, (iii) Hannan-Quinn information criterion, (iv) Corrected Akaike's information criterion, and (v) Bayesian information criterion. In comparison to all other competitive models, the estimated values of the log-likelihood function $-\ell(\hat{\underset{\sim}{\zeta}})$, BIC, AIC, HQIC, and CIAC of proposed model are least. Hence, the suggested model is significantly superior to other competitive models (Table 5).

Table 5. Goodness of fit value of competitive models and proposed models.

Models	$-\ell(\hat{\underset{\sim}{\zeta}})$	AIC	BIC	CIAC	HQIC
KwU	-58.029	110.059	185.787	110.5589	124.458
LEW	-211.08	416.168	644.952	416.6684	430.568
CR	-154.92	305.845	321.544	306.1448	315.444
ERP	-67.464	128.920	214.092	129.4200	143.327
EC	-57.110	108.219	183.029	108.7193	122.619

CONCLUSIONS

This study is based on a proposed new distribution having three parameters a, b and, θ called Kumaraswamy Uniform distribution. The proposed distribution derived from the Kumaraswamy family of distribution with uniform distribution, whereas θ is scale parameters and a and b, are the shape parameter. We have derived some important properties like cumulative probability density function, probability density function, reliability function, hazard rate function, quintile and median, mode. The parameters are estimated by loglikelihood function which has been used MLE technique. We have concluded that the proposed distribution is negatively skewed distribution unimodal and inverted bathtub, and bathtub hazard rate function model. After analyzing the sample data, we concluded that the proposed model provides an admirably better fit than some other well-known models. Therefore, the proposed distribution can be applied as an alternative model life-testing model.

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AUTHOR CONTRIBUTIONS

RP Tharu played a role in conceptualizing the research, developing the methodology and contributed to writing the manuscript. GP Dhungana also developed the concept and utilized computer applications for data analysis as well as contributed to writing the manuscript. RK Joshi was involved in editing the entire manuscript.

CONFLICT OF INTEREST

The authors declare no conflict of interests.

DATA AVAILABILITY STATEMENT

All the data relevant to the study are presented in the manuscript and can be provided upon request from the corresponding author.

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