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Evidences of Hierarchy of Brahmi Numeral System

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Abstract: The numeral system developed in South Asian Subcontinent in third century B. C. E. as the ancestor of the Hindu Arabic, Lichhavi, Kharosthi and other different numeral systems is popular by Brahmi numeral system. Ashoka prepared the pillar to preserve the Brahmi inscription with consisting numerals. The Brahmi numerical symbols are found at Lumbini of Nepal, for example a symbol used there tells the division by eight (Athabhagiya) and conversely multiplication of eight. Ashoka pillar with different inscriptions were found at Bihar, Uttarpradesh, Delhi, Madhyapradesh of India and different parts of Nepal like Niglihawa and Lumbini. In this system numerals are written from left to right. This system was very popular in South Asian Subcontinent for a long time and it impacts to the development of other numeral systems. The aim of this paper is to explore the hierchy and the existence of symbols of Brahmi numeral on the basis of document analysis and symbols found at different manuscript and monuments.

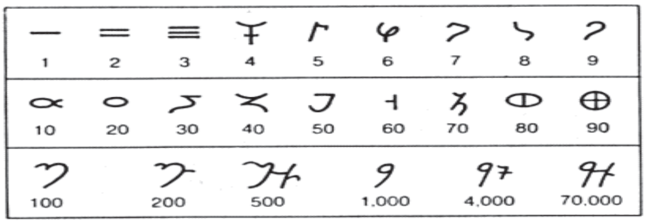
Key words: Brahmi numeral, hierachy, Ashoka, Athabhagiya, ligatures

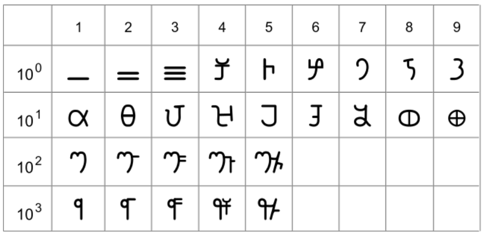
1. Introduction

Numeral systems are the fundamental elements of development of every branch of mathematics. Every numeral system impacts to each phases of the development of mathematics in every civilizations. The Brahmi numerals, appear on inscriptions dating back to third century B.C. E. It was the earliest than Hindu Arabic numeral system [4]. The pillars prepared by Ashoka (300 B. C. E.) was the most reliable source for Brahmi and other inscriptions [8]. In Brahmi numeral system not only the symbols for 1 through 9 but also the symbols for 10, 20, 30,… 90, 100, 200 and so on. In this context, it would be worthy to introduce few words about Hindu Arabic numeral system. The ingenious method of expressing every possible number using a set of ten symbols emerged in Bharatavarsha is popularised as Hindu Arabic numeral system. The idea seems in Hindu Arabic numeral system so simple and nowadays its significance and profound importance is appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful system on the basis of its ancestor 'Brahmi Numeral System'.

The positional notation was not known on Brahmi numeral system. There were no symbol for a zero in Brahmi numeral system but due to lack of the symbol of zero there was no any huddle in this system [10]. The Brahmi numerals looks like as following table, even though it were followed many phases for development [15].

Table 1 : Symbols used in Brahmi Numeral System





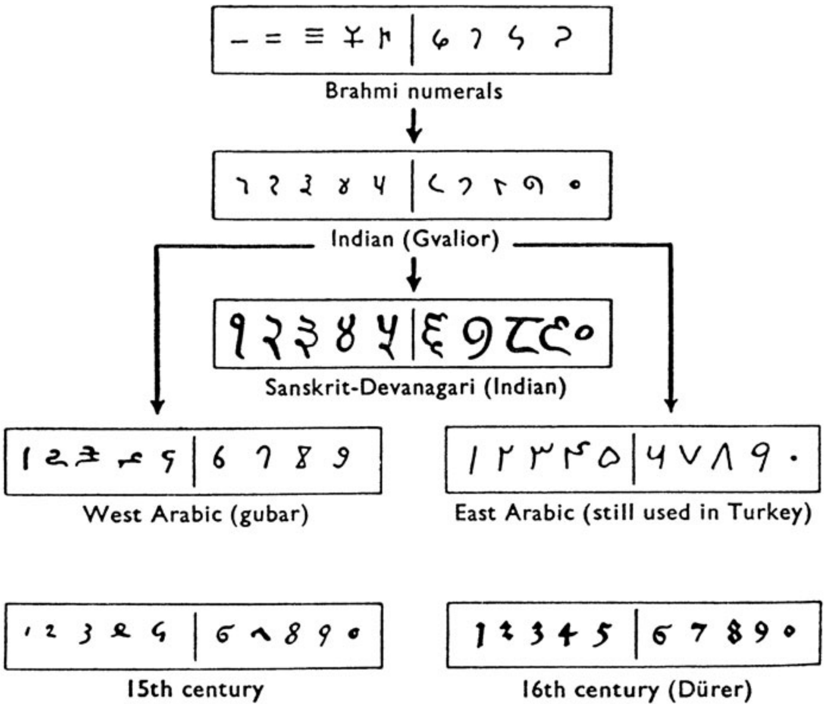
The Brahmi numeral system was practiced before the development of Hindu Arabic numeral system. Around 500-700 A. D., the Hindu Arabic numeral system was developed with 10 symbols including a zero then Brahmi numeral system was replaced, even though it is the as the ancestor of most of all the numerals in the world. Brahmi numeral system's numeric symbols were found as the first written mathematical documents on Ashoka pillar at Lumbini, Niglihawa of Nepal and Delhi, Uttar Pradesh, Madhya Pradesh and Bihar of India. Ashoka inscription in Brahmi is dated to 232 B. C. E. India and Nepal is known to have many ancient stone inscriptions written in Brahmi script. One of them is the Ashoka Pillar (249 B. C. E.). In a pillar of Ashoka at Lumbini, there were found the word numeral "*Atha Bhagiya*". This word "*Atha Bhagiya*" is used for the division by eight. In 1937 James Prinsep read Brahmi inscription and Georg Bühler prepared a complete *Varnamala Chart* (Appendix-A) of Brahmi inscription [12]. A survey report found that 198 scripts were derived from Brahmi script. In context of the development of numerals and mathematics, the South Asian Subcontinent is very popular. The exact location cannot declared but about the South Asian Subcontinent and its cultural/lingual development, 'Kim Plofker' described as,

The geographical locus of classical Indian culture is the South Asian Subcontinent, encompassing most of the modern nations of India, Pakistan, Nepal, Bangladesh, and Sri Lanka. Evidence concerning the historical roots of this culture is quite sparse. The earliest known texts in an Indian language are the collections of religious hymns and rituals called the Vedas, composed in an archaic form of Sanskrit known as Vedic Sanskrit, or Old Indo-Aryan [11]. Around that territory Brahmi numerals were developed.

2. Development and Hierarchy of the Brahmi Numeral System

Nearly 1500 B. C. E. the Sindhu inscription was disappeared then after there were appeared two inscription systems in 3rd century B. C. E. These two new systems were Brahmi inscription and Kharosthi inscription system. In these two systems, the inscription system which is popular in India and Nepal was recognized by the Brahmi inscription and another was popular in some district of Pakistan by the Kharosthi inscription system [12]. Both of these two inscription systems were popular in Bharatavarsha that is in South Asian Subcontinent. Bharatavarsha is popular for development of Hindu numeral system with zero, Kharosthi and Brahmi numeral systems. The development of zero becomes a manifesto of mathematics. David Burton described the different phases of changes of Brahmi numerals to become Hindu Arabic numerals and other systems [5, 6]. J. Katz has also illustrated the changes of Brahmi numeral system to form other numeral systems. It is illustrated in the following table.

Table 2: Brahmi Numeral System and its Successor [9]



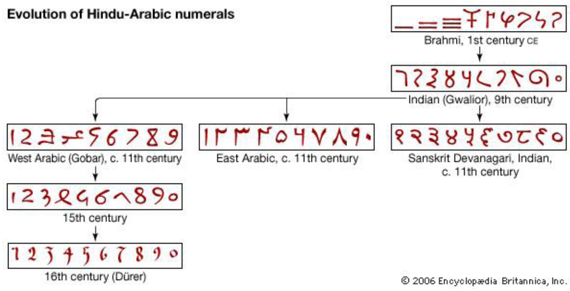
Roger Cooke expressed hierarchy of Brahmi numerals as Katz's hierarchical development of numeral system in South Asian Subcontinent as shown below from top to bottom [7].

Table 3: The Hierarchy of Development of Numeral System

Brahmi Numeral System
Indian (Gwalior)
Sanskrit (Devanagari-Indian)
West Arabic Gobar and East Arabic Gobar
11^{th} centuries (A Palaces)
15^{th} century and 16^{th} century (Durer)

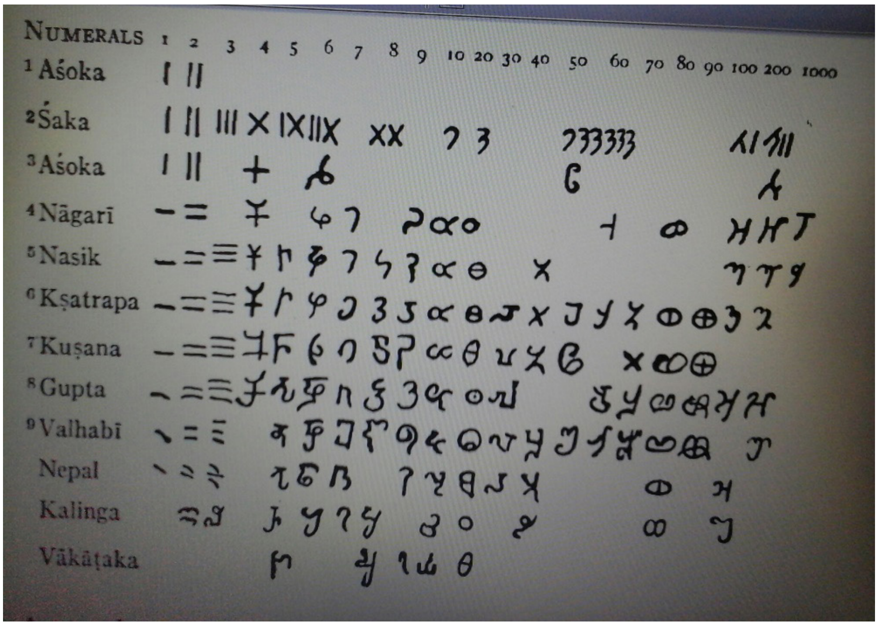
The following table also shows the changes of numerals systems from Brahmi system. These numerals were found in the region as described above which follow the law of evolution in the development of numeral systems that varies into different classical numeral system as antique form [1].

Table 4 : Development of different numerals from Brahmi Numeral System [16]



David Eugene Smith has given emphasis to 'Nepal which can be seen from the following table. He focused the variants of the Hindu forms of numerals in the invention of the zero.

Table 5 : Variant of Numerals from Brahmi Numeral System [13]



Almost at the same time, there existed a well-organized set of decimal based numeral system. This system is now known as the Lichhavi numeral system. Inscription on the base of the Jaya Varma-statue at Mali village, Kathmandu of Nepal is illustrated below. The symbols used in this stone states Saka Samvats 107. This would be A. D. 185/186 [2]. It is related to Brahmi numeral system.

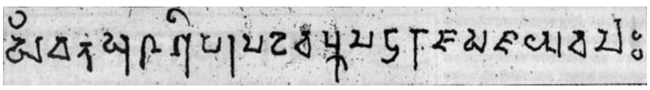
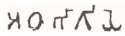


Fig. 1: A glimpse of Lichhavi Numeral [2, 3, 12, 14]

Fig. 2: Brahmi Numeral on Ashoka Pillar at Lumbini [12, 14]

The symbols (*Atha Bhagiya*: 1/8)  is found on Ashoka pillar at Lumbini. This gives the concept of division by 8 and conversely multiple of 8. The different symbols were used to denote numbers. They practiced mathematics in different bases like base 10 and 20. They also used the word '*Wisati* (20)'.

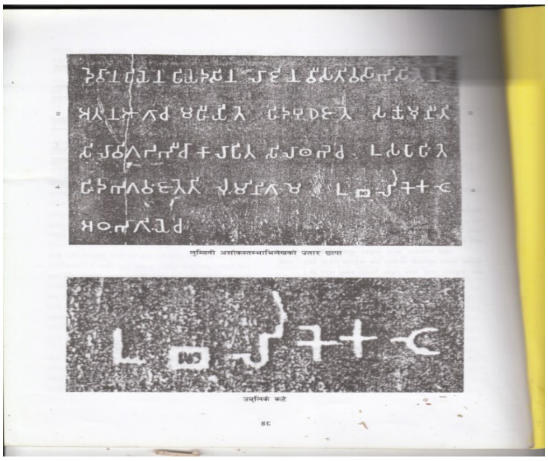


Fig. 3: Illustration of Some Brahmi Numerals [2, 12, 14]

3. Characteristics of Brahmi Numerals

On the basis of the observations to the above evidences of Brahmi numeral system, it is the strong one among the many numeral systems. It is as the milestone for the development of numeral systems in different civilizations. A peculiar property of Brahmi numeral system is using of ligatures. The main characters of Brahmi numeral system are as listed below.

i. Brahmi numerals system is an indigenous South Asian Subcontinental numeral system attested from the 3rd century B. C. E. It is an ancient system for writing numerals and are the direct graphic ancestors of the modern Indian and Hindu-Arabic numerals.

ii. They were conceptually as the root for the other systems, they were not used as a positional system with a zero. There were separate numerals for each of the tens (10, 20, 30, etc.). There were also symbols for 100 and 1000 which were combined in ligatures with the units to signify 200, 300, 2000, 3000, etc.

iii. The source of the first three numerals seems clear; they are collections of 1, 2, and 3 strokes, in Ashoka's era (300 B. C. E.) vertical I, II, III like Roman numerals, but soon becoming horizontal like the modern Chinese numerals. In the oldest inscriptions, 4 is a (+), reminiscent of the X of neighbouring Kharosthi, and perhaps a representation of 4 lines or 4 directions.

iv. Sometimes it come from collections of strokes, run together in cursive writing in a way similar to that attested in the development of Egyptian Hieratic and Demotic numerals and the system resembled that of the Greeks [4], but this is not supported by any direct evidence.

4. Conclusion

Brahmi numerals system is an indigenous mathematical development of South Asian Subcontinent. This numeral system was appeared in the 3rd century B. C. E., which was popular in South Asian Subcontinent. Many evidences supported that it is as the ancestor of other numeral systems. Ashoka pillar at Lumbini, Niglihawa of Nepal and Bihar, Madhya Pradesh, Uttar Pradesh and Delhi of India that supports the illustration of Brahmi numeral system and its highest hierarchical position in the development of numeral systems. Using Ligatures to represent the larger numbers like 100, 200, 1000, 2000, etc. is the important feature of Brahmi numeral system.

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Appendix-A

https://commons.wikimedia.org/wiki: Brahmi evolution from 250 BCE to 800 CE.jpg