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DIFFERENCE SEQUENCE SPACES AND MATRIX TRANSFORMATIONS

Geeta Bhakta Joshi, Ph.D.
Prof. Mathematics Department, Pulchowk Campus, Institute of Engineering, Tribhuvan University

Shailendra Kumar Mishra, Ph.D.
Prof. Mathematics Department, Pulchowk Campus, Institute of Engineering, Tribhuvan University

Shankar Rajpant, Ph.D.
Ex-Chairman, Prof. Central Department of Mathematics, Tribhuvan University, Kirtipur

Abstract: In this paper, we extend the work of Gaur, A.K. and Mursaleen [6] and also extend the work of Mursaleen, Gaur, A.K. and Saifi, A.H.[16]. We characterize the matrices that map $S_r(p,\Delta)$, $\Delta\lambda_\infty(p)$, $\Delta c_0(p)$, $\Delta c(p)$ and $\lambda_\infty(\Delta_r p)$ into $\Omega(t)$.

1. INTRODUCTION

Let λ_∞ , c and c_0 be the sets of all bounded, convergent and null sequences of $x=(x_k)$ respectively. Let ω denote the set of all complex sequences and let λ_1 denote the set of all convergent and absolutely convergent series.

If $p=(p_k)$ is a bounded sequence of strictly positive real numbers, and if $\Delta x=(x_k - x_{k-1})$, then we have

$$\lambda(p) = \left\{ x=(x_k) : \sum_k |x_k|^{p_k} < \infty \right\} ;$$

$$\lambda_\infty(p)=\left\{ x=(x_k)\in \omega : \sup_k |x_k|^{p_k} <\infty \right\} ;$$

$$\Delta\lambda_\infty(p)=\{x=(x_k) : \Delta x\in\lambda_\infty(p)\} ;$$

$$\Delta c(p)=\{ x=(x_k) :\Delta x\in c(p) \} ;$$

$$\Delta c_0(p)=\{ x=(x_k) : \Delta x\in c_0(p)\}$$

If all the terms of $p=(p_k)$ are constant and $p>0$, then $\Delta\lambda_\infty(p)=\Delta\lambda_\infty$, $\Delta c(p)=\Delta c$ and $\Delta c_0(p)=\Delta c_0$. The classes $\Delta\lambda_\infty$, Δc, Δc_0 are normed spaces under the norm

$$\| x\| = \| \Delta x\|_\infty$$

where $\|.\|_\infty$ is the usual norm on λ_∞, c or c_0. It is known that if $(p_k)\in\lambda_\infty$ then $\Delta c_0(p)$ is a paranormed space paranormed by $g^*(x)=g(\Delta x)$; $\Delta\lambda_\infty(p)$ and $\Delta c(p)$ are paranormed by $g^*(x)=g(\Delta x)$ if and only if $\inf p_k>0$, where g is the usual paranorm on $\lambda_\infty(p)$, $c(p)$ and $c_0(p)$.

Let z be any sequence and Y be any subset of ω. Then

$$z^{-1}.Y=\{x\in \omega : zx=(z_k x_k)_1^\infty \in Y\}$$

For any subset X of ω, the sets

$$X^\alpha = \bigcap_{x\in X} (x^{-1}.\lambda_1) \text{ and } X^\beta = \bigcap_{x\in X} (x^{-1}.cs)$$

are called the α^- and β^- duals of X .

We define the linear operators $\Delta, \Delta^{-1} : \omega \to \omega$ by

$$\Delta x = (\Delta x_k)_1^\infty = (x_k - x_{k+1})_1^\infty ,$$

and

$$\Delta^{-1} x = (\Delta^{-1} x_k)_1^\infty = \left(\sum_{j=1}^{k-1} x_j \right)_1^\infty ,$$

$$\Delta^{-1} x = 0 .$$

Let,

$$S_r(\Delta) = \left\{ x \in \omega : \left(k^r \left| \Delta x_k \right|\right)_{k=1}^\infty \in c_0 \right\}$$

Let $p = (p_k)_1^\infty$ be an arbitrary sequence of positive reals and $r \geq 1$, then Gaur, A.K. and Mursaleen [6] have defined a new sequence space

$$S_r(p, \Delta) = \left\{ x \in \omega : \left(k^r \Delta x_k\right)_{k=1}^\infty \in c_0(p) \right\}$$

where

$$c_0(p) = \left\{ x = (x_k) \in \omega : \lim_{k \to \infty} \left| x_k \right|^{p_k} = 0 \right\}$$

If $p = e = (1,1,1,\ldots)$, then the set $S_r(p, \Delta)$ reduces to the set $S_r(\Delta)$. For $r = 0$, $S_r(p, \Delta)$ is the same as $\Delta c_0(p)$. In [16] Mursaleen, Gaur, A.K. and Saif, A.H. has defined

$$\lambda_\infty(\Delta_r p) = \{x = (x_k) : \Delta_r x \in \lambda_\infty(p),\ r < 1\}$$

where

$$\Delta_r x = \left(k^r \, \Delta x_k\right)_1^\infty .$$

1.2 MATRIX TRANSFORMATIONS

For any infinite complex matrix $A = (a_{nk})_{n,k=1}^\infty$, we write $A = (a_{nk})$ for the sequence in the n^{th} row of c. Let X and Y be two subsets of ω. By (X, Y) we denote the class of all matrices of A such that the series $A_n(x) = \sum_{k=1}^\infty a_{nx} x_k$ converges for all $x \in X$ and $n \in N$, and the sequence $Ax = (A_n(x))_{n=1}^\infty \in Y$ for all $x \in X$.

Fricke and Fridy [8] introduced a new sequence space $\Omega(t)$. We shall give the definition of $\Omega(t)$ and some results from [8].

For each $r = (r^k)$ in the interval $(0,1)$ let

$$G(r) = \{x = (x_k) \in \omega : x_k = O(t_k)\}.$$

We define the set of geometrically sequences as

$$G = \bigcup_{r \in (0,1)} G(r)$$

The analytic sequences are defined by

$$A = \left\{ x = (x_k) \in \omega : \limsup_n \left| x_n \right|^{\frac{1}{n}} < \infty \right\}$$

Obviously $G \subseteq A$

In [8] , Fricke and Fridy replaced the geometric sequence (r^k) with a non-negative number sequence *t* and defined.

$$\Omega(t) = \{x = (x_k) \in \omega : x_k = O(t_k)\}.$$

Here for a given matrix A , the sequence $\sigma = (\sigma_n)_{n=1}^\infty$ is defined by

$$\sigma_n = \sum_{k=0}^\infty \left| a_{nk} \right|$$

Corollary 1.2.1. (see [8], Corollary 2B) : If A is an infinite matrix and *t* is nonnegative number sequence, then A maps λ_∞, c, c_0 into $\Omega(t)$ if and only if $\sigma \in \Omega(t)$.

Corollary 1.2.2. (see [16], Theorem 2.1. and Theorem 3.1.): $A \in (X, Y(\Delta_r))$ if and only if

(i) $A_1 \in X^\beta$ where,

$$X^\beta = D_2(p)$$

$$= \bigcap_{N \geq 2} \left\{ a \in \omega : \sum_{k=1}^\infty k^{-r} a_k \sum_{j=1}^{k-1} N^{\frac{1}{p}} \text{ converges } , \sum_{k=1}^\infty k^{-r} N^{\frac{1}{p_k}} \left| R_k \right| < \infty \right\},$$

$$R_k = \sum_{m=k+1}^\infty a_m$$

(ii) $B \in (X, Y)$,

where $B = (b_{nk})$ is defined by b_{nk} $= k^r (a_{nk} - a_{n+1,k})$ for $r < 1$ and $n, k = 1,2,\ldots$

Remark 1.2.1. If one wishes to have a matrix *A* that transforms every null sequence into a sequence that conveges at least as rapidly as some $t_n \downarrow 0$, thus *A* must satisfy $\sigma \in \Omega(t)$. Similarly, if t is a nonzero constant sequence, then $\Omega(t) = \lambda_\infty$, and in this case Corollary 1.2.1.reduces to the well known result that *A* preserves boundedness if and only if σ is bounded.

Remark 1.2.2. This remark is about obtaining a "given rate of convergence" by mapping c_0 into $\Omega(t)$. The work [4,9] has shown that regular matrices can not accelerate the rate of convergence of every null sequences. Therefore we emphasize that having *A* map c_0 into $\Omega(t)$ does not say that every sequence in c_0 is accelerated, even if $t_n \downarrow 0$ vary rapidly ; some sequences that are already in $\Omega(t)$ may map into other members of $\Omega(t)$ that converge at same rate or slower.

Now we characterize the matrices that map $S_r(p,\Delta)$, $\Delta\lambda_\infty(p)$, $\Delta c_0(p)$, $\Delta c(p)$ and $\lambda_\infty(\Delta_r p)$ into $\Omega(t)$.

Theorem 1.2.1. $A \in \left(S_r(p,\Delta), \Omega(t) \right)$ if and only if

(i) $\left(\sum_{k=1}^{\infty} | a_{nk} | \frac{N^{\frac{-1}{p_k}}}{k^r} \right) \in \Omega(t)$ for some integer $N > 1$.

(ii) $R \in \left(S_r(p,\Delta), \Omega(t) \right)$

where, $R = (r_{nk}) = \left(\sum_{\nu=k}^{\infty} a_{n\nu} \right)$.

Proof. If $A \in \left(S_r(p,\Delta), \Omega(t) \right)$ then the series $\sum_{k=1}^{\infty} a_{nk} x_k$ is convergent and $\left(A_n(x) \right)_{n=1}^{\infty} \in \Omega(t)$ for each $n \in N$ and $x \in \left(S_r(p,\Delta) \right)$.

In order to see that the condition (i) is necessary, we assume that for some $N > 1$,

$$\left(\sum_{k=1}^{\infty} | a_{nk} | \frac{N^{\frac{-1}{p_k}}}{k^r} \right) \notin \Omega(t)$$

Let the matrix C be defined by

$$\left(C = \left(C_{nk} \right) = \left(a_{nk} \frac{N^{\frac{-1}{p_k}}}{k^r} \right) \right)$$

Then from corollary 1.2.1 , it follows that $C \notin \left(c_0, \Omega(t) \right)$. But as, $S_r(p,\Delta) = \left\{ x \in \omega : \left(k^r \Delta x_k \right)_1^\infty \in c_0(p) \right\}$, $C \notin \left(S_r(p,\Delta), \Omega(t) \right)$. Hence there is a sequence $x \in c_0$ such that $\sum_{k=1}^{\infty} C_{nk} x_k \neq O(1)$.

We now define a sequence $v = (v_k)$ by

$$v_k = \frac{N^{\frac{-1}{p_k}}}{k^r} x_k \ ;$$

so that $v_k \left(k^r N^{\frac{1}{p_k}} \right) = x_k$. Then $v \in S_r(p,\Delta)$ and

$$\sum_{k=1}^{\infty} a_{nk} v_k = \sum_{k=1}^{\infty} C_{nk} x_k \neq O(1)$$

This contradicts that

$A \in \left(S_r(p,\Delta), \Omega(t) \right)$.

Thus the condition (i) is necessary.

In order to prove that the condition (ii) is necessary we assume that (ii) is false. Then there is a sequence $x = (x_\nu) \in S_r(p,\Delta)$ with $\left| k^r \Delta x_k \right| = 1$ such that

$$\sum_{\nu=1}^{\infty} r_{n\nu} x_\nu \neq O(1)$$

We now define a sequence $y = (y_k)$ by

$$y_v = \sum_{i=1}^{v} x_i$$

Then $y \in S_r(p, \Delta)$ and

$$\sum_{v=1}^{\infty} a_{nv} y_v = \sum_{v=1}^{\infty} r_{nv} x_v \neq O(1)$$

This contradicts the fact that $A \in (S_r(p,\Delta), \Omega(t))$. Thus the condition (ii) is necessary.

We now prove the sufficiency part of the theorem. Suppose that the given condition of the theorem is satisfied. Then there exists a $\mu > 0$ such that

$$\left(\sum_{k=1}^{\infty} | a_{nk} | \frac{N^{\frac{-1}{p_k}}}{k^r} \right) \leq \mu t^n \text{, for each } n \in N .$$

Let $x \in S_r(p, \Delta)$. Then $\left(k^r |\Delta x_k| \right) < \frac{1}{N^{p_k}}$, for sufficiently large value of *k*.. Now we write,

$$A_n(m,x) = \sum_{k=1}^{\infty} a_{nk} x_k = \sum_{k=1}^{m} r_{nk} \Delta x_k - r_{n+1,m} \sum_{k=1}^{m} \Delta x_k, m \in N$$

,

Since,

$$\sum_{k=1}^{\infty} | r_{nk} | \, |\Delta x_k| \leq \sum_{k=1}^{\infty} | r_{nk} | \frac{1}{k^r N^{\frac{1}{p_k}}} \quad \text{, where}$$

$$\sum_{k=1}^{\infty} | r_{nk} | \frac{1}{k^r N^{\frac{1}{p_k}}} \in \Omega(t)$$

$\leq \mu t^n$, for each $n \in N$.

Therefore the convergence of

$$\sum_{k=1}^{\infty} | a_{nk} | \frac{1}{k^r N^{\frac{1}{p_k}}}$$

implies that

$$r_{n+1,m} \sum_{k=1}^{m} \frac{1}{k^r N^{\frac{1}{p_k}}} = o(1)$$

Hence,

$$A_n(x) = \sum_{k=1}^{\infty} a_{nk} x_k = \sum_{k=1}^{\infty} r_{nk} \Delta x_k$$

Since $x \in S_r(p, \Delta)$ if and only if $\Delta_r x \in c_0(p)$, where $\Delta_r x = (k^r \Delta x_k)$. Therefore by condition (ii) it follows that $A_n(x)$ exists for each $x \in S_r(p, \Delta)$ and $Ax \in \Omega(t)$. Thus $A \in (S_r(p,\Delta), \Omega(t))$.

Theorem 1.2.2. $A \in (\Delta\lambda_\infty(p), \Omega(t))$ if and only if

(i) $$\left(A_n \left(\sum_{m=1}^{k} N^{\frac{1}{p_m}} \right) \right)_{n=1}^{\infty} \in c, \; N > 1 \; ;$$

(ii) $R \in (\lambda_\infty(p), \Omega(t))$, where $R = (r_{nk}) = \left(\sum_{v=k}^{\infty} a_{nv} \right)$.

Proof. If $A \in (\Delta\lambda_\infty(p), \Omega(t))$ then the series $\sum_{k=1}^{\infty} a_{nk} x_k$ is convergent and $A_n(x) \in \Omega(t)$ for some $n \in N$ and $x \in \Delta\lambda_\infty(p)$. Since,

$$x = \left(\sum_{m=1}^{k} N^{\frac{1}{p_m}} \right)_{k=1}^{\infty} \in \Delta\lambda_\infty(p)$$

Then it follows that

$$\sum_{k=1}^{\infty} a_{nk} \left(\sum_{m=1}^{k} N^{\frac{1}{p_m}} \right)$$

converges for each $n \in N$. Therefore (i) is necessary.

In order to see that the condition (ii) is necessary, we assume that (ii) is false. Then there is a sequence $x=(x_v)\in \lambda_\infty(p)$ with $\sup_k |x_v|^{p_k}=1$ such that

$$\sum_{v=1}^{\infty} r_{nv} x_v \neq O(1) .$$

We now define a sequence $y=(y_v)$ by

$$y_v=\sum_{i=1}^{v} x_i .$$

Then $y\in \Delta\lambda_\infty(p)$ and $\sum_{v=1}^{\infty} a_{nv} y_v = \sum_{v=1}^{\infty} r_{nv} x_v \neq O(1)$. This contradicts that $A\in (\Delta\lambda_\infty(p),\Omega(t))$. Thus the condition (ii) is also necessary.

We now prove the sufficiency part of the theorem. Suppose that the given conditions of the theorem are satisfied. Let $x\in \Delta\lambda_\infty(p)$. Then there is an integer $N>\max\left(1,\sup_k |\Delta x_k|^{p_k}\right)$.

Now we write ,

$$A_n(m,x)=\sum_{k=1}^{\infty} a_{nk} x_k = \sum_{k=1}^{m} r_{nk}\,\Delta x_k - r_{n+1,m}\sum_{k=1}^{m} \Delta x_k ,\ m\in N$$

Since,

$$\sum_{k=1}^{\infty} |r_{nk}|\,|\Delta x_k| \le \sum_{k=1}^{\infty} |r_{nk}|\, N^{\frac{1}{p_k}}$$

and

$$\left(\sum_{k=1}^{\infty} |r_{nk}|\, N^{\frac{1}{p_k}}\right)_{n=1}^{\infty} \in \Omega(t)$$

Therefore the convergence of

$$\sum_{k=1}^{\infty} a_{nk}\left(\sum_{i=1}^{k} N^{\frac{1}{p_i}}\right)$$

implies that

$$r_{n+1,m}\sum_{k=1}^{m} N^{\frac{1}{p_k}} = o(1)$$

Hence,

$$A_n(x)=\sum_{k=1}^{\infty} a_{nk} x_k = \sum_{k=1}^{\infty} r_{nk}\,\Delta x_k$$

Since, $x\in \Delta\lambda_\infty(p)$ if and only if $\Delta x\in \lambda_\infty(p)$. Therefore by condition (ii) it follows that $A_n(x)$ exists for each $x\in \Delta\lambda_\infty(p)$ and $Ax\in \Omega(t)$. Thus, $A\in (\Delta\lambda_\infty(p),\Omega(t))$.

Theorem 1.2.3. Let $(p_k)\in \lambda_\infty$. Then $A\in (\Delta c_0(p),\Omega(t))$ if and only if

(i) $$\left(A_n\left(\sum_{m=1}^{k} N^{\frac{-1}{p_m}}\right)\right)_{n=1}^{\infty} \in c,\quad N>1;$$

(ii) $R\in (c_0(p),\Omega(t))$ with R as above.

This follows from the arguments given in Theorem (1.2.2) and [pp.80, 13].

Theorem 1.2.4. Let $(p_k)\in\lambda_\infty$. Then $A\in (\Delta c(p),\Omega(t))$ if and only if

(i) $A\in (\Delta c_0(p),\Omega(t))$;

(ii) $$\left(\sum_{k=1}^{\infty} k a_{nk}\right)_{n=1}^{\infty} \in \Omega(t)$$

This follows from Theorem (1.2.3) and [pp.80, 15].

Theorem 1.2.5. $A\in (\lambda_\infty(\Delta_r p),\Omega(t))$ if and only if

(i) $$\left(\sum_{k=1}^{\infty} |a_{nk}|\, k^{-r} N^{\frac{1}{p_k}}\right)\in \Omega(t)$$ for every integer $N>1$.

(ii) $R\in (\lambda_\infty(\Delta_r p),\ \Omega(t))$

where $R=(r_{nk})=\left(\sum_{v=k}^{\infty} a_{nv}\right)$

Proof. Let us assume that $A\in(\lambda_\infty(\Delta_r p),\Omega(t))$ but $\left(\sum_{k=1}^{\infty}|a_{nk}|k^{-r}N^{\frac{1}{p_k}}\right)_{n=1}^{\infty}\notin\Omega(t)$ for every integer $N>1$. Then from corollary 1.2.1 and [8], it follows that the matrix

$$B=(b_{nk})=\left(a_{nk}\,k^{-r}N^{\frac{1}{p_k}}\right)\notin(\lambda_\infty(\Delta_r),\Omega(t))$$

.

Therefore there exists an $x\in\lambda_\infty(\Delta_r)$ with $\sup_k|x_k|=1$ such that

$$\sum_{k=1}^{\infty}a_{nk}k^{-r}N^{\frac{1}{p_k}}x_k\neq O(1).$$

Now define a sequence $u=(u_k)$ by

$$u_k=\sum_{i=1}^{k}k^{-r}N^{\frac{1}{p_i}}x_i\ .$$

It is clear that $u\in\lambda_\infty(\Delta_r p)$ and

$$\sum_{k=1}^{\infty}a_{nk}u_k=\sum_{k=1}^{\infty}a_{nk}k^{-r}N^{\frac{1}{p_k}}x_k\neq O(1).$$

This contradicts the fact that $A\in(\lambda_\infty(\Delta_r p),\Omega(t))$.

Hence, we must have ,

$$\left(\sum_{k=1}^{\infty}|a_{nk}|k^{-r}N^{\frac{1}{p_k}}\right)_{n=1}^{\infty}\in\Omega(t).$$

In order to see that the condition (ii) is necessary let us assume that (ii) is false. Then there exists a sequence $x=(x_v)\in\lambda_\infty(\Delta_r p)$ with

$$\sup_v|x_v|^{p_v}=1\quad\text{i.e.}\sup_k\left|k^r\Delta x_k\right|^{p_k}=1$$

such that

$$\sum_{v=1}^{\infty}r_{nv}x_v\neq O(1)\ .$$

We now define a sequence $y=(y_v)$ by

$$y_v=\sum_{i=1}^{v}x_i\ .$$

Then $y\in\lambda_\infty(\Delta_r p)$ and

$$\sum_{v=1}^{\infty}a_{nv}y_v=\sum_{v=1}^{\infty}r_{nv}x_v\neq O(1)$$

This contradicts the fact that $A\in(\lambda_\infty(\Delta_r p),\Omega(t))$. Thus the condition (ii) is necessary.

Next, suppose that the given conditions are satisfied. Then there exists a constant $M>0$ such that

$$\sum_{k=1}^{\infty}|a_{nk}|k^{-r}N^{\frac{1}{p_k}}\leq M\,t_n\text{, for each }n\in N.$$

Let $x\in\lambda_\infty(\Delta_r p)$. Then there is a positive number $N>\max\left(1,\sup_k|x_k|^{p_k}\right)$

Now we write,

$$A_n(m,x)=\sum_{k=1}^{\infty}a_{nk}x_k=\sum_{k=1}^{m}r_{nk}\,\Delta x_k-r_{n+1,m}\sum_{k=1}^{m}\Delta x_k\,,m\in N$$

,

Since,

$$\sum_{k=1}^{\infty}|r_{nk}|\,|\Delta x_k|\leq\sum_{k=1}^{\infty}|r_{nk}|k^{-r}N^{\frac{1}{p_k}}\ ,\text{ where}$$

$$\sum_{k=1}^{\infty}|r_{nk}|k^{-r}N^{\frac{1}{p_k}}\in\Omega(t)$$

$$\leq\mu\,t^n,\text{ for each }n\in N.$$

Therefore the convergence of

$$\sum_{k=1}^{\infty}|a_{nk}|k^{-r}N^{\frac{1}{p_k}}$$

implies that

$$r_{n+1,m}\sum_{k=1}^{m}k^{-r}N^{\frac{1}{p_k}}=o(1)$$

Hence,

$$A_n(x)=\sum_{k=1}^{\infty}a_{nk}x_k\ =\sum_{k=1}^{\infty}r_{nk}\ \Delta x_k$$

Since $x\in\lambda_\infty(\Delta_r p)$ if and only if $\Delta_r x\in\lambda_\infty(p)$. Therefore by condition (ii) it follows that $A_n(x)$ exists for each $x\in\lambda_\infty(\Delta_r p)$ and $A_n(x)=O(t_n)$. Then $Ax\in\Omega(t)$ for arbitrary $x\in\lambda_\infty(\Delta_r p)$. Thus $A\in(\lambda_\infty(\Delta_r p),\Omega(t))$.

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