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A Comparative Study on Objective Functions of Product Rate Variation and Discrete Apportionment Problems

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ABSTRACT

Setting the proper objective functions to optimize the decision making situations is prevalent in most of the mathematical programming problems. In this paper, we formulate the mathematical models of product rate variation and discrete apportionment problems. Furthermore, a brief comparative study of the objective functions to both the problems is reported in terms of inequality measures, precisely indicating the equitably efficient frontier for production rate variation problem via discrete apportionment. The largest reminder algorithm and rank-index algorithm for the apportionment problem are discussed briefly.

Key words: Objective function, product rate variation, sequencing, discrete apportionment

INTRODUCTION

The product rate variation problem (PRVP) is the mixed-model just-in-time (JIT) sequencing problem which is widely studied during the latter half of the 20^{th} century. The JIT production system is popularly known as Toyota production system developed and perfected by T. Ohno while working as assembly manager in Toyota company around 1970s. It is a management philosophy that requires to produce only the necessary products in the needed quantities at the required times and in desired quality, which is based on the planned elimination of all wastages, continuous improvement of productivity and reduction of inventories in all levels. To achieve this goal, the early-tardy operations are penalized considering the negligible switch over or setup costs from one model to other model, and hence underproduction and overproduction are minimized to increase profits as well as to decrease costs. The PRVP reduces the rate at which different models are produced on the production line by minimizing the discrepancies between the actual and ideal productions (Thapa, 2012). It is single-level problem which is studied in perspective of the two types of objective functions, namely maximum deviation and sum deviation. The PRVP with the objective of minimizing the maximum deviation between the actual and the ideal productions is called the bottleneck PRVP. Similarly, the PRVP with the objective of minimizing the total deviations between the actual and the ideal productions is called the total PRVP. The total PRVP with a general objective function has been solved in a pseudo-polynomial time (Kubiak & Sethi 1994). The bottleneck PRVP is solved via perfect matching approach and bottleneck analysis (Steiner & Yeomans 1993; Brauner & Crama 2004). The total PRVP is efficiently handled via discrete apportionment approach (Dhamala *et al.* 2012).

The discrete apportionment problem (DAP) plays an important role in modern democracies, a classical example being the U.S. presidential election, which is the problem of translating an election outcome to a number of seats from a fixed-sized political house. Mathematically, it is the problem of translating a sequence of real numbers to a sequence of integers, while ensuring that the sum of the sequence sums up to a pre-determined number, called the house size. The problem arises because seats are indivisible (integers), whereas an election outcome generally gives rise to fractional remainders. The main problem is to minimize the difference between these two quantities as close as possible. DAP occurs in all kinds of electoral systems (Thapa & Dhamala, 2009a), for example, in *federal system* (regional representation based on population as in USA), in *proportional system* (political representation based on votes as in Israel) and *mixed system* (mixture of federal and proportional systems as in Nepal). The DAP is a complex kind of discrete fair division problem, since all possible apportionment methods contradict the principle of fairness criteria (Brams, 1976). In fact, no method equalizes states under the fixed house size allocating minimum requirement of one seat and states not crossing the house size. Mainly two fairness ideas have been studied in the literature: the first is each state should get either its lower quota or upper quota, and the second is to look at pair-wise equity between states. In any case, the philosophy of apportionment must obey political legitimacy and the solutions must be acceptable to nation. It naturally appears that finding a perfect apportionment method is a difficult job. Some methods of apportionment are practiced in different time and situations. For an excellent historical note, mathematical formulations and the apportionment methods, we refer the seminal monograph by Balinski and Young (Balinski & Young, 2001).

Product Rate Variation Formulation

The first step in solving the real-world problem is to model it in mathematical form. Assume n products to be produced within the given time horizon 1 through k with integer demands $d_1, d_2, \cdots, d_n$ such that $\sum_{i=1}^{n} d_i = D$ is the total demand. The time horizon is partitioned into D equal units. If $r_i = \frac{d_i}{D}$ is the ideal production rate for the parts of type i such that $\sum_{i=1}^{n} r_i = 1$, then the scheduling goal for the assembly line is to maintain the total cumulative production of product i to the total production as close to r_i as possible. This means exactly kr_i units of product i should be produced in the first k time periods.

Let x_{ik} be the actual production of product i in the time period 1 through k and kr_i be the ideal production in the same time horizon where $i = 1, 2, \cdots, n; k = 1, 2, \cdots, D.$ If $x_{ik} - kr_i > 0$, then overproduction occurs creating inventories and $x_{ik} - kr_i < 0$ implies the underproduction creating the shortages. The ideal case would be $x_{ik} - kr_i = 0$, which is very rare in general. For a convex penalty function f_i with minimum $f_i(0) = 0$, the bottleneck PRVP and total PRVP are formulated respectively as follows:

$$F_{\max} = \min \max_{i,k} f_i(x_{ik} - kr_i) \quad \text{.......... ....(1)}$$

$$F_{sum} = \min \sum_{i=1}^{n} \sum_{k=1}^{D} f_i(x_{ik} - kr_i) \quad \text{.......... .... (2)}$$

subject to $$\sum_{i=1}^{n} x_{ik} = k,\ k = 1, 2, \cdots, D \quad \text{.......... .....(3)}$$

$$x_{i(k-1)} \le x_{ik},\ i = 1, 2, \cdots, n;\ k = 1, 2, \cdots, D \quad \text{........(4)}$$

$$x_{iD} = d_i,\ i = 1, 2, \cdots, n \quad \text{.......................................(5)}$$

x_{ik} is a non-negative integer ....(6)

The constraint (3) ensures that exactly k units are sequenced in time periods 1 through k, the constraint (4) represents the monotone condition for production sequence which is clearly non-decreasing function. The constraint (5) ensures that production requirements are met for each product whereas the constraint (6) represents the integrality of the product. These four constraints jointly indicate that exactly one product is produced during each stage. The formulation (1) to (6) is an integer programming problem with cardinality, monotonicity and integrality constraints. The objectives (1) and (2) are studied as squared and absolute deviation objectives in terms of inequality measures (Thapa & Dhamala 2009a; Thapa, 2012) which are discussed in the section right after the following section.

Discrete Apportionment Formulation

Suppose there are s states or parties indexed by $j = 1, 2, \cdots, s$ that are supposed to receive representatives or seats according to the size of their population or votes from the congressional integer house size h, a pre-defined fixed number. Suppose the state j has a population p_j such that the total population of the nation is $\sum_{j=1}^{s} p_j = p.$ The DAP is to apportion a_{jh} integer seats to state j under the constraints $\sum_{j=1}^{s} a_{jh} = h$ and $a_{jh} \in Z^+$, set of positive integers. An ideal apportionment is assumed to satisfy the equation $\frac{a_{jh}}{h} = \frac{p_j}{p}$ for all states giving $a_{jh} = \frac{p_j h}{p}$, called the *ideal quota* or *fair share* for state j denoted by q_{jh} which is not necessarily an integer. The number a_{jh} is called actual apportionments for the state j associated with house size h. Since only the integral a_{jh} can be assigned to any state, the crucial point is how to handle the problem fairly. An immediate idea is rounding: for each state, ideal apportionment should either be rounded down to the next lower integer or rounded up to the next higher integer; but should never exceed these bounds (Thapa 2012). The lower and upper bounds are defined by the floor and ceiling values of the fractional number q_{jh} as follows: $\lfloor q_{jh} \rfloor \le q_{jh} \le \lceil q_{jh} \rceil$. The apportionment vector a satisfies the quota if and only if $\lfloor q_{jh} \rfloor \le a_{jh} \le \lceil q_{jh} \rceil$ for each state j. This idea of rounding is not unique. On top of the rounding concept, there are five traditional divisor methods from Huntington family (Balinski & Young 1977) which are proposed in different political situations and intervals of time. Largest reminder methods are another type of methods. With the above notations, the mathematical models of DAP are formulated as follows: For the fixed house size h, the maximum and the sum deviation global indices of discrete apportionment to be minimized are (Thapa & Dhamala, 2009b; Thapa 2012):

$$G_{\max} = \min_j \max_h g_j(a_{jh} - q_{jh}) \qquad (7)$$

$$G_{sum} = \min \sum_{j=1}^{s} g_j(a_{jh} - q_{jh}) \qquad (8)$$

subject to $\sum_{j=1}^{s} a_{jh} = h$ .... (9)

$$a_{jh} \le a_{j(h+1)},\ j = 1, 2, \cdots, s \qquad (10)$$

$a_{jh} \ge 1$, an integer .... (11)

This is a constrained integer programming problem seeking for integer allocations a_{jh} in such a way that the sum of them does not exceed the house size h and they remain near to the fair shares q_{jh} as close as possible. Also the allocations must never be less than unity, since it is the minimum requirement.

PRVP verses DAP

These two problems are considered to be similar due to their generic common properties. Establishing the relation between PRVP and DAP, Bautista *et al.* (1996) stated that the former problem can be seen as a constrained sequential apportionment problem. Józefowska *et al.* (2006) characterized some of the algorithms of PRVP via apportionment theory with suitable transformation of the problems. Adding some similar properties, we present the notational interrelation of the problems as follows:

Number of products n ⇔ number of states s

Product i ⇔ state j

Demand vector d ⇔ population vector p

Demand d_i for product i ⇔ population p_j of state j

Position in sequence k ⇔ size of fixed house h

Actual production x_{ik} ⇔ actual apportionment a_{jh}

Ideal production kr_i ⇔ exact quota q_{jh}

Total demand $D = \sum_{i=1}^{n} d_i$ ⇔ total population $p = \sum_{j=1}^{s} p_j$

Monotonicity in PRVP ⇔ house monotone in apportionment

Thus, the two problems can be observed from the same window and handled in similar ways in most of the cases, such as the parametric divisor methods of apportionment generate cyclic just-in-time sequences (Kubiak, 2009). The optimality of many instances of either problem indicates the optimality of the other one. In the next Section, we discuss the inequality measures in PRVP and DAP.

Inequality Measures of PRVP and DAP Objectives

Equality is essentially an abstract socio-political concept that implies fairness and justice. However, the equality is usually quantified with the so-called inequality measures to be minimized. According to the theory of equity measurement, the preference model should satisfy the principle of transfers which states that a transfer of small amount from an outcome to any relatively worse-off outcome results in a more preferred achievement vector. The comparison and reduction of amount of inequalities between the intended quantities or shares of states to reach as near as to equity are the main ideas of the problem. To reach near to equity means to obtain fairness and justice. In the following, we present the inequality measures of PRVP followed by the same measures in case of DAP.

PRVP Objectives

The bottleneck PRVP is studied with absolute and squared objective functions to minimize the maximum deviation between actual and ideal productions (Lebacque *et al.* 2007; Thapa & Dhamala 2009b; Thapa, 2012). These discrepancy functions are as follows:

$$F^a_{\max} = \min \max_{i,k} |x_{ik} - kr_i| \qquad (12)$$

$$F^s_{\max} = \min \max_{i,k} (x_{ik} - kr_i)^2 \qquad (13)$$

where a and s over $F_{\max}$ stand for absolute and square respectively. The mostly studied total PRVP as absolute deviation and square deviation objectives are (Miltenburg 1989; Thapa, 2012) as follows:

$$F^a_{sum} = \min \sum_{i=1}^{n} \sum_{k=1}^{D} \left| \frac{x_{ik}}{k} - r_i \right| \qquad (14)$$

$$F^s_{sum} = \min \sum_{i=1}^{n} \sum_{k=1}^{D} \left(\frac{x_{ik}}{k} - r_i \right)^2 \qquad (15)$$

These measures are further deduced as the following standard total PRVP objectives (Miltenburg 1989):

$$F^a_{sum} = \min \sum_{i=1}^{n} \sum_{k=1}^{D} |x_{ik} - kr_i| \qquad (16)$$

$$F^s_{sum} = \min \sum_{i=1}^{n} \sum_{k=1}^{D} (x_{ik} - kr_i)^2 \qquad (17)$$

The tractability of either type of these objectives is equivalent mathematically. The discrepancy functions

(14) and (15) keep the actual proportions of the production mix $\frac{x_{ik}}{k}$ close to the desired proportions r_i at all times k, whereas (16) and (17) attempt to keep the actual production close to the desired production at all times. Both types of objectives yield reasonably similar schedules. Miltenburg (1989) proposed three algorithms with two supporting heuristics for good solution of the objective (17). A concise survey on the heuristics for PRVP is recently reported (Thapa & Silvestrov 2014).

On defining the ideal production time $t_{ik} = \frac{2k-1}{2r_i}$, $i = 1, 2, \cdots, n$; $k = 1, 2, \cdots, D$ and required production time y_{ik} of each product, Inman & Bulfin (1991) proposed the min-sum squared sequencing objective $f(y) = \sum_{i=1}^{n}\sum_{k=1}^{D}(y_{ik} - t_{ik})^2$ to be minimized and developed a pseudo-polynomial heuristic with complexity $O(D)$ via an efficient algorithm, namely earliest due date (EDD) algorithm. The problem is reduced into single-machine scheduling with due date t_{ik}. The optimal sequences are found by ordering the jobs following the EDD rule. Likewise, PRVP objectives with weight factor are also considered and discussed in many research works, for example in Thapa (2012).

Another type of deviation for the pair of products i_1 and i_2 is proposed (Balinski and Shahidi 1998), which aims to minimize the variation of production rates from product to products. The production rates of the products i_1 and i_2 are defined by $\frac{x_{i_1k}}{r_{i_1}}$ and $\frac{x_{i_2k}}{r_{i_2}}$ respectively. If $\frac{x_{i_1k}}{r_{i_1}} = \frac{x_{i_2k}}{r_{i_2}}$ for all i_1, i_2, then perfection will be gained. However, this ideal case is very rare in practice. Therefore, the objective minimizing the inequality between these two ratios is measured as

$$\min_{i_1}\max_{i_2}\left|\frac{x_{i_1k}}{r_{i_1}} - \frac{x_{i_2k}}{r_{i_2}}\right| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

This is an interesting objective that minimizes the difference of product rates between two products. We have developed equitably efficient frontier for this objective function relating with DAP defining the objective function for state to state variation in the number of representatives (Dhamala *et al.* 2012). Both the objectives are studied in terms of their relative differences. The similar inequalities can be observed in the DAP which are discussed in the following subsection.

DAP Objectives

There are several apportionment objectives to be minimized in terms of global and local measures of deviations (Thapa, 2012). The global index is the overall discrepancy function to be minimized whereas the local index is the discrepancy function that minimizes the pair-wise injustice between the two states. The global indices of apportionment objectives (7) and (8) can be studied under the same constraints in absolute and square deviation perspectives as follows:

$$G^a_{\max} = \min \max_{j \quad h} \left|a_{jh} - q_{jh}\right| \quad \ldots\ldots\ldots\ldots \quad (19)$$

$$G^s_{\max} = \min \max_{j \quad h} \left(a_{jh} - q_{jh}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (20)$$

$$G^a_{sum} = \min \sum_{j=1}^{s}\left|a_{jh} - q_{jh}\right| \quad \ldots\ldots\ldots\ldots \quad (21)$$

$$G^s_{sum} = \min \sum_{j=1}^{s}\left(a_{jh} - q_{jh}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (22)$$

It is observed that the global indices of the PRVP and the DAP objectives are of similar nature and an efficient frontier is established (Thapa, 2012). Hamilton presented the largest reminder (LR) method for the global index of DAP in 1792, in which the apportionments are made easily as follows: Calculate the quota and assign to each state its integer part. Distribute unapportioned seats to the states ordering with the largest remainders until the house is full. This method was used in the U.S. from 1850 to 1900, and so it is important from a historical perspective. Moreover, this method seems to be natural and simple considering the quota approach. The quota method of apportionment is elaborately reported in Balinski and Young (1975). The following algorithm shows a formal description of how the apportionment process is carried out in Hamilton method (*i.e.,* the LR method):

Algorithm 1. The Largest Reminder Algorithm:

Step 1. Compute $q_{jh} = \frac{p_j h}{p}$, the ideal (fractional) value that gives perfect proportionality.

Step 2. Set $r_j = q_{jh} - \lfloor q_{jh} \rfloor$, the fractional remainder of q_{jh}.

Step 3. Assign $a_{jh} = \lfloor q_{jh} \rfloor$ for $j = 1, 2, \cdots, s$.

Step 4. Let $R = h - \sum_{j=1}^{s} a_{jh}$ be the number of seats that remain to be allocated.

Step 5. If $R > 0$, assign one more seat to states/parties with the largest fractional remainders r_j.

The LR method is studied in various form by different mathematical and political scientists. This method satisfies quota rule, however suffers Alabama, population and new states paradoxes. To avoid the effect of the apportionment paradoxes, *house-monotone* methods are developed which are called divisor methods.

Let a and p denote apportionment vector and population vector. A divisor function depending upon a is a monotone increasing function $d(a)$ such that $a \le d(a) \le a+1$. In light of this divisor function, a systematic study of the divisor methods has been carried out (Huntington, 1921) and thereby investigated several local measures of inequity in DAP based on pair-wise comparisons of states' relative representation as the house-monotone methods (Thapa 2012). This family of methods is based upon *rank-index* $r(p, a) = \frac{p}{d(a)}, d(a) \neq 0$ and *fairness measure* $\frac{a_{j_1h}}{p_{j_1}} > \frac{a_{j_2h}}{p_{j_2}}$, minimizing pair-wise measures of inequity between two states j_1 and j_2 (Balinski & Young 1977, 2001). The state having maximum of $\frac{p}{d(a)}$ receives the $(h+1)^{th}$ seat. For given house size h, $\frac{p_j}{a_{jh}}$ and $\frac{a_{jh}}{p_j}$ represent the average district size and the share of representatives respectively for state j. If $\frac{a_{j_1h}}{p_{j_1}} > \frac{a_{j_2h}}{p_{j_2}}$, then state j_1 is said to be better off than the state j_2. An apportionment method M is said to be house monotone if for every apportionment solution $f \in M$, $f(p,h) \le f(p,h+1)$ is satisfied. There always exists a certain inequality in practice between two states, which gives one of the states a slight advantage over the other. An apportionment a is stable if no transfer of one seat from a better off state j_1 to a less well off state j_2 reduces the value of the inequality. The main idea of divisor (rank-index) method is:

Algorithm 2. Rank-index Algorithm:

Step 1. Start with $f(p, 0) = 0$, that is,

$$a_j = 0, \quad j = 1, 2, \cdots, s$$

Step 2. Find a state t such that

$$r(p_t, a_t) = \max_j r(p_j, a_j)$$

Then, $a_t' = a_t + 1$ and $a_{t_1}' = a_{t_1}$ for $t \neq t_1$

Step 3. Repeat step 2 until all h seats are allocated.

Theorem: An apportionment method M is a house monotone and consistent if and only if it is a Huntington method.

The measures of amount of inequalities between the states defined as the fairness measures, are not unique. The inequality $\frac{a_{j_1h}}{p_{j_1}} > \frac{a_{j_2h}}{p_{j_2}}$ can be rearranged in $2^4 = 16$ different ways by taking different combinations of $p_{j_1}, p_{j_2}, a_{j_1}, a_{j_2}$. Several cases of the fairness measures between two states can be found in the literature (Balinski & Young 2001, Thapa & Dhamala, 2009a). For any two states j_1 and j_2, one of the local measures of inequalities in DAP similar to the PRVP objective (18) is:

$$\min_{j_1} \max_{j_2} \left| \frac{a_{j_1h}}{p_{j_1}} - \frac{a_{j_2h}}{p_{j_2}} \right| \quad \text{.......... ....} (23)$$

We have established an equitably efficient frontier for the two objectives (18) and (23) with respect to their relative differences (Dhamala *et al.* 2012). These pair-wise variation problems in PRVP and DAP are equivalent and have if and only if condition for the optimality of each other. Note that balancing the inter-state apportionment is equivalent to balancing sub-products of a product. The complexity issues of the problems are also equivalent in both the local and global deviations (Thapa, 2012). The PRVP is studied in terms of parametric methods of apportionment relating with rounding and production (Balinski & Ramirez, 1999). On top of all above discussions, we can clearly observe that the PRVP and DAP are closely interlinked in terms of global measures of deviation and in terms of pair-wise variations.

CONCLUSION

The PRVP and DAP both are the promising research problems independently and simultaneously as well. A number of research works has been carried out in both directions. The goal of both the problems is to minimize certain type of discrepancy functions under pre-assigned constraints. The comparative study of the inequality measures as objective functions of both the problems is reported in this short paper and hence if and only if conditions are indicated. The problem whether balancing global deviation measures in each problem can balance the local deviation measures and vice-versa remains open. It is really difficult to find a perfect apportionment method meeting all the desired requirements. The famous Impossibility Theorem of apportionment due to Balinski and Young (2001) states that there are no perfect apportionment methods. Moreover, it is impossible for an apportionment method to be population monotone and stay within the quota at the same time for any reasonable instance of the problem. This implies that there are still many open research issues in the discrete apportionment domain.

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