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***Research Article***

# ANALYSIS OF BLOOD FLOW THROUGH ARTERY WITH MILD STENOSIS

**Puskar R. Pokhrel**[1], **Jeevan Kafle**[2*], **Parameshwari Kattel**[3], **Hari Prasad Gaire**[2]

[1]*Department of Mathematics, Ratna Rajya Laxmi Campus, Tribhuvan University, Kathmandu, Nepal*
[2]*Central Department of Mathematics, Tribhuvan University, Kirtipur, Kathmandu, Nepal*
[3]*Department of Mathematics, Tri-Chandra Multiple Campus, Tribhuvan University, Kathmandu, Nepal*
[*]*Correspondence author: jeevankafle@yahoo.com*

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## ABSTRACT

Arterial stenosis is an abnormal condition in arteries due to the deposition of fats and other substances, called atherosclerosis. As it restricts the blood flow, it may induce a heart attack. Employing the Navier-Stokes equations, we consider the blood flow in an artery with the presence of a stenosis in an axisymmetric shape. We analyze the blood flow dynamics in cylindrical form by evaluating pressure, pressure drop against the wall, shear stress on the wall. We also analyze the dynamics by evaluating the ratio of pressure drop with stenosis to the pressure drop without stenosis against the wall, and the ratio of maximum to minimum shear stresses with the ratios of various thicknesses of stenosis to radius of the artery.

**Keywords:** Artery, Blood flow, Pressure drop, Shear stress, Stenosis.

## INTRODUCTION

Stenosis is a pathological restriction of an artery, usually due to fat deposition, which alters different mechanisms involved in blood circulation. Stenosis in blood vessels, especially in arteries involve narrowing or constriction of the inner surfaces. It is the main cause of well-known serious diseases such as atherosclerosis. Therefore, the study of blood flow in a stenotic artery is useful for the understanding of circulatory disorders (Hye, 2012; Ku, 1997; Phaijoo, 2013).

Blood behaves as a Newtonian fluid when it flows through arteries with a larger diameter at a high shear rate, whereas it exhibits a non-Newtonian fluid while flowing through arteries with smaller diameter at a low shear rate (Darcy, 1993; Eldesoky, 2012; Jain *et al.*, 2010; Pedlosky, 1987). One of the major causes of the deaths in the world is due to heart diseases, and the most commonly heard names among the same are ischemia, atherosclerosis, and angina pectoris. Ischemia is the deficiency of oxygen in a part of the body, usually temporary. It can be due to a constriction (stenosis) or obstruction in the blood vessel supplying the blood in that part (Phaijoo, 2013; Pralhad & Schultz 2004).

The mathematical investigation of blood flow in the human circulatory system is one of the major challenges from the past few decades to many years to come. The development of more effective and accurate numerical simulation techniques could provide a better understanding of the hemodynamical abnormalities due to stenosis (Phaijoo, 2013). Blood flow under atherosclerosis which together with flow pulsatility can be the cause of some periodic turbulence (Varghese & Frankel, 2003). Turbulence in blood flow might affect some physiological processes such as the flow resistance, high shear stress on the blood vessel wall, tensile stress in endothelial cell membrane, change in blood rheology due to deformability of red blood cells, the surface cell loss as well as internal cell motion due to pressure and shear stress (Brinkman, 1949; Fung, 1993; Varghese & Frankel, 2003). Human blood consists of plasma, red blood cells, white blood cells, and platelets to form a colloidal suspension (Guyton & Hall, 2000). The human circulatory system is a closed cardiovascular type flowing in the network of arteries, veins, and capillaries (Chakraborty *et al.*, 2011; Jasit, 2016). The flow of the blood remains laminar within the range of Reynolds numbers 5,000-10,000 (Phaijoo, 2013).

Stenosis is generally an abnormal narrowing or contraction of a body passage or opening (Keane & O' Toole, 2003). Stenosis can lead to serious circulatory disorders, affecting many hydrodynamic factors such as resistance to flow, wall shear stress, and apparent viscosity. Aortic stenosis, Hypertrophic subaortic stenosis, Mitral stenosis, Pulmonary stenosis, Renal artery stenosis, Spinal stenosis, Subaortic stenosis, Tracheal stenosis, Tricuspid stenosis are some common types of stenosis (Keane & O' Toole, 2003; Sherwood, 2016).

Many existing literatures focused on the study of blood flow in human arteries with stenosis. Varghese and Frankel (2003) numerically modeled the pulsatile turbulent flow in a stenotic vessel using the Reynolds-averaged Navier-Stokes equation approach. Srivastava *et al.* (2010) studied the increased impedance and other flow characteristics during artery catheterization with a composite stenosis assuming that the flowing blood behaves like a Newtonian fluid. Jain *et al.* (2010) developed a mathematical model for studying the oscillatory flow of blood in a stenosed artery under the influence of a transverse magnetic field through a porous medium. They solved the equation of motion of blood flow analytically by deriving the expressions for axial velocity, volumetric flow rate, pressure gradient, resistance to blood flow, and shear stress. Phaijoo (2013) analyzed the N-S equation in blood flow in cylindrical form by using different parameters. Moreno and Bhanagar (2013) developed the model in the case of realistic physiological flow conditions accounting for the unsteady flow conditions (systole/diastole) as well as the transition from laminar to turbulent state. Their studies clearly showed that, for the same degree of stenosis, (a) the presence of turbulence, (b) location of transition to turbulence, (c) turbulence intensity, and (d) region of turbulence are type-dependent.

Argyropoulos and Markatos (2015) reviewed the recent advances and success of computing turbulent flows. Their review was primarily concerned with the most recent methods for computer predictions such as Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES) to flows in pipes and free-surface flows. They noticed that the LES was the most accurate among the methods available for practical computations. Hye and Paul (2015) proposed that the spiral effect should be incorporated to get a better insight into the transition-to-turbulence flow of blood through the arterial stenosis. Their results showed that the spiral flow affected the turbulence kinetic energy in the post stenosis region and the wall pressure and shear stress remained almost unchanged by the spiral velocity. Mahalingam *et al.* (2016) studied the nature of blood flow through stenosed coronary arteries by numerical analysis of the effect of turbulence transition on the hemodynamics parameters. They found that the primary biological effect of blood turbulence is the change in wall shear stress (WSS) on the endothelial cell membrane, while the local oscillatory nature of the blood flow influences some physiological changes in the coronary artery. Shah and Shukla (2017) studied some curvature properties of quasi-conformal curvature tensor on Sasakian manifolds. Thakur *et al.* (2018) used a fluid hydrodynamic model in the magnetized plasma sheath in a cylindrical coordinate system. Shah (2018) used the curvature conditions on Kenmotsu manifolds.

In this work, the blood flow inside an artery was described by Navier-Stokes (N-S) equations. So, we presented N-S equations along with the equation of continuity in cylindrical form as the model employed. We also modeled a stenosis in an artery giving a constriction in the flow. We described the pressure drop, and the ratio of maximum and minimum shear stresses due to stenosis. A simple model for the stenosis in the curved artery was also constructed.

## MODEL EQUATIONS

Blood flow in arteries can be modeled by Navier-Stoke's equations for fluid flow inside a cylinder (Kapur, 1985). Let r be the radius of the artery, p be the pressure; three components $v^r$, $v^\theta$ and $v^z$ be the of velocities along the radius vector, perpendicular to the radius vector, and parallel to the axis of *z*, respectively. The continuity equation and the equations of motion are given by (Kapur, 1985).

$$\frac{1}{r}\frac{\partial}{\partial r}(r\, v^r) + \frac{\partial}{\partial z}(v^z) = 0, \tag{1}$$

$$\rho\left(\frac{\partial v^r}{\partial t} + v^r\frac{\partial v^r}{\partial r} + v^z\frac{\partial v^r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v^r}{\partial r^2} + \frac{\partial^2 v^r}{\partial z^2} + \frac{1}{r}\frac{\partial v^r}{\partial r} - \frac{v^r}{r^2}\right) \tag{2}$$

$$\rho\left(\frac{\partial v^z}{\partial t} + v^r\frac{\partial v^z}{\partial r} + v^z\frac{\partial v^z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v^z}{\partial r^2} + \frac{\partial^2 v^z}{\partial z^2} + \frac{1}{r}\frac{\partial v^z}{\partial r}\right), \tag{3}$$

In the axisymmetric case, we assume $v^\theta = 0$, and $v^r$, $v^z$, and p are independent of θ. For the steady flow of blood, we assume a constant viscosity (μ) and constant density (ρ) for the in a cylindrical artery of radius and length inclined at an angle represented in Fig.1.

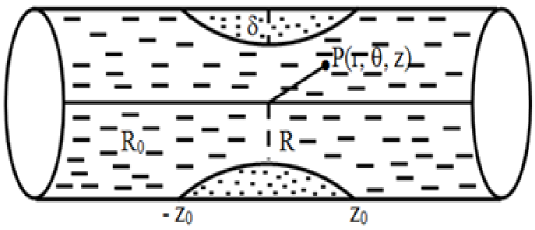


**Fig. 1. Section of an artery with mild stenosis**

Considering steady flow, and the velocity component parallel to the axis, so that $v^r = 0$, $v^\theta = 0$, and $v^z = v$, equations (1)-(3) reduce to $v^z = v(r)$, $0 = -\frac{\partial p}{\partial r}$,

$$0 = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right) \tag{4}$$

Denoting $P(z) = -\partial p / \partial z$, (4) reduces to

$$-P(z)\frac{r}{\mu} = \frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \tag{5}$$

We consider the boundary condition (Kapur, 1985):

$$v = \begin{cases} 0 & \text{at } r = R(z), \quad -z_0 \le z \le z_0 \\ 0 & \text{at } r = R_0, \quad |z| \ge z_0. \end{cases} \tag{6}$$

The shape function R(z) for the radial structure of the surface of cylindrical pipe as shown in Fig. 1 was given below by equation (7) (Kapur, 1985).

$$\frac{R}{R_0} = 1 - \frac{\delta}{2R_0}\left(1 + \cos\frac{\pi z}{z_0}\right) \tag{7}$$

Integrating (5) with respect to r, taking z as constant gives

$$r\frac{\partial v}{\partial r} = -P(z)\frac{r^2}{4\mu} + C(z),$$

Where, C(z) is the constant of integration.

Applying $\partial v/\partial r = 0$ at r = 0 gives C(z) = 0. Integrating it again and using the boundary condition given by (6) results in

$$v(r) = -P(z)\frac{r^2}{4\mu} + D(z),$$

where, D(z) is another constant of integration.

Applying v = 0 at r = R gives $D(z) = P\,R^2/4\mu$, and so the velocity will be

$$v = \frac{P}{4\mu}(R^2 - r^2).$$

This shows that velocity is maximum along the axis, and it vanishes on the surface of the artery. As v is a function of r and z, the flux through the cylindrical tube can be obtained through Kapur (1985).

$$Q = \int_0^{R(z)} 2\pi r v\, dr = \frac{P\pi}{2\mu}\int_0^{R(z)} (rR^2 - r^3)\, dr,$$

$$\therefore \quad Q = \frac{P\pi}{2\mu}\left[\frac{r^2R^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\pi P(z)}{8\mu} R^4(z).$$

Since Q is independent of z, this equation gives P (z) as a function of z showing that pressure gradient varies inversely as the fourth power of the surface distance of the stenosis from the axis of the artery, and thus pressure gradient is minimum at the middle of the stenosis and maximum at the ends (Kapur, 1985).

Blood flow in an artery is considered as the laminar flow of non-Newtonian fluid in a circular tube under a constant pressure gradient. As pressure is equal to force per unit area, i.e., P = F/A, force due to pressure is now

$$F = P.2\pi r.dr = 2\pi Prdr \tag{8}$$

Let $\tau(r)$ be shear stress at a distance R from the axis. Force due to stress on the inner cylinder is $F = \tau.2\pi r.1 = 2\pi\tau r$. Therefore, force on the cylindrical surface is

$$F = 2\pi\tau r + 2\pi\frac{d}{dr}(r\tau)dr \tag{9}$$

Now, from (8) and (9), $Pr = \frac{d}{dr}(r\tau)$. Integrating it with respect to *r* gives $r\tau = \frac{P(z)r^2}{2} + E(z)$, Where, E(z) is the constant of integration. Since the shear stress is finite at r= 0, E(z)= 0. So, $\tau = \frac{1}{2}P(z)r$.

For the fluid power law, $\tau = \mu e^n$, where $\mu$ is viscosity (assumed constant) and $0.68 < n < 0.80$ for blood (Kapur, 1985) and thus

$$e = (\tau/\mu)^{1/n} \text{ gives } -\frac{dv}{dr} = \left(\frac{1}{2\mu}Pr\right)^{1/n}.$$

Integrating it from r = R to any r, the velocity becomes

$$v = \left(\frac{P}{2\mu}\right)^{1/n}\frac{n}{n+1}\left[R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right],$$ and the flux Q is obtained by (Kapur, 1985)

$$Q = \int_0^R 2\pi r v\, dr = \frac{n\pi}{3n+1}\left(\frac{P}{2\mu}\right)^{1/n} R^{\frac{1}{n}+3}.$$

$$\text{Further, } P(z) = \left(\frac{3n+1}{n\pi}\right)^n \frac{2\mu Q^n}{R^{3n+1}}.$$

The pressure drop across the length of the stenosis is denoted by (Kapur, 1985)

$$\Delta P = \int_{-z_0}^{z_0} P(z)\, dz = \int_{-z_0}^{z_0} \left(\frac{3n+1}{n\pi}\right)^n \frac{2\mu Q^n}{R^{3n+1}}\, dz.$$

For mild stenosis, using the radial surface given by (7), the pressure drop across the length is given by (Kapur, 1985):

$$\Delta P = \int_{-z_0}^{z_0}\left(\frac{3n+1}{n\pi}\right)^n \frac{2\mu Q^n dz}{R_0^{3n+1}(a - b\cos(\pi z/z_0))},$$

Where, $a = 1 - \delta/2R_0$, $b = \delta/2R_0$. Putting $\pi z/z_0 = u$, so that $\pi\, dz = z_0\, du$.

When $z = -z_0$, $u = -\pi$, and when $z = z_0$, $u = \pi$.

$$\Delta P = \frac{4\mu z_0 Q^n}{\pi R_0^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n \int_0^{\pi} \frac{du}{(a - b\cos u)^{3n+1}}.$$

When there is no stenosis, $\delta = 0$; but $f(\delta/R_0) = 1$, then the pressure drop across the stenosis length is given by

$$(\Delta P)_p = \frac{4\mu z_0 Q^n}{R_0^{3n+1}}\left(\frac{3n+1}{n\pi}\right)^n.$$

The ratio of pressure drop across the stenosis is as (Kapur, 1985):

$$\frac{\Delta P}{(\Delta P)_p} = \frac{1}{\pi}\int_0^{\pi} \frac{du}{(a - b\cos u)^{3n+1}} \qquad (10)$$

We note that

$$\int_0^{\pi} \frac{du}{a - b\cos u} = \int_0^{\pi} \frac{\sec^2(u/2)\,du}{a - b + (a+b)\tan^2(u/2)}.$$

Putting $\tan(u/2) = t$, $\sec^2(u/2) = 2dt$, to get

$$\int_0^{\pi} \frac{du}{a - b\cos u} = \int_0^{\infty} \frac{2t\,dt}{a - b + (a+b)t^2} = \frac{\pi}{\sqrt{a^2+b^2}}.$$

For Newtonian fluid, one can put n = 1 in equation (10). For this, applying Leibnitz's rule, partially differentiating with respect to a thrice, and replacing the values of a and b gives

$$\frac{\Delta P}{(\Delta P)_p} = \frac{1}{\pi}\int_0^{\pi} \frac{du}{(a - b\cos u)^4} = \left(1 - \frac{\delta}{2R_0}\right)$$
$$\times\left(1 - \frac{\delta}{R_0} - \frac{5\delta^2}{8R_0^2}\right)\left(1 - \frac{\delta}{R_0}\right)^{-7/2} \qquad (11)$$

The shear stress at wall $\tau = P(z)R(z)/2$ gives

$$\tau = \mu Q^n \left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R(z)^{3n}}.$$

When $\delta = 0$, i.e., there is no stenosis, $f(\delta/R_0) = 1$.

The shear stress across the stenosis length is

$$(\tau)_p = Q^n \left(\frac{3n+1}{n\pi}\right)^n \frac{1}{R_0^{3n}}.$$

The ratio of the shear stress at the wall is

$$\frac{\tau}{\tau_p} = \frac{1}{\left(1 - \frac{\delta}{2R_0} - \frac{\delta}{2R_0}\cos\frac{\pi z}{z_0}\right)^{3n}}.$$

The ratio of the maximum stress to the minimum stress is

$$\frac{\tau_{max.}}{\tau_{min.}} = \frac{1}{\left(1 - \frac{\delta}{2R_0} - \frac{\delta}{2R_0}\right)^{3n}} = \left(1 - \frac{\delta}{R_0}\right)^{-3n}. \qquad (12)$$

**Mild stenosis through curved artery**

Previous studies have found the effect of arterial curvature on blood flow in arteriovenous fistulae (Buradi *et al.*, 2016; Grechy *et al.*, 2017; Iori *et al.*, 2015). Stenosis most often occurs in the bending part of the human body due to the contraction and stressing of the artery. So, to make closer to this scenario, we consider the case of blood flow in the twisted artery, as shown in Fig. 2.

When blood is passing through a twisted artery containing stenosis, we consider the important aspect of curvature. Then, writing $P(z) = -\partial p/\partial z$, we propose the model equation of the blood flow in cylindrical form in generalized form as

$$-P(z) = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r} + \lambda\kappa\, v^2\right), \qquad (13)$$

Where, $\lambda\kappa$ is the local curvature, and appropriate boundary conditions can similarly be provided for Fig. 2. But, the analysis of the blood flow in a curved artery with stenosis is not a focus here and will be presented in some other contribution.

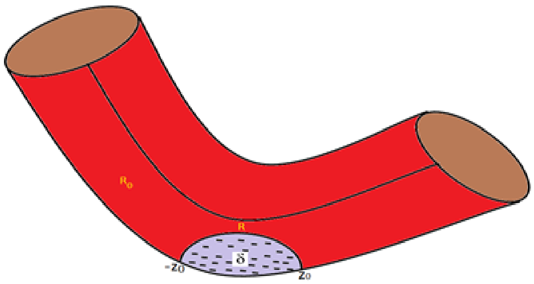


**Fig. 2. Surface of the mild stenosis with given radius δ in a curved artery as given in equation (13)**

## RESULTS AND DISCUSSION

Figure 3 shows that the ratio of the pressure drops across the stenosis increase exponentially against the ratio of the stenosis thickness to the radius of the artery, for different radii of stenosis: R = 3.0 mm, R = 3.1 mm, R = 3.2 mm, R = 3.3 mm. The rate of increment of the ratio is larger for the narrower artery.

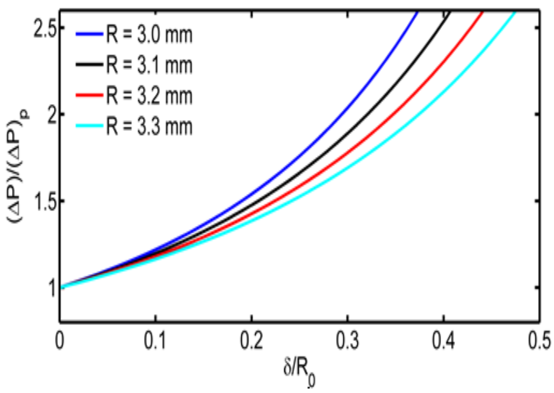


**Fig. 3. Pressure drop across the length of stenosis given by the equation (11)**

Figure 4 shows that the ratio of maximum to minimum shear stresses increases almost linearly against the ratio of the stenosis thickness to the radius of the artery with different (index of) power law of the shearing for n = 0.10, n = 0.30, n = 0.50, n = 0.70, n = 1.0. As the index n increases, the ratio increases.

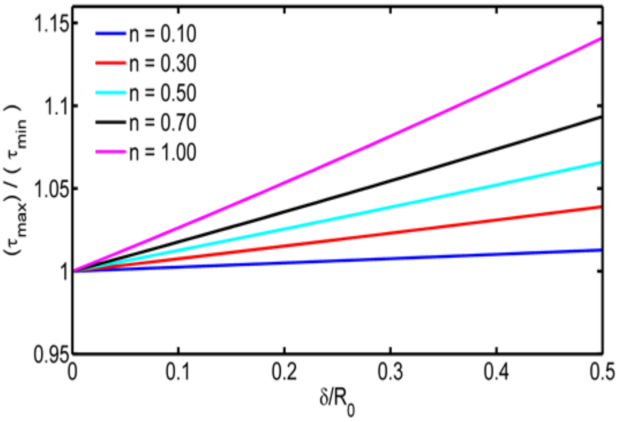


**Fig. 4. Ratios of maximum to minimum of the shear stresses for different values of n as given by the equation (12)**

## CONCLUSIONS

Here, we presented the Navier-stokes equations in cylindrical form for the blood flow of the artery, and analyzed the pressure drop in the artery with various radii of stenosis. It was observed that the ratio of the pressure drop of the blood flow decrease along with the increase in the radii of the stenosis. Meanwhile in case of increment of stenosis thickness to the artery radii , the pressure ratio of the pressure drop increase. The shear stress of the blood flow increased with the indices of the power law of the stress. Also, we extended a model equation to include curvature effects on the blood flow.

## REFERENCES

Argyropoulos, C. D., & Markatos, N. C. (2015). Recent advances on the numerical modeling of turbulent flows. *Applied Mathematical Modeling*, *39*, 693-732.

Brinkman, H. C. (1949). A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. *Applied Scientific Research*, *1*(1), 27-34.

Buradi, A., Araki, T., Ikeda, N., Nicolaides, A., Laird, J. R., Saba, L., & Suri, J. S. (2016). Numerical analysis of the effect of turbulence transition on the hemodynamic parameters in human coronary arteries. *Cardiovascular Diagnosis and Therapy*, *6*(1), 208-220.

Chakraborty, U.S., Biswas, D., & Paul, M. (2011). Suspension model blood flow through an inclined tube with an axially non-symmetrical stenosis. *Korea-Australia Rheology Journal*, *23*(1), 25-32.

Darcy, H. (1993). *The flow of fluids through porous media*. McGraw-Hill, New York, NY, USA.

Eldesoky, I. M. (2012). Slip effects on the unsteady MHD pulsatile blood flow through porous medium in an artery under the effect of body acceleration. *International Journal of Mathematics and Mathematical Sciences*. Article ID 860239, 26 pages.

Fung, Y. C. (1993). *Biomechanics: mechanical properties of living tissues*. Springer-Verlag, New York.

Grechy, L., Iori, F., Corbett, R. W., Gedroyc, W., Duncan, N., Caro, C. G., & Vincent P. E. (2017). The effect of arterial curvature on blood flow in arterio-venous fistulae: Realistic geometries and pulsatile flow. *Cardiovascular Engineering and Technology*, *8*(3), 313-329.

Guyton, A., & Hall, J. (2000). A *textbook of medical physiology*. Saunders, USA.

Hye, Md. A. (2012). *Simulation of transient blood flow in models of arterial stenosis and aneurysm* (PhD Thesis). School of Engineering, University of Glasgow, UK.

Hye, Md. A., & Paul, M. C. (2015). A computational study on spiral blood flow in stenosed arteries with and without an upstream curved section. *Applied Mathematical Modelling*, *39*, 4746-4766.

Iori, F., Grechy, L., Corbett, R. W., Gedroyc, W., Duncan, N., Caro, C. G., & Vincent, P. E. (2015). The effect of in-plane arterial curvature on blood flow and oxygen transport in arteriovenous fistulae. *Physics of Fluids*, *27*(3), 031903.

Jain, M., Sharma, G. C., & Singh, R. (2010). Mathematical modeling of blood flow in a stenosed artery under the MHD effect through porous medium. *International Journal of Engineering*, *Transactions B*, *23*(3-4), 243-251.

Jasit, S. S. (2016). Numerical analysis of the effect of turbulence transition on the hemodynamics parameters in human coronary arteries. *Cardiovascular Diagnosis and Therapy*, *6*(3), 208-220.

Kapur, J. N. (1985). *Mathematical Models in Biology and Medicine*. Affiliated East-West Press Pvt. Ltd., India, p. 520.

Keane, M., & O'Toole, M. (2003). *Miller-Keane encyclopedia & dictionary of medicine*. Nursing & Allied Health, Elsevier.

Ku, D.N. (1997). Blood flow in arteries. *Annual Review of Fluid Mechanics*, *29*(1), 399-434.

Mahalingam, A., Gawandalkar,U.U., Kini, G., Buradi, A., Araki, T., Ikeda, N., Nicolaides, A., Laird, J.R., Saba, L., & Suri, J.S. (2016). Numerical analysis of the effect of turbulence transition on the hemodynamic parameters in human coronary arteries. *Cardiovascular Diagnosis and Therapy*, *6*(1), 208-220.

Moreno, C., & Baghanagar, K. (2013). Modeling of stenotic coronary artery and implications of plaque morphology on blood flow. *Modeling and Simulation in Engineering*, *2013*, 1-14.

Pedlosky, J. (1987). *Geophysical fluid dynamics*. New York, USA: Springer-Verlang Inc.

Phaijoo, G. R. (2013). *Mathematical study of blood flow characteristics during catheterization in the stenosed artery* (MPhil Thesis). School of Science, Kathmandu University, Nepal.

Pralhad, R. N., & Schultz, D. H. (2004). Modeling of arterial stenosis and its applications to blood diseases. *Mathematical Biosciences*, *190*(2), 203-220.

Shah, R. J. (2018). On kenmostsu manifolds satisfying certain curvature conditions. *Journal of Institute of Science and Technology*, *22*(2), 56-62.

Shah, R. J., & Shukla, N. V. C. (2017). Quasi-conformal curvature on sasakian manifolds. *Journal of Institute of Science and Technology*, *22*(1), 94-98.

Sherwood, L. (2016). *Human physiology: from cells to systems*. USA: Cengage Learning, p. 955.

Srivastava, V. P., Mishra, S., & Rastogi, R. (2010). Non-Newtonian arterial blood flow through an overlapping stenosis. *Application and Applied Mathematics*, *5*(1), 225-238.

Thakur, P. K., Pokhrel, R. R., & Khanal, R. (2018). Fluid analysis of magnetized plasma sheath in a cylindrical geometry. *Journal of Institute of Science and Technology*, *23*(1), 26-29.

Varghese, S. S., & Frankel, S. H. (2003). Numerical modeling of pulsatile turbulent flow in stenotic vessels. *Journal of Biomechanical Engineering*, *125*(4), 445-460.