ISSN: 2469-9062 (print), 2467-9240(e)

*Journal of Institute of Science and Technology, 2015, **20(1):** 51-58, © IOST, Tribhuvan University*

Analysis of Retrospective Time to First Pregnancy Data in Uttar Pradesh under Various Dimensions

Sonam Maheshwari[1], Brijesh P. Singh[2] and P. S. Pudir[3]

[1]*Department of Statistics, Banaras Hindu University, Varanasi, India*

[2]*Faculty of Commerce & DST-CIMS, Banaras Hindu University, Varanasi, India*

[3]*Department of Statistics, University of Allahabad, Allahabad, India*

E-mail:brijesh@bhu.ac.in

ABSTRACT

The present study deals with the estimation of the mean value of fecundability by fitting a theoretical distribution from the observed distribution of first conception of the women, who did not use any contraceptive method before their first conception. It is assumed that fecundability is fixed for a given couple, but across couples it varies according to a specified distribution. Under the classical approach, methods of moment and maximum likelihood are used while for Bayesian approach, empirical Bayes method used. A real data analysis from the third National Family Health Survey (NFHS-III, 2005-06) is analyzed as an application of model for various age at marriage groups of women. Finally, a simulation study is performed to access the performance of the several of methods used in this paper.

Keywords: Fecundability, Geometric distribution, Beta distribution, prior distribution, VGAM Package.

INTRODUCTION

The size and composition of population is highly related with the tempo and quantum of fertility rate and fertility is governed by the terms fecundity and fecundability. Fecundability and marital fertility are linked through the frequency of unprotected sexual intercourse, capability to conceive and exposure time (Bongaarts 1975). Fecundability can be interpreted as the transition probability for the shifting from the susceptible state to pregnancy (Perrin & Sheps 1964). In a homogeneous population, fecundability is equal to the reciprocal of its mean conception wait (Sheps & Menken 1973) but for heterogeneous populations, the mean fecundability is modeled with two parameters (Potter & Parker 1964). Thus the concept of fecundability is one of the principal determinates of fertility and in human reproductive behavior.

Gini (1924) first considered birth intervals as waiting time problems dependent on fecundability. In fact, there is enough evidence that couples vary in their fecundability. About 30% of sexually active couples achieve pregnancy in their first non contraception cycle, a smaller proportion of the remaining couples achieve pregnancy in the second, and with each additional unsuccessful cycle, the conception rate continues to decline, as the risk sets become further depleted of relatively fecund couples (Weinberg & Gladen 1986).

The fecundability varies from women to women thus one can assume it follow certain distribution lies in the parameter space [0, 1]. Beta distribution denoted as *Beta* (*a*, *b*), where *a* and *b* are the two shape parameters, is the most commonly used mixing distribution to model the random variable defined on the standard unit interval [0,1] due to its ability of accommodating wide range of shapes. Thus the beta-geometric (BG) distribution is considered as a very versatile distribution in modeling human fecundability data in literature originally proposed by Henry (1957).

The parameters of this mixed distribution have practical utility, with this distribution one can obtain the distribution of the fecundibility which is not possible to observe directly. According to Sheps (1964), fecundability affects fertility through its relationship with the average time required for a conception to occur and can also be considered as the transition probability for the passage from the susceptible state to pregnancy. In a homogeneous population, fecundability is equal to the reciprocal of its mean conception delay but for heterogeneous populations, the mean fecundability is usually modeled on two parameters (Jain 1969, Chowdhury & Dale 2012). Weinberg and Gladen (1986) considered that the decrease in conception probability over time is a sorting effect in a heterogeneous population, rather than a time effect. Paul (2005) develops tests of heterogeneity in the fecundability data through goodness of fit of the geometric model against the beta-geometric model along with a likelihood ratio statistic and a score test statistic. Islam *et al.* (2005) also made an attempt to compare the two methods of estimation of the mean value of fecundability. The aim of this paper is to estimate the mean fecundability for homogeneous as well as heterogeneous group of women by the methods of moments, maximum likelihood and Bayesian methods and also compute the variation of mean fecundability. A simulation study has been done to know the suitability of Bayesian procedure.

THE MODEL

Let X be the random variable denotes time required for first conception after marriage of the woman and follows a geometric distribution with parameter θ, which stays constant over time for a given couple and represent the fecundability. The distribution of X is as follows

$$P\left(X = x \mid \theta\right) = \theta\left(1-\theta\right)^{x}; 0 \le \theta \le 1; x = 0,1,2,3... \quad (1)$$

$E(x) = \frac{(1-\theta)}{\theta}$ and mean fecundability is $\theta = \frac{1}{E\left(x\right)+1}$.

This is known as the conditional distribution of conception delay. Now if θ varies among couple to couple according to beta distribution, and θ has the following density function

$$f(\theta) = \frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}; \alpha,\beta > 0$$

Where $B(\alpha,\beta) = \frac{\lceil\alpha+\beta}{\lceil\alpha\lceil\beta}$ is the beta function and (α,β) are two unknown non-negative parameters and the unconditional distribution of the conception delay X is given by

$$P(X = x) = \int_0^1 f(x,\theta)d\theta = \int_0^1 P(X = x \mid \theta) f(\theta)d\theta = \frac{B(\alpha+1, x+\beta)}{B(\alpha,\beta)} \quad (2)$$

This distribution is known as beta-geometric distribution.

ESTIMATION OF PARAMETER

In this section, we obtain the estimate of parameter of the heterogeneous model 2 by using methods of moment, method maximum likelihood and also an attempt has been made to obtain the estimate of the parameter of the model 1 under Bayesian paradigm considering heterogeneity.

METHOD OF MOMENT

The corresponding population moment of X about origin, conditional on θ, as given by the simple geometric distribution are

$$E\left(x\right) = \sum_{x=0}^{\infty} x\theta\left(1-\theta\right)^{x} = \frac{\left(1-\theta\right)}{\theta} \text{ and}$$

$$E\left(x^2\right) = \sum_{x=0}^{\infty} x^2\theta\left(1-\theta\right)^{x} = \frac{\left(1-\theta\right)}{\theta} + \frac{2\left(1-\theta\right)^2}{\theta^2}$$

To obtain the unconditional moment of X, we have to put the value of

$$E\left(\frac{(1-\theta)^r}{\theta^r}\right) = \int_0 \frac{(1-\theta)^r}{\theta^r} p(\theta)d\theta = \frac{\beta^{[r]}}{(\alpha-1)^{(r)}} \quad ;where \begin{cases} x^{[r]} = x(x+1)(x+2).....(x+r-1) \\ x^{(r)} = x(x-1)(x-2).....(x-r+1) \end{cases}$$

to get

$$\mu_1' = \frac{\beta}{\alpha-1} = m_1\,(say); \qquad \mu_2' = \frac{\beta(2\beta+\alpha)}{(\alpha-1)(\alpha-2)} = m_2\,(say)$$

Thus after solving we get

$$\hat{\alpha} = \frac{2\left(m_2 - m_1^2\right)}{m_2 - m_1 - 2m_1^2}; \qquad \hat{\beta} = m_1\left(\hat{\alpha}-1\right) \quad (3)$$

From equation (3), we can easily obtain the moment estimate of α and β, hence moment estimate of the mean fecundability θ can be obtained.

METHOD OF MAXIMUM LIKELIHOOD

Suppose that data are available on N individuals as $x_i; i = 1,2,......,n$. The likelihood function (Rao 1952) for data based on beta geometric distribution is given as

$$L = \prod_{i=1}^{n} \frac{B(\alpha+1, x_i+\beta)}{B(\alpha,\beta)} \quad (4)$$

and the corresponding log-likelihood $L\left(\Theta\right); \Theta = \left(\alpha,\beta\right)$ is given as

$$\log L = L\left(\Theta\right) = \sum_i \log B(\alpha+1, x_i+\beta) - n\log B(\alpha,\beta) \quad (5)$$

The score function $U\left(\Theta\right)$ is defined as the gradient of $L\left(\Theta\right)$, derived by taking the partial derivatives of $L\left(\Theta\right)$ with respect to α and β. The components of the score function $U(\Theta) = \left(U_\alpha(\Theta), U_\beta(\Theta)\right)^T$ are given below

$$U_\alpha(\Theta) = \frac{\partial L(\Theta)}{\partial\alpha} = N\psi(\alpha+1) + N\psi(\alpha+\beta) - \sum_i \psi(x_i+\alpha+\beta+1) - N\psi(\alpha) \quad (6)$$

$$U_\beta(\Theta) = \frac{\partial L(\Theta)}{\partial\beta} = \sum_i \psi(x_i+\beta) + N\psi(\alpha+\beta) - \sum_i \psi(x_i+\alpha+\beta+1) + N\psi(\beta) \quad (7)$$

The maximum likelihood estimates α and β can be obtained either by directly maximizing the above log likelihood function with respect to Θ or by solving the two simultaneous equations obtained by equating $U(\Theta)=0$. From equations (6) and (7) we can see that the MLEs of $\Theta=(\alpha,\beta)$ cannot be obtained in closed form. Therefore, we need some numerical iterative procedures such as Newton-Raphson method. One can also use the vglm function of R-Environment to obtain the MLE of $\Theta=(\alpha,\beta)$. Using the invariance property of MLEs, one can easily obtain the MLEs of fecundability parameter θ.

METHOD OF BAYES

In many practical situations, it is observed that the behavior of the parameters representing the various model characteristics cannot be treated as fixed constant throughout the life period. In the introductory section, we have already discussed that the fecundability parameter should not be assumed constant rather it is assumed to be random. Keeping this fact in mind, we have also conducted an empirical Bayesian study by assuming the following beta prior for fecundability parameter θ

$$h(\theta)=\frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1} \tag{8}$$

The hyper parameters α and β are assumed to be known real numbers. Based on the above prior assumption, the joint density function of the sample observations and θ becomes

$$L(\underset{\sim}{x},\theta)=\theta^{n}(1-\theta)^{\sum_i x_i}\frac{1}{B(\alpha,\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1} \tag{9}$$

Thus, the posterior density function of θ, given the data is given by

$$\pi(\theta|\underset{\sim}{x})=\frac{L(\underset{\sim}{x}|\theta)h(\theta|\alpha,\beta)}{\int_0^1 L(\underset{\sim}{x}|\theta)h(\theta|\alpha,\beta)d\theta} \tag{10}$$

Putting the expression of equation (8) and (9) in equation (10), we get the posterior density of θ

$$\pi(\theta|\underset{\sim}{x})=\frac{1}{\beta\left(n+\alpha,\sum_i x_i+\beta-n\right)}\theta^{n+\alpha-1}(1-\theta)^{\sum_i x_i+\beta-n-1} \tag{11}$$

For the squared error loss, the Bayes estimator is the posterior mean and the mean fecundability is $\theta^*=\frac{n+\alpha}{\sum_i x_i+\beta-n}$ Without loss of generality, one can assume the value of α and β obtained by method of maximum likelihood.

APPLICATION OF THE MODEL

Here in this study data from National Family Health Survey-III, which is conducted in 2005-06 is used for the application of the theoretical distribution considered. In this Study, only women who are currently married in age group 15-49 are used. In order to estimate the fecundability for women, we have extracted 3767 women out of 12183 women who have had at least one recognizable conception (regardless of outcome). We have excluded women who were pregnant before marriage. Since our study is based on birth history data, we exclude those conceptions of women occurring more than 5 years preceding the survey to avoid memory lapse of the respondents. Finally, we have also excluded those women who did not conceive during their first 15 years or 180 months of marriage, because women who fail to conceive within 15 years of their marriage are considered to be the primarily sterile. So from the above data, we have the following:

$\sum X=78112$; n= 3767. Here, E[x]=20.73586 and hence the mean fecundability is 0.46. Now the estimated values of the parameters involved in the model obtained by using the different method of estimation are given as follows:

- Method of Moment:
  $\hat{\alpha}=20.4669,\ \hat{\beta}=403.663 \Rightarrow \hat{\theta}=0.04826$
- Method of Maximum Likelihood:
  $\hat{\alpha}=20.94735,\ \hat{\beta}=413.6093 \Rightarrow \hat{\theta}=0.04820$
- Method of Bayes: $\hat{\theta}=0.04602$

Estimated mean value of fecundability by Bayesian estimate is much closer to the true value, and hence, we can say that Bayes procedure is best for the above data set followed by maximum likelihood estimate and method of moment estimate. When we are able to obtain the estimate of α and β for the model 2 we can get the estimate of mean fecundability and also obtain the distribution of fecundability which can't observe directly (the distribution of fecundability for various age marriages is shown in Fig. 1). The estimated values of the average fecundability for various group of women's age at marriage is also obtained by using the different methods of estimation to know the variation in mean fecundability over the various age at marriage. The following table shows that the mean fecundability is increasing with the increasing age at marriage. This means that for higher age at marriage women the duration of first birth interval is shorter.

Estimation method	Estimate of θ for various age at Marriage in years			
	Less than 16	16-18	18-20	20+
Moment	0.03960795	0.04290674	0.04460279	0.05175923
ML	0.04015898	0.04368629	0.04473479	0.05175477
Bayesian	0.03703094	0.04135400	0.04293600	0.04918200
Average month required for first conception	25.24	22.31	21.42	18.32

A SIMULATION STUDY

Here, we assess the performance of the method of moment estimate, maximum likelihood estimate and Bayes estimate of mean fecundability with respect to varying sample size *n*. The model parameters along with Biases and Mean Square Errors (MSEs) for various estimation methods have been used for comparison purpose. For each of the following options, we simulated six sets of data with samples of sizes 100, 200, 400, 500, 750 and 1,000 respectively, and based on each set of data we computed the above mentioned measures.

For fixed α and varying β	For fixed β and varying α
α=20; β=1.40$\Rightarrow$ θ=0.125	α=10.25; β=400$\Rightarrow$ θ=0.025
α=20; β=180$\Rightarrow$ θ=0.1	α=21.05; β=400$\Rightarrow$ θ=0.05
α=20; β=380$\Rightarrow$ θ=0.05	α=44.44; β=400$\Rightarrow$ θ=0.1
α=20; β=780$\Rightarrow$ θ=0.025	α=57.14; β=400$\Rightarrow$ θ=0.125

The above assessments are based on following algorithm have been done:

- Generate 5,000 samples of size *n* from beta-geometric distribution using VGAM package of R-environment.
- Compute the moment, maximum likelihood and Bayes estimate for the 5,000 samples.
- Compute the average estimates (AE), biases and mean-squared errors given by

$$Bias(\theta) = \frac{1}{5000}\sum_{i=1}^{5000}(\hat{\theta}_I - \theta) \text{ and}$$

$$MSE(\theta) = \frac{1}{5000}\sum_{i=1}^{5000}(\hat{\theta}_I - \theta)^2$$

- We repeat these steps for *n* =100, 200 . . . , 1000 with various values of θ hence computing AE, bias and *MSE* for *n* = 100, 200, . . . , 1000.

CONCLUSION

Study clearly indicates that the estimate of mean fecundability by various classical and non-classical methods is ranged from 0.046 to 0.048. One reason for this almost stable value of mean fecundability is attributed to the large value of sufficient statistics i.e. the sample size is 3767. Form the figures and tables, it is observed that for all the choices of the mean fecundability θ, the magnitude of the Bias and MSE decreases as the sample size *n* increases thereby leading to increased precision. We also note that the Bayesian analysis seems to fit better and are eager to see it applied in further as an alternative way. The biases are negative for the method of moment and maximum likelihood while it is positive for Bayesian method of estimation. Bayesian method provide better estimate of the mean fecundability particularly for smaller samples.

REFERENCES

Bongaarts, J. 1975. A method for the estimation of fecundability. *Demography* **12**: 210-227.

Gini, C. 1924. Premieres recherches sur la fecondabilite de la femme. Proceedings of the International Mathematics Congress, Toronto.

Henry, L. 1957. Fecondite et famille-models mathematiques. *Population* **12**: 413-444.

Islaam, S.M.S., Karim, M.A., Chowdhury, M.S. and Hossain, M.S. 2005. Fitting and estimation of type-I geometric distribution by using C-language. *Asian Journal of Information Technology* **4(1)**: 112-126.

Jain, A.K. 1969. Fecundability and its relation to age in a sample Taiwanese women. *Population Studies* **23**: 69-85.

Chowdhury M. R. and Dale U. 2012 Some Bayesian Analyses of Fecundability. *Pakisthan Journal of Statistics* **28(3)**: 293-302.

Paul, S.R. 2005. Testing goodness of fit of the geometric distribution: an application of the human fecundability data. *Journal of Modern Applied Statistical Methods* **4(2)**: 25-33.

Perrin, E.B. and Sheps, M.C. 1964. Human Reproduction: A Stochastic Process. *Biometric* **20**: 28-45.

Potter, R.G. and Parker, M.P. 1964. Predicting the time required to conceive. *Population Studies* **18**: 99-116.

Rao, C.R. 1952. Advanced statistical methods in biometric research, Newyork, Willey.

Sheps, M. C. 1964. On the time required for conception. *Population Studies* **18**: 85-97.

Sheps, M.C. and Menken, J.A. 1973. Mathematical models of conception and birth. University of Chicago Press, Chicago

Weinberg, P. & Gladen, B.C. 1986. The beta-geometric distribution applied to comparative fecundability studies. *Biometrics* **42**: 547-560.

Table 1. Average Estimate of θ with their Bias and MSE for α=20; β=140; θ=0.125 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.09812	0.10271	0.11234	-0.0821	0.0802	0.0794	0.0021	0.0020	0.0019
200	0.09874	0.10459	0.11832	-0.0801	-0.0792	0.0762	0.0013	0.0013	0.0012
300	0.09901	0.11329	0.11932	-0.0789	-0.0760	0.0687	0.0010	0.0009	0.0008
400	0.09987	0.11855	0.12091	-0.0761	-0.0695	0.0619	0.0007	0.0006	0.0006
500	0.10173	0.11936	0.12191	-0.0692	-0.0631	0.0586	0.0006	0.0005	0.0005
750	0.10632	0.12133	0.12421	-0.0621	0.0592	0.0522	0.0003	0.0002	0.0002
1000	0.11262	0.12452	0.12492	-0.0574	-0.0463	0.0460	0.0002	0.0001	0.0001

Table 2. Average Estimate of θ with their Bias and MSE for α=20; β=180; θ=0.1 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.09467	0.09529	0.09634	-0.0976	0.0954	0.0936	0.0021	0.0020	0.0019
200	0.09506	0.09695	0.09703	-0.0912	-0.0901	0.0882	0.0011	0.0010	0.0009
300	0.09583	0.09782	0.09792	-0.0865	-0.0832	0.0791	0.0009	0.0008	0.0007
400	0.09631	0.09788	0.09826	-0.0743	-0.0721	0.0692	0.0006	0.0005	0.0005
500	0.09671	0.09818	0.09913	-0.0612	-0.0593	0.0549	0.0004	0.0004	0.0003
750	0.09691	0.09845	0.09993	-0.0578	0.0533	0.0525	0.0004	0.0003	0.0002
1000	0.09712	0.09912	0.10028	-0.0432	-0.0392	0.0388	0.0002	0.0002	0.0001

Table 3. Average Estimate of θ with their Bias and MSE for α=20; β=380; θ=0.05 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.04641	0.04711	0.04759	-0.0997	0.0989	0.0981	0.0024	0.0023	0.0022
200	0.04691	0.04754	0.04803	-0.0967	-0.0951	0.0943	0.0013	0.0012	0.0012
300	0.04721	0.04801	0.04853	-0.0891	-0.0871	0.0862	0.0009	0.0008	0.0007
400	0.04787	0.04822	0.04881	-0.0801	-0.0787	0.0711	0.0005	0.0004	0.0005
500	0.04831	0.04891	0.04911	-0.0745	-0.0710	0.0691	0.0004	0.0003	0.0003
750	0.04887	0.04906	0.04954	-0.0682	0.0699	0.0622	0.0003	0.0002	0.0002
1000	0.04911	0.04987	0.04994	-0.0574	-0.0609	0.0587	0.0002	0.0001	0.0001

Table 4. Average Estimate of θ with their Bias and MSE for α=20; β=780; θ=0.025 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.02138	0.02178	0.02199	-0.0998	0.0994	0.0991	0.0028	0.0028	0.0027
200	0.02189	0.02209	0.02257	-0.0971	-0.0967	0.0961	0.0021	0.0019	0.0018
300	0.02219	0.02259	0.02288	-0.0897	-0.0891	0.0882	0.0013	0.0011	0.0010
400	0.02291	0.02311	0.02341	-0.0805	-0.0799	0.0791	0.0008	0.0007	0.0005
500	0.02232	0.02387	0.02412	-0.0761	-0.0753	0.0742	0.0004	0.0003	0.0003
750	0.02386	0.02431	0.02464	-0.0699	0.0682	0.0669	0.0002	0.0002	0.0002
1000	0.02413	0.02487	0.02495	-0.0586	-0.0571	0.0532	0.0002	0.0001	0.0001

Table 5. Average Estimate of θ with their Bias and MSE for α=10.25; β=400; θ=0.025 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.04641	0.04711	0.04759	-0.0997	0.0989	0.0981	0.0024	0.0023	0.0022
200	0.04691	0.04754	0.04803	-0.0967	-0.0951	0.0943	0.0013	0.0012	0.0012
300	0.04721	0.04801	0.04853	-0.0891	-0.0871	0.0862	0.0009	0.0008	0.0007
400	0.04787	0.04822	0.04881	-0.0801	-0.0787	0.0711	0.0005	0.0004	0.0005
500	0.04831	0.04891	0.04911	-0.0745	-0.0710	0.0691	0.0004	0.0003	0.0003
750	0.04887	0.04906	0.04954	-0.0682	0.0699	0.0622	0.0003	0.0002	0.0002
1000	0.04911	0.04987	0.04994	-0.0574	-0.0609	0.0587	0.0002	0.0001	0.0001

Table 6. Average Estimate of θ with their Bias and MSE forα=21.05; β=400; θ=0.05 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.04619	0.04792	0.04749	-0.0998	0.0992	0.0989	0.0026	0.0025	0.0023
200	0.04675	0.04729	0.04719	-0.0978	-0.0971	0.0958	0.0018	0.0017	0.0015
300	0.04711	0.04792	0.04992	-0.0899	-0.0891	0.0869	0.0011	0.0009	0.0010
400	0.04754	0.04811	0.04793	-0.0821	-0.0811	0.0702	0.0008	0.0007	0.0005
500	0.04821	0.04861	0.04902	-0.0760	-0.0751	0.0734	0.0007	0.0006	0.0004
750	0.04865	0.04893	0.04945	-0.0699	0.0680	0.0661	0.0004	0.0003	0.0002
1000	0.04892	0.04971	0.04991	-0.0611	-0.0601	0.0591	0.0003	0.0002	0.0001

Table 7. Average Estimate of θ with their Bias and MSE for α=44.44; β=400; θ=0.1 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.09481	0.09531	0.09639	-0.0998	0.0978	0.0958	0.0029	0.0029	0.0027
200	0.09511	0.09699	0.09717	-0.0921	-0.0902	0.0889	0.0020	0.0019	0.0017
300	0.09575	0.09787	0.09799	-0.0878	-0.0831	0.0797	0.0013	0.0012	0.0011
400	0.09629	0.09792	0.09849	-0.0759	-0.0726	0.0712	0.0007	0.0006	0.0005
500	0.09673	0.09827	0.09920	-0.0634	-0.0599	0.0565	0.0004	0.0003	0.0003
750	0.09689	0.09860	0.09998	-0.0587	0.0521	0.0511	0.0003	0.0002	0.0002
1000	0.09717	0.09915	0.10017	-0.0447	-0.0398	0.0375	0.0002	0.0001	0.0001

Table 8. Average Estimate of θ with their Bias and MSE forα=57.14; β=400; θ=0.125 and varying sample size n:

n	Average Estimate of θ			Bias of θ			MSE of θ		
	MME	MLE	Bayes	MME	MLE	Bayes	MME	MLE	Bayes
100	0.02138	0.02138	0.02138	-0.0999	0.0995	0.0993	0.0033	0.0032	0.0031
200	0.02189	0.02189	0.02189	-0.0981	-0.0975	0.0969	0.0023	0.0022	0.0021
300	0.02219	0.02219	0.02219	-0.0921	-0.0910	0.0901	0.0018	0.0017	0.0015
400	0.02291	0.02291	0.02291	-0.0892	-0.0831	0.0801	0.0010	0.0009	0.0008
500	0.02232	0.02232	0.02232	-0.0855	-0.0792	0.0711	0.0006	0.0005	0.0003
750	0.02386	0.02386	0.02386	-0.0811	0.0722	0.0692	0.0003	0.0003	0.0002
1000	0.02413	0.02413	0.02413	-0.0756	-0.0701	0.0634	0.0002	0.0001	0.0001

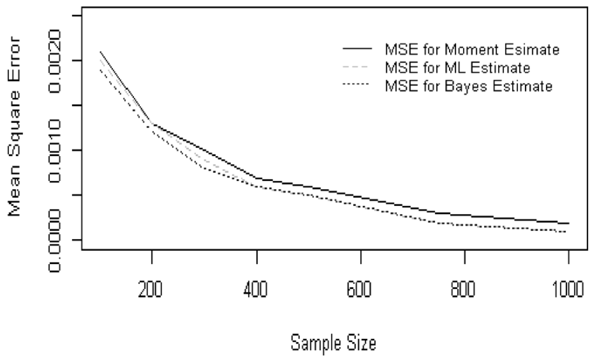

Fig. 1. Plots of trend in MSE of θ varying n at θ=0.125

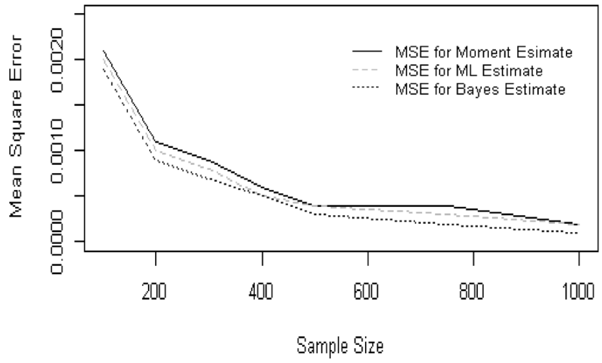

Fig. 2. Plots of trend in MSE of θ varying n at θ=0.1

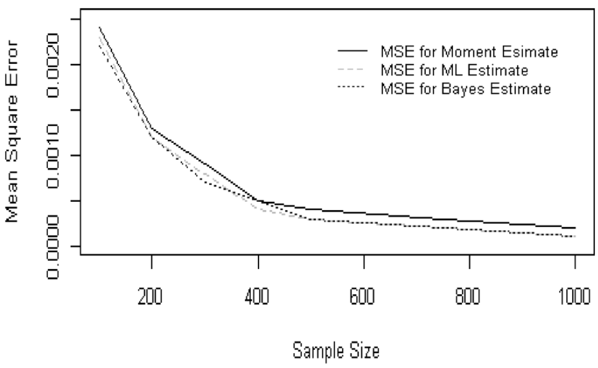

Fig. 3. Plots of trend in MSE of θ varying n at θ=0.05

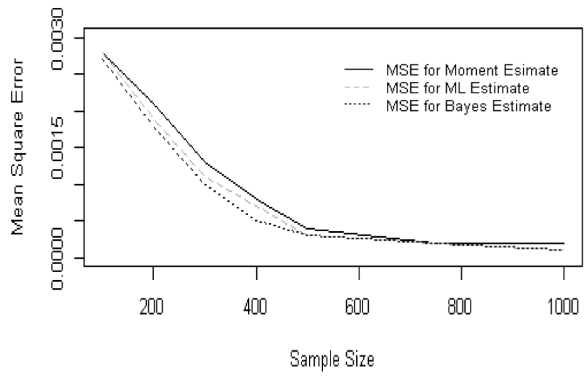

Fig. 4. Plots of trend in MSE of θ varying n at θ=0.025

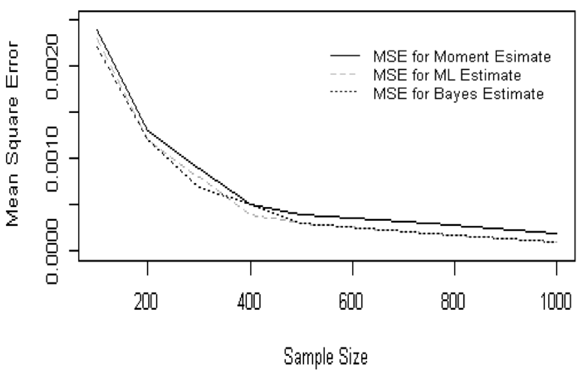

Fig. 5. Plots of trend in MSE of θ varying n at θ=0.025

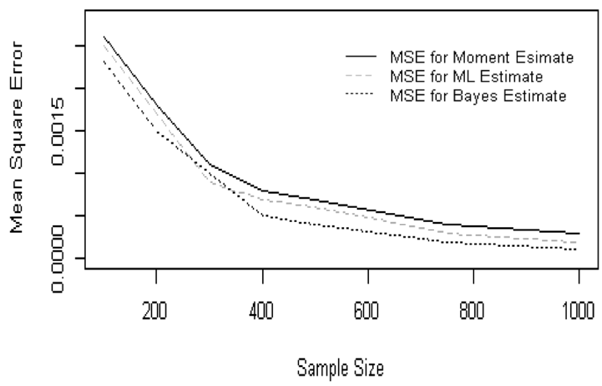

Fig. 6. Plots of trend in MSE of θ varying n at θ=0.05

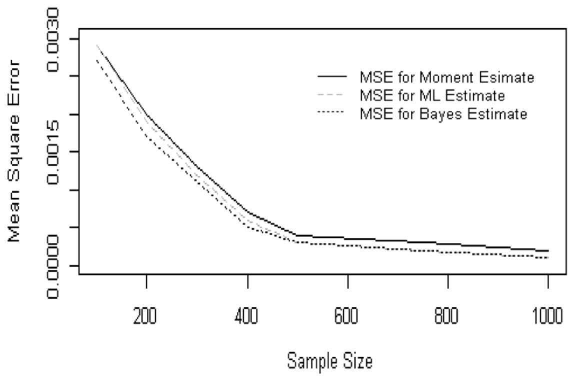

Fig. 7. Plots of trend in MSE of θ varying n at θ=0.01

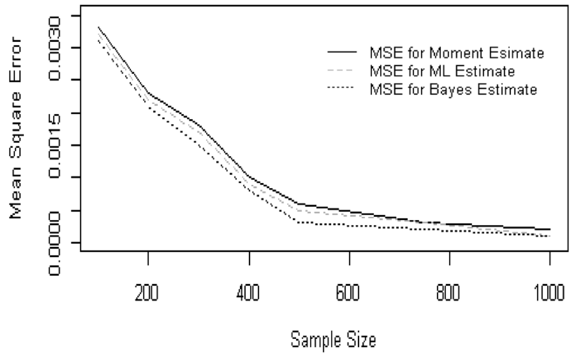

Fig. 8. Plots of trend in MSE of θ varying n at θ=0.125

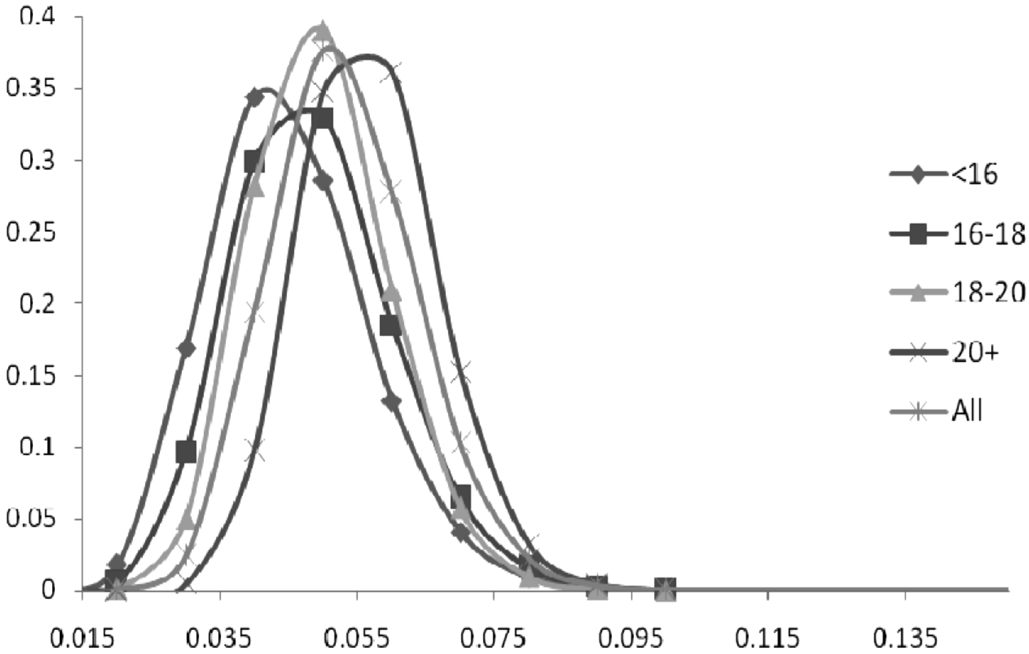

Fig. 9. The distribution of risk of conception for various ages at marriage group