

# A Note on Teaching and Learning Linear Algebra in Tribhuvan University

**Tulasi Prasad Nepal**

Central Department of Mathematics, IOST, Tribhuvan University, Nepal

Corresponding author: *tpnepall@gmail.com*

**Received:** May 2, 2019

**Revised:** June 22, 2019

**Accepted:** June 25, 2019

---

**Abstract:** The teaching of linear algebra has always been a challenge for teachers of mathematics, because it is extremely important that students become introduced into complex and abstract mathematical system of linear algebra and learn concepts which can be successfully applied later in other mathematical topics. It is necessary that teachers better understand how students learn, and recognize and allow that the appropriate content, methods and context could be different in different environments. As mathematicians, we are aware of the significant interconnections of different ideas and concepts, which is difficult to recognize and understand. We should not forget that understanding of these kinds of interconnections develops through active and hard exploration of mathematical topics through permanent discovering of new interconnections and relations.

Thus, primary role of a teacher is to try to move students to take an active part during the class concerning important and difficult concepts, either through the form of individual opinion or through the form of group discussions. It is not easy to suggest teaching methods, especially in comparison to traditional lectures, which would be effective and would actively, engage students and generate stimulating learning.

**Key words:** Linear algebra, teachers of mathematics, traditional lectures, qualitative

---

## 1. Introduction

Linear algebra is the study of vectors and linear functions. Linear algebra has in recent years become an essential part of the mathematical background required by mathematician and mathematics teachers, engineers, computer scientists, physicists, economists, and statisticians, among others. The requirement reflects the importance and wide applications of the subject matter. Many difficult problems can be handled easily once relevant information is organized in a certain way. This paper aims to teach how to organize information in cases where certain mathematical structures are present. Linear algebra is, in general, the study of those structures.

## 2. Organizing Information

Functions of several variables are often presented in one line such as  $f(x, y) = 2x + 7y$ .

But let us think carefully; what is the left hand side of this equation doing? Functions and equations are different mathematical objects so why is equal sign necessary? If some says "Consider the function of two variables  $5\alpha - 7\beta$ ." We do not quite have all the information we need to determine the relationship between inputs and outputs. Example of organizing and reorganizing information is as follows:

You stock in three banks: Himalayan bank, Everest bank and Nabil bank. The value  $V$  of stock portfolio  $S_H, S_E, S_N$  of these banks is

$$24S_H + 80S_E + 35S_N.$$

Here is an ill-posed question: what is  $V \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ?

The column three numbers is ambiguous! It is meant to denote

1 share of H, 2 shares of E and 3 shares of N?

1 share of E, 2 shares of H and 3 shares of N?

Do we multiplying the first number of the input by 24 or by 35? No one has specified an order for the variables, so we do not know how to calculate an output associated with a particular input. A different notation for  $V$  can clear this up; we can denote  $V$  itself as an ordered triple of numbers that reminds us what to do each number from the input. Denote  $V$  by and thus we write  $(24 \ 80 \ 35)$  and thus we write

$V \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}_{\beta} = (24 \ 80 \ 35) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  to remind us to calculate  $24(1) + 80(2) + 35(3) = 289$  because we

choose the order  $(H \ E \ N)$  and named that order  $\beta$ , so that inputs are interpreted as

$$\begin{pmatrix} S_H \\ S_E \\ S_N \end{pmatrix}$$

If we change the order for the variables, we should change the notation for  $V$ .

Denote  $V$  by  $(35 \ 80 \ 24)$  and thus we write

$V \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}_{\beta'} = (35 \ 80 \ 24) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  to remind us to calculate  $35(1) + 80(2) + 24(3) = 264$  because we

choose the order  $(N \ E \ H)$  and named that order  $\beta'$ , so that inputs are interpreted as

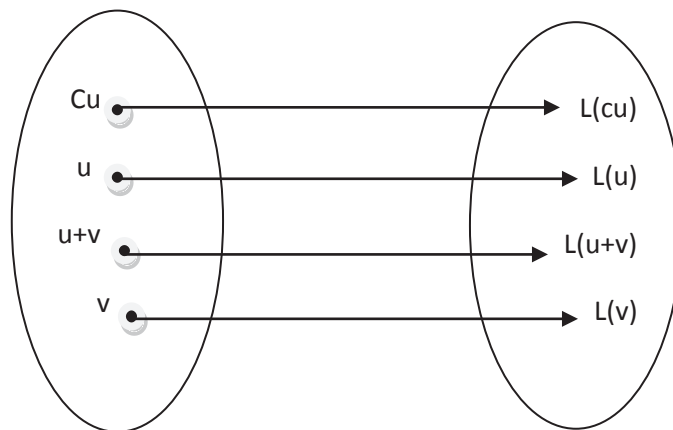
$$\begin{pmatrix} S_H \\ S_E \\ S_N \end{pmatrix}$$

The subscripts  $\beta$  and  $\beta'$  on the columns of numbers are just symbol reminding us how to interpret the column of numbers. But the distinction is critical; as shown above  $V$  assign completely different numbers to the same columns of numbers with different subscripts. There are six different ways to order the three banks. [3] Each way will give different notation for the same function  $V$ , and a different way of assigning numbers to columns of three numbers. Thus, it is critical to make clear which ordering is used if we are to understand what is written. Doing so is a way of organizing information. This is a hint at an idea of basis.

### 2.1. Linear Functions

Let us use the letter  $L$  to denote an arbitrary linear function and think about vector addition and scalar multiplication. Also, suppose that  $v$  and  $u$  are vectors and  $c$  is a number. Since  $L$  is function from vectors to vectors, if we input  $u$  into  $L$ , the output  $L(u)$  will also be some sort of vector. The same goes for  $L(v)$ . Because vectors are things that can be added and scalars multiplied,  $u + v$  and  $cu$  are also vectors, and so they can be used as inputs. The essential characteristic of linear functions is what can be said about  $L(u + v)$  and  $L(cu)$  and  $L(u)$  and  $L(v)$  [2].

Before we tell this essential characteristic, ruminant on this picture



**Fig. 1: Linear Transformation**

The 'blob' on the left represents all the vectors that we are allowed to input into the function  $L$ , the blob on the right denotes the possible outputs and the line tell us which inputs are turned into which outputs. Now we think about adding  $L(u)$  and  $L(v)$  to get another vector  $L(u) + L(v)$  or of multiplying  $L(u)$  by  $c$  to obtain the vector  $cL(u)$ , and placing both on the right blob of the picture above.

Are we certain that these are possible outputs? Here is the answer:

Additivity:  $L(u + v) = L(u) + L(v)$

Homogeneity:  $L(cu) = cL(u)$  [1]

Most functions of vectors do not obey this requirement. For example if  $f(x) = x^2$ , then

$f(1+1) = f(2) = 4$ , but  $f(1) + f(1) = 1^2 + 1^2 = 2$ . So  $f(1 + 1) \neq f(1) + f(1)$ .

When a function of vectors obeys the additivity and homogeneity properties we say that it linear, this is the linear of linear algebra. Together, additivity and homogeneity are called linearity. Are there other, equivalent, names for linear functions? Yes. They are linear operator, homomorphism, linear map, and linear transformation. So, function = transformation = operator.

### 3. Conclusion

The essence of teaching is to help students to learn the material that they need and want to learn since different students want to learn different material in different way, we should expect to change the way of our teaching. The integration of ICTs and theoretical mathematics is natural in linear algebra, so that students can use their experience with linear algebra as a starting point for seeking similar integration in other mathematical areas and understanding of mathematics. Students have learned new techniques and have been able to model and evaluate a situation that was challenging, interesting, and real. Technology brings to students and teachers the opportunity to individualize learning to generate illustrative examples, to follow interesting topics to the desired depth, to choose their own problems and appropriate tools for solving them.

### References

- [1] Cheney W and Kincaid D (2010), *Linear Algebra, Theory and Applications*, Jones and Bartlett Publishers.
- [2] Ronlanson D and Falvo C (2008), *Elementary Linear Algebra*, Houghton Mifflin Harcourt Publishing Company.
- [3] Sepideh JH and Thomas SM (2016), Teaching Mathematics and its Applications, *An International Journal of the IMA*, **35(4)**: 216 – 235.