JER
KUSOED

Journal of Education and Research
August 2013, Vol. 3, No. 2, pp. 15-38
DOI: http://dx.doi.org/10.3126/jer.v3i2.8396

Beliefs about Teaching Geometric Transformations with Geometers' Sketchpad: A Reflexive Abstraction

Shashidhar Belbase*

College of Education, University of Wyoming, Laramie, Wyoming, USA

Abstract

A teacher's belief plays a significant role in the quality of teaching mathematics. In a fictive way, I changed my role from a researcher to a research participant in an imaginative interview. My interior other (David) interviewed me as a researcher. A single interview session was held lasting for about three hours. The interview text was used for the analysis and interpretation using the grounded theory method. I invented a substantive theory of my beliefs about teaching geometric transformations with Geometer's Sketchpad. The theory is "reflexive abstraction of my beliefs about teaching Geometric Transformations with Geometer's Sketchpad" as a personal theory characterized by some basic categories of - beliefs about the advancement of pedagogy, beliefs about pedagogical environment, beliefs about the role of students' and teacher, beliefs about self as a future teacher, beliefs about teaching learning activities, and beliefs about transitions in teaching learning. I reconstructed a synthesis of the characteristics of these beliefs. While constructing these layers of interpretive accounts, I used radical constructivist grounded theory as a theoretical base.

Keywords: teaching beliefs, self-interview, radical constructivist grounded theory, invention of local theory

Introduction

James Raths and Amy McAninch's (2003) questions in relation to teachers' beliefs and their intimate connection to mathematics teacher education in general provide a context to introduce the study of my beliefs about teaching geometric transformations (GTs) with Geometer's Sketchpad (GSP). Their questions are related to the mechanism for changing teacher beliefs, consensus about better beliefs, ethical issues in relation to changing teachers'

* Author Email: belbaseshashi@gmail.com
ISSN: 2091-0118(Print)/2091-2560(Online)

beliefs, problems of changing beliefs, and issues related to studies of preservice or inservice teachers' beliefs (Raths & McAninch, 2003).

Current mathematics teacher education focuses a little on teacher beliefs as an important aspect and it aims to change their beliefs despite challenges in traditional practices of teacher education (Richardson, 2003). This indicates that teacher educators have some sense of necessity to change the preservice and inservice mathematics teachers' beliefs. Many researchers in mathematics education (e.g., Peterson, Fennema, Carpenter, & Loaf, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1992) purported to influence teachers' beliefs about the subject matter and pedagogy for change in mathematics education. These researchers emphasized the role of teacher education programs to have such influence to prospective teachers' beliefs and their subsequent practices (Fenstermacher, 1979; Green, 1971). Fenstermacher (1979) argued that the goal of teacher education is to transform teachers' beliefs about teaching and learning.

Some researchers (e.g., Lundeberg & Levin, 2003) advocated case studies, action researches, problem-based teaching and learning, and technology integration for changing the teachers' beliefs. They suggested multiple courses and efforts in teacher education for effective and lasting change in teachers' beliefs. This indicates that teacher educators have some sense of responsibility to change the preservice and inservice mathematics teachers' beliefs. Many mathematics education researchers (e.g., Fenstermacher, 1994; Pajares, 1992; Richardson, 1996) consider teacher belief as an important aspect of teacher education and it needs a change in a positive direction. They also admitted modification and formation of belief systems as an important endeavor in teacher education (Green, 1971).

For some teacher educators and researchers (e.g., Raths, 2001), teachers' beliefs and change of their beliefs in teacher education programs are related to the sense of ethics. Raths (2001) raises an important question, "What ethics are involved in making a concerted effort to change the beliefs of another person?" (Three Central Questions section, para 11). He characterized such effort as 'brainwashing'. He claimed that it is not ethical to change one's belief by another person who is in the position of power. However, the purpose of mathematics teacher education is and *should* be to develop a positive belief system among the preservice and inservice mathematics teachers to be informed practitioners of different

methods, and to discern the best possible teaching and learning, and technology integration in mathematics education.

These scenarios reflect the present context of preservice and/or inservice secondary mathematics teachers' beliefs about teaching and learning mathematics and it also relates to their beliefs about teaching GTs with GSP implicitly and other technology rich mathematics classrooms (Cavin, 2007). The development of this paper comes from the context of a broader study on preservice mathematics teachers' beliefs and hence the purpose of this paper is to report my beliefs about teaching GTs with GSP through description, analysis, and interpretation of a self-interview.

The aim of this paper is to create a basic pathway for a broader study of the preservice and/or inservice mathematics teachers' beliefs about teaching GTs with GSP. The self-reflection, that is a reflection upon my own beliefs (von Glasersfeld, 1994), and self-reflexivity, that is a way to create a trajectory of my beliefs (Steffe, 1995) on my own way of teaching GTs with GSP, are some initial steps to prepare myself to understand others' (participants') beliefs. Self-reflection is related to developing an understanding of my belief system in relation to teaching GTs with GSP. I want to go back to my belief system and see how it may influence my real and anticipated practices. Whereas, through self-reflexivity I would like to review my existing belief system in relation to teaching GTs with GSP and contemplate a more appropriate belief system that may help me to be a teacher or teacher educator with more eclecticism (i.e. an ability to draw upon multiple epistemic paradigms) and humility (i.e. an ability to be adaptive to changing contexts). To me, self-reflexivity is retrospection (i.e. looking back), introspection (i.e. looking inside) and prospection (i.e. looking further) on one's beliefs. Understanding of others' beliefs begins with understanding of one's own beliefs at the first hand and then using the lens or perspective to see the world (of others' beliefs). The scope of this paper seems narrow at this point. However, I expect that the process of writing this paper will lead me to frame a broader study. The next aim of this paper is to create an "epistemic sensibility" (Belbase, 2012b, p. 4) with an integrated approach of using constructivism and grounded theory together in a more creative, flexible and yet a feasible way of doing a qualitative research.

Constructivist Grounded Theory: A Theoretical Lens

There may be a variety of constructivism integrated with grounded theory. The variety maybe concerned with the integration of two theories ontologically, epistemologically, and methodologically. Charmaz (2006) purported a constructivist grounded theory. She focused on collection of rich data, layers of coding of the data, memo writing, theoretical sampling, and theoretical saturation in flexible, constructive, and adaptive ways. In this context, we can look at the integration of trivial, social, and radical constructivism with grounded theory to form a constructivist grounded theory. We could also integrate other forms of constructivism- sociocultural, constructionist, and critical- with grounded theory to form other variety of grounded theories.

In radical constructivist grounded theory (RCGT), the researcher invents a theory or a knowing with his or her interpretation of the concepts, categories, and theory from the data (Belbase, 2013). The data on teacher beliefs come from self-reflexivity of participants on their retrospections, introspections, prospections, and idiosyncratic beliefs in tasks, situations, and interactions within those situations. According to radical constructivism, one's knowing or theory of beliefs is actively constructed by the cognizing "subjects" (i.e. the researcher and participants); one's knowing or theory on beliefs is autopoietic (Maturana & Varela, 1980), that is self-adaptive and generative; and the quality of one's knowing or a theory of beliefs is judged through the principle of viability (von Glasersfeld, 1978). The act of constructing a local theory within this paradigm may lead to the invention of a new theory or re-invention of the existing theory in a modified version. The metaphor of construction and invention can be understood with an example of a house. A contractor can construct a house with a new design. However, he or she does not invent the design. Invention carries a deeper sense of developing a new theory than a mere construction. The metaphor of invention has been used as a softer version of creating a new local theory. It is not a hardcore scientific connotation of INVENTION of a new thing or idea. At a broader level, the local theories can be merged together to invent or form a formal theory but that is not the purpose of this study.

For this study, I would like to contemplate RCGT. The key motives for choosing RCGT for the study are based on my idiosyncratic beliefs and experiences of learning and teaching mathematics. I believe that RCGT serves my purpose of inventing or re-inventing a local theory of mathematics teachers' beliefs about teaching GTs with GSP with a deeper sense of epistemic sensibility and responsibility. RCGT seems to be a more viable form of grounded

theory than others in the sense that it allows individual inventions of grounded theory from the local contexts. The process of inventing a grounded theory is self-regulated (autopoietic), and teachers' beliefs (as a social and psychological reality) may exist, but I do not have direct access to their idiosyncratic beliefs. My invention or re-invention of the theory of their beliefs could be based on their expressed beliefs through narratives. That means I might be able to invent a local theory of their beliefs with an interpretation of their idiosyncratic beliefs through self-reflexivity.

To me, the notion of self-reflexivity maybe used to understand their beliefs through their conscious reflections and my second order interpretation of their beliefs. Here, the second order interpretation of their beliefs means my interpretation of other's beliefs based on what they (research participants) reveal to me (i.e. to the author or researcher) as their beliefs (Atkins, Lewin, Smith, Engel, Fretheim, & Volmink, 2008; Steffe, von Glasersfeld, Richards, & Cobb, 1983). Self-reflexivity helps me in identifying the weak and strong aspects of my beliefs and then motivating myself to change the beliefs to emancipatory ones. Here, my sense of emancipatory beliefs relates to those transformative beliefs which empower me in terms of logics, ethics, approaches, and actions of teaching GTs with GSP. Self-reflexivity keeps me aware of my beliefs through conscious reflection and adaptation to new beliefs.

A theory of teacher beliefs invented within RCGT seems to be stronger than a theory constructed within the trivial-constructivist-grounded-theory or social-constructivist-grounded-theory because it uses teachers' self-reflexivity helping them turn inward to reflect on their beliefs. Such reflection on one's beliefs is deeper than superficial reflections on the actions or interactions. Therefore, the theoretical bases of this study are the three principles of radical constructivism integrated with the grounded theory where the theory of beliefs is actively invented by the researcher and participants. This theory of beliefs is self-organized and self-adaptive with the new experiences; and it is established based on the principle of viability (von Glasersfeld, 1978). Furthermore, the invention of the theory follows through theoretical sampling, theoretical memoing, theoretical sensitivity, and theoretical coding. This process is involved with the active construction of meanings with each category and the process is adaptive to the research context. The notion of the active construction of meaning of the conceptual codes or categories through unfolding processes of RCGT relates to the reconciliation of these codes or categories to a central and thematic construct towards the invention of a local theory. The process of making meaning out of data (i.e. codes and

categories) is a dynamic and active function with multiple back and forth movements until a better sense is achieved. I would like to go a little further in the process of conceptualization of RCGT based on the historical emergence of grounded theory methodology.

Strauss and Corbin (1990, 1998) and Corbin and Strauss (2008) challenged the conventional pathway of grounded theory toward a flexible, reflexive, subjective, interpretive, and constructive form (Belbase, 2013). However, the notion of constructive does not mean that it is the same as the notion of constructivism. Charmaz (2006) moves further to give this movement a new trend with the constructivist standpoint of grounded theory. She claims further that her approach is founded on Glaser and Strauss's (1967) affirmation that grounded theory is *emergent* and it can be used by researchers in a *flexible*, yet in a *rigorous* way. To me, Charmaz's (2006) call for a constructivist approach of grounded theory is not just a consequence of elasticity of the method to construct a theory. Nonetheless, it seems to be a general call to recognize the subtle interplay between the researcher and the research, the researcher and the research participants, and different selves of the researcher (Belbase, 2013).

Moreover, Charmaz's (2006) constructivist grounded theory is a progressive adaptation of the classical grounded theory of Glaser and Strauss (1967), Glaser (1978, 1992) and interpretive grounded theory of Strauss (1987), Strauss and Corbin (1990, 1998) and Corbin and Strauss (2008). The interpretive grounded theory (Corbin & Strauss, 2008; Strauss; 1987; Strauss & Corbin, 1990, 1998) clearly outlined the interrelations of researcher and participants and researcher and data. Strauss and Corbin (1990, p. 7), outlining their interpretive views in the preface, raised a few questions: "How can I make sense out of all of these materials? How can I have a theoretical interpretation while still grounding it in the empirical reality reflected in my materials?" These questions clearly reflect their sense of interpretive grounded theory. They claimed that the purpose of the book (Strauss & Corbin, 1990) was to help researchers "build theory at the substantive level" (p. 8). They also suggested maintaining a balance between researcher's creativity in developing a theory and doing a good science during the process. Construction of conditional matrix itself is an interpretive phenomenon. They did not use the term interpretive in their grounded theory, nonetheless, their grounded theory was interpretive in terms of developing a theory from the data with the use of researcher's subjectivity and personal creativity. The act of interpretation

within a transactional system was a leap from the notion of 'discovery' of grounded theory towards the generation of a theory.

The transition moved further with Charmaz's (2006) constructivist notion within grounded theory. This transition shows a social constructivist dimension clearly articulated in Charmaz's (2006) constructivist grounded theory. She claims that "how researchers use guidelines in grounded theory texts is not neutral; nor are assumptions they bring to their research and enact during the process" (p. 9). This means she advocates a vision of contextually viable approach to be adopted by the researchers while constructing a grounded theory from the data. However, the progressive adaptation of grounded theory in a constructivist frame is not complete with that. The very notion of construction begins from individual researchers within their struggles, reflections (i.e., looking behind), reflexive abstractions (i.e., abstracting from self-reflexivity), and critical social dialectics (i.e., a form of self-other interactions). This paper is an attempt in that direction beginning from self-reflexivity and then aiming to move further with dilemmas, dialogues, dialectical and critical co-construction of grounded theory of mathematics teachers' beliefs about teaching GTs with GSP.

Method

Conceptualization of this study began with thinking about my beliefs in relation to teaching mathematics. I considered unveiling my beliefs as a starting point to conceptualize a broader study of mathematics teachers' beliefs about teaching GTs with GSP. In this context, I would like to discuss how I studied my beliefs through a self-interview.

Key Aspects of My Beliefs

I wanted to identify some focal aspects of my beliefs in relation to teaching GTs with GSP. Based on my prior experiences, pedagogical metamorphosis in my career as a teacher and teacher educator (Belbase, 2006; Belbase, Luitel, & Taylor, 2008), and rationale for teaching mathematics, I identified some stages of teaching GTs with GSP as focal areas for the study. These focal areas are - (i) beliefs about content of teaching GTs with GSP, (ii) beliefs about the nature of teaching GTs with GSP, (iii) beliefs about activities and situations of teaching GTs with GSP, and (iv) beliefs about concerns and issues of teaching GTs with GSP. These preselected focal aspects have been considered as beginning steps for the systematic revelation of my beliefs. However, these focal aspects are also helpful in

designing questions, tasks, and reflections during task-based interviews to be conducted in the next level of study (in near future).

With these focal aspects and related questions for interviews, I considered myself an interviewee and imagined an interviewer (David) asking me these questions. I thought of my role as an interviewee for the moment. I virtually switched my role from a researcher into a research participant. This process brought me into the research, not only as a researcher but also as a research participant.

An Interview with Myself

Some authors (e.g., Allett, Keightley, & Pickering, 2011) discussed self-interview as a process in qualitative research. They used this approach to involve their research participants to record their voices about the issues or questions in the research. They used this approach over one-on-one interview. Johnson (2009) used self-interview as a part of a one-on-one interview to develop a sense of respect, openness, and empathy in the context and participants. Since the self - interview and response process can be the first steps before interviewing other participants, it can be a basis to understand the interview protocol and frame and possible modifications in the design for an interview to others. Therefore, self-interview can be a very powerful reflective and reflexive tool for the researcher to envision his or her further interviews either one-on-one or focused group. I considered this possibility as an important aspect of this part of study because I was considering it as a part of a broader study but not as an end in itself.

Autoethnography in qualitative research has been considered as a part of self-interview (Crawley, 2012). Likewise, Boufoy-Bastick (2004) discussed auto-interviewing as a tool for conceptualizing one's worldview in relation to the research problem. I used self-interview as a beginning part of a broader research, not as a complete process in itself. This process assumed the researcher's responsibility in terms of "reflexivity and critical awareness" (Boufoy-Bastick, 2004, para. 4). Self-interview was an initial phase in a broader study that I am not reporting in this paper. The self-interview as a phase has been conceptualized as an independent part of this paper.

I formulated potential interview questions for an imaginary interview episode with myself. These questions were based on the focal areas identified. Once the formulation of questions was over, I considered myself a potential interviewee and imagined an interviewer

(David) asking me these questions. I wrote my responses to the questions. I finished writing responses to those questions in one session (about 3 hours). While writing these responses, I tried to be open and revealed my idiosyncratic beliefs as a graduate student and a prospective mathematics teacher/educator without concealing my prior experiences. I also tried to be as truthful as possible to reveal my beliefs about teaching GTs with GSP. The imaginary interview episode was used for an analysis and interpretation.

Analyzing the Self Interview

I used RCGT for the analysis and interpretation of the self-interview data. RCGT is a hybrid of radical constructivism as an epistemology and grounded theory as a methodology. The former provided me with an eye to look at the world of data and the latter provided me with a hand to act on the data (both collection and analysis). Radical constructivism itself does not prescribe how to collect and analyze the data. However, it provided me with a worldview of looking at and knowing from the data. The data themselves are nothing unless we derive meanings out of them. How do we derive meanings out of data depends upon how we look at the data and what we see in them. The positivists, interpretivists, and constructivists may have different interpretations of the same data. Hence, the use of RCGT cannot be justified at the level of data and analysis, but at the level of interpretation. Layers of interpretation are what this approach demands and makes sense of. I have tried to use this approach in this sense.

A positivist or interpretivist grounded theory methodology may find a common ground at the beginning stages, but they depart with RCGT at the interpretation level. A theory is discovered from data from the viewpoint of the classical positivistic grounded theory of Glaser and Strauss (1967). A theory is emerged or developed from data from the viewpoint of the interpretive grounded theory of Strauss and Corbin (1990, 1998) and Corbin and Strauss (2008). A theory is constructed from data from the viewpoint of the constructivist grounded theory of Charmaz (2006). In my view, a theory is subjectively invented from the data. This invention has been conceived as an individual construction with subject to reorganization producing a viable interpretation of a theory. This may lead to the construction of belief narratives, layers of codes and categories, and invent a local (personal) belief theory about teaching GTs with GSP. To me, the entire process was not a forceful but a generative practice.

A belief narrative was constructed based on the revelations of my beliefs about teaching GTs with GSP. The narrative was then sliced into pieces of concepts and named with codes. I read the interview text and identified meaningful units (words, phrases, and sentences) and labelled them with conceptual codes using review and comment menu of Microsoft Word 2010. I used track change option to label these codes at the right margin of the pages containing interview data. The coding was done by keeping all possibilities for interpretation of the codes and meanings open. I re-read the interview text and the codes to ensure the consistency and relevancy of the data. I created another Word Document to list all the open codes obtained from the open coding.

At the next stage, I sought all possible and meaningful connections of these open codes from the initial analysis to get a broader construct or code with a name. This process of axial coding helped me to regroup the open codes into larger meaningful units. These units helped me to see a big picture of my beliefs about teaching GTs with GSP. However, these units were still vague and too many. I condensed them into a few, yet, more meaningful. I regrouped the axial codes to form final categories. This analysis was different from conventional or interpretive grounded theory in the sense that a local theory on my beliefs was actively invented by myself (i.e., by the researcher) as a cognizing subject. The coding at different levels helped me to revisit my beliefs. In some cases, I had to reorganize them in a more coherent and consistent way. My belief system as a whole was challenged when I went through my beliefs about teaching GTs with GSP. Then I considered my beliefs at the time when I was teaching GTs as impoverished method due to lack of new technology, pedagogy, and visions. Nevertheless, I found these beliefs reorganized with new experiences with the use of GSP with preservice teachers and hence they were self-adaptive. The relationship of old beliefs with new beliefs seemed to contradict in many ways. I found these beliefs retained as viable until they contradicted with new experiences.

The analysis of the belief data text, the belief narrative, was complemented with a comparison of each meaning units or concepts with other concepts and categories. The clustering of these concepts into categories was based on semantics (i.e., basic meanings) and pragmatics (i.e., general practices in a context) of the codes at different levels. Theoretical sampling was an important aspect of grounded theory. This was accomplished by revisiting my beliefs while analyzing and conceptualizing the categories with further details of the narrative. Although the interview session was held only once, the theoretical sampling and

constant comparison was observed throughout the analysis and interpretation phase. This seems to be a tricky method. From the radical constructivist viewpoint, revisiting a conceptual category was possible through retrospection, introspection, and prospection. This also made the reorganization of those conceptual categories possible until they seemed viable description and interpretation of my beliefs. The construction of theoretical categories was based on my subjective abstraction of meanings from the narrative. The process of abstraction was very subtle. The conceptual categories seemed to have a link with different crosscutting themes (major categories). However, I tried to see which concepts had better connections to which categories.

Construction, reconstruction, reorganization, invention, and viability were the key aspects of coding, categorizing, theoretical sampling, and constant comparison of concepts of the self-interview data. This is how the analytical and interpretive approach is different from the conventional grounded theory.

I came up with eleven such selective codes as my final belief categories in relation to my beliefs about teaching GTs with GSP. I discussed only five categories as the major themes due to space limitation.

Dealing with Quality Issues

The quality of a qualitative study can be dealt with the classic triple crisis issues as discussed by Denzin and Lincoln (2005). This way of dealing with the quality of a study in terms of the crises of representation, legitimation, and praxis can be one approach. However, I would like to focus on RCGT approach to deal with the quality of this study in terms of fit and viability (von Glasersfeld, 1978). The idea of fit and viability can be extended to the criteria of transferability, dependability, and confirmability. These criteria are further related to originality, usefulness, and resonance of data as suggested by Charmaz (2005).

I maintained the originality of the data by considering new insights, concepts, and theoretical significance, and challenge to the existing ideas and practices (Charmaz, 2005) that shows a viable means to shape the quality. I also addressed the issue of usefulness by considering the applicability of the concepts in mathematics education, generative processes, extension of ideas to different contexts, and contribution to the wider society (Charmaz, 2005). Likewise, I tried to address the issue of resonance through the fullness of explanation of experiences, the revelation of biases, links between larger social and personal interests, and

a deep sense of lives and worlds (Charmaz, 2005) in relation to participant's (i.e., my) beliefs about teaching GTs with GSP. The notion of resonance, applicability, and extension of ideas brought a good fit to the theory and the data.

Results and Discussion: A Reflexive Abstraction of My Beliefs

Now, I would like to present the results of the analysis and interpretation of my beliefs from the first person perspective and the third person perspective. The first person perspective comes from my belief narrative as a first order description and analysis. The third person perspective comes from a discussion of belief categories as a second order interpretation. These two perspectives helped me in engendering my beliefs from the view point of RCGT.

Belief Narrative: First Order Description and Analysis

When I think of my 'self' as a future teacher educator, I find myself well prepared in terms of content knowing of geometric transformations: reflection, translation, rotation, glide, and dilation geometrically, algebraically, and analytically. I have already taught these contents in high schools and also to preservice teachers at the college level. Actually, I taught these contents without using any kind of technology. I simply used paper and pencil activities and demonstration of constructions on the blackboards. I even did not use any kind of manipulatives in those activities. However, I was conscious of teaching concepts and procedures from the simple introductions to the complex analytical proofs.

I also believe that teaching GTs with or without GSP is a complex process. Also, I assume that teaching GTs with GSP is challenging. I think it involves a lot of things going on at the same time. Engaging all the students in high quality learning experiences is a complicated process. Different students have different styles of learning, different paces of learning, and different interests in learning. As a teacher, I need to cope with their style, pace, and quality of learning. To me, a classroom has a very complex environment in terms of interactions, pedagogical processes, and students' engagement in the learning of mathematics (GTs). As I mentioned earlier, the individual differences of students make teaching GTs with GSP a complicated process.

While teaching GTs with GSP, a teacher decides whether to introduce GTs with paper and pencil or manipulatives or GSP or other tools. These decisions influence the quality of learning experiences of students. This may depend on the nature of the class and classroom

dynamics. Classroom dynamics are related to various things going on in the classroom. When a teacher is teaching GTs with or without GSP, the way students are engaged, the way teacher facilitates their learning, and the way GSP is tied with particular contents of GTs create classroom dynamics. The momentum of learning, teaching, and interacting goes either smoothly toward a desired goal or goes in an unwanted direction.

I consider that I need to play different roles while teaching GTs with GSP. I need to be a facilitator, a guide, an instructor, a designer, and an artist. These different roles vary considerably according to the situations in the class. When students are working on their tasks, I am simply a facilitator to help them organize their tasks. If some students are struggling, I need to play the role of a guide. I need to go to them and help them identify their problems and guide them toward solutions. Sometimes, I need to be an instructor to have a focused demonstration and lecturing of new topics or concepts. I think that students' roles also change with the changing role of a teacher. However, their prime role is to engage themselves as learners of mathematics. Their other roles can be co-learners, co-teachers, collaborators, and role models.

Learning environment plays a vital role in creating equal opportunity to learn in the class. I agree that a teacher should create such an environment in which students can learn mathematics at their own pace, within their ability, and within the best access to contents. I cannot treat all students as having similar learning ability. I believe the view '*One size does not fit all'*. Therefore, I need to create an environment in which students have an opportunity to make progress in learning based on their cognitive ability. As I already mentioned, an environment of equal opportunity is a part of any mathematics class and possibly other disciplines, too. Then, teaching GTs with GSP is a part of the system and hence it should provide such an opportunity to the students. The broader goal of teaching GTs with GSP is related to access and equity to all students in the learning of mathematics in general and the specific contents in particular.

Prior knowing of geometry, that I mentioned earlier, is a necessary condition to begin teaching geometric transformation. However, only prior knowing is not sufficient. I have to motivate students. I have to engage them in different activities to connect their prior knowing to the current topic. I have to engage them in explorations and discussions of different geometric properties that connect to teaching and learning of geometric transformations.

Well, learning GTs by students depends on their prior knowing of geometry, especially attributes of two dimensional geometric objects like triangles, quadrilaterals, and other polygons. Also, they need to know Cartesian coordinates to analyze geometric properties of these objects. Probably, I would focus on these concepts before teaching them reflection or any transformation. I would revise the earlier lessons on these concepts (congruency, similarity, parallelism, perpendicularity, etc.). I think these concepts would bridge the concept of reflection and other transformations. At the same time, students' interest in the topic and in learning has a vital role in leading to successful and high quality learning experiences.

Obviously, the nature of content affects the choice of instructional methods. The nature of teaching reflection would be different from the nature of teaching dilation. If the contents are broadly different (like in algebra and geometry), instructional approach also differs significantly in terms of the choice of resources, use of resources, activities, and tools used in teaching and learning. Teaching is also a part of learning. I believe that we learn more as a teacher from teaching than as a student from learning activities. To me, teaching is a more conscious, effortful, and careful activity than learning as a student. The greater is the consciousness, effort, and carefulness, the higher is the quality of experiences and deeper is the cognitive achievement. Hence, to me, every teaching episode is also a learning episode for a teacher. This makes one adjust and re-adjust his or her teaching style. Therefore, every teaching is different from the earlier teaching even if the content is the same and one uses the same resources and technology.

These beliefs portrayed in this narrative are my unfolded idiosyncratic beliefs at the time of self-interview. Although their backdrops might be my past experiences, these beliefs are no way my past beliefs. These beliefs were expressed during the self interview and these beliefs are limited to that particular episode. Also the narrative of my beliefs about teaching GTs with GSP is a shortened version and it may not have all the elements of my actual beliefs portrayed in the full interview episode. This selection was forceful due to the constraints of space for the manuscript. Now, I would like to discuss the second order interpretations of the selected belief categories.

Beliefs about Teaching GTs with GSP: Second Order Interpretation

From the belief narrative, eleven key categories were abstracted. These key categories characterized my beliefs about teaching GTs with GSP in terms of beliefs about self as a future teacher, knowing for the future teacher, pedagogical environment, the role of teacher and students, pedagogical relationship, teaching goals and approaches, classroom interactions, teaching learning activities, environment for equity and access, transitional phases, and advancement of teaching and learning. These categories form a substantive (local) theory of my beliefs. This theory is named "Reflexive Abstraction of My Beliefs about Teaching GTs with GSP". Due to the constraint of space in this manuscript, I would like to highlight only five key belief categories.

Self as a future teacher. The beliefs about self as a future teacher seemed to be grounded on my experiences as a classroom teacher and as a mathematics educator. The sense of preparedness to teach GTs revealed confidence both in content and technology. However, from the belief narrative, one can see that there was a lack of use of manipulative and technological tools during my past teaching. There was no explication of why they were not used. Either there was a general lack of such materials for teaching GTs or there were other factors for not using manipulatives and technology (GSP). The narrative also revealed that I learned using GSP only in the graduate study. This showed that I did not have a knowing of how to use GSP when I was teaching GTs.

My unfolding beliefs showed a sense of readiness to teach GTs with GSP. However, these beliefs lacked clarity in terms of Kaiser, Schwarz, and Krackowitz's (2007) four aspects: formalism, schema, application, and process. Probably, I should have an appreciation of the formal and logical aspects of teaching and learning mathematics while dealing with GTs and GSP. At the same time, I should also have clarity about the nature of teaching GTs with GSP as a "conglomerate of special rules, formula, and procedures" (Kaiser et al., 2007, p. 101). Likewise, the beliefs expressed in relation to readiness as a future teacher might also capture the extension of formal mathematical ideas into non-formal or informal everyday life. The belief statement should also have explicated the "heuristic and creative activity" (Kaiser et al., 2007, p. 101). Then it indicates that my beliefs about self as a future teacher were still weak at the time of interview. The beliefs expressed during the interview session seemed to critically fall short of self-other, self-subject-matter, and self-self (multiple selves, for

example: self as a student, self as a teacher, self as a learner, self as a motivator, self as a creator, self as a follower, etc.) relationships.

Pedagogical environment. The narrative of beliefs about the pedagogical environment indicated toward the complexity of classroom environment, teacher's ability to cope with such an environment, the sequence of activities, dynamics of pedagogical activities in the class, empowerment of teachers and students in such an environment, and quality of learning experiences in the environment as such. One way, we can view classroom dynamics as an emerging phenomenon although we have plans for activities and we may assume some kinds of flow of teaching and learning activities. The idea of emergence was not clearly explicated and thus it was implicit.

The teaching of GTs with or without GSP is a complex phenomenon and this has been recognized in literatures (e.g., Kosnik & Beck, 2003). The mathematics teachers are "overwhelmed with work, begin to doubt their profession, and many question whether their teacher education program prepared them well to meet the challenges they face" (Kosnik & Beck, 2003, p. 18). Beliefs about the pedagogical environment seemed to capture the essences of the complexity of dealing within the classroom environment, task situations and priorities, control over the learning activities, empowerment of teacher and students through increased confidence, a sense of creative thinkers, and high quality learning experiences through exploration and extensions of mathematical ideas. These essences were related to Kosnik and Beck's (2003) four components that mathematics teachers could go deeper into - specific teaching area (GTs), school culture, school programs and activities, and their own enthusiasm as teachers.

At the same time, preservice teachers need to be aware of difficulties that they might face in their future teaching in terms of pedagogical environment, especially while using technology (i.e. GSP) in teaching GTs. In this regard, Ma, Lai, Shouldiams, and Prejean (2008) highlighted two kinds of barriers that the mathematics teachers may face. The first-order barrier is related to availability of technological tools and related technical support. The second-order barrier relates to their own interest, belief, and confidence in the use of technology in teaching GTs. Overcoming these barriers is a part of the teacher education program and also the self-effort of the mathematics teachers in order to create a dynamic teaching and learning environment for both the teacher and students. The narrative of beliefs

about the pedagogical environment seemed to lack commitment towards an ethic of care in the mathematics classroom, students' mathematical thinking, mathematical language, mathematical communication, and extensions and connections. These are some of the critical elements in mathematics education that one needs to be aware of in terms of beliefs and actions.

Roles of students and teachers. These beliefs outlined the multiple roles of a mathematics teacher as a facilitator, a guide, an instructor, a designer, and an artist. The sense of the different roles of a teacher did not spring from simply literatures or pedagogical discussions in formal classes, but they were deeply rooted in my experiences as a teacher and teacher educator. These beliefs were founded on many efforts to create a classroom environment that fostered a comfortable learning environment for students and teaching environment for myself. I cannot claim that I was highly successful to assume these multiple roles while I was a classroom teacher or educator. However, my experiences with success and failure taught me different lessons at different times and each of them added a new brick onto my beliefs as a teacher. Probably, I changed my decisions and plans many times while coping with situations in the classes and even changed my beliefs in relation to my roles as a teacher and students' roles as learners.

Ernest (1988) discussed three major philosophically based roles of a mathematics teacher that can be tied to the role of preservice teachers, too, and also to my role as a future teacher or teacher educator. The first role that a mathematics teacher can assume is the role of an instructor. This role seems to be more authoritative that allows one way communication between teacher and students. Ernest (1988) mentioned the next role of the teacher as an explainer in Platonist view. Within this view, a teacher explains things to the students. He or she explains the rules, formulas, and procedures and asks students to follow them. He or she can also demonstrate an example and the students can simply follow it to solve other mathematical problems. The third role Ernest (1988) described was a facilitator. While assuming this role, a teacher may create an environment for students to learn from contexts. He or she may provide minimum guidelines and support to the students. He or she may try to help students work on their own pace and on their own at first. He or she may facilitate those students who really struggle and who do not get a way out to solve problems. These roles of a mathematics teacher clearly signify corresponding roles of the students, too. My belief narrative also indicated some of these elements in a vague way.

A teacher may build a learning community within the classroom in which students are engaged in inquiry of mathematical relations and solving mathematical problems (Applefield, Huber, & Moallem, 2001). In this sense, the role of a teacher is believed to create a conducive learning environment in which students construct their meaning of what they learn based on their existing knowledge. To me, the teacher also promotes interactions in the class within an authentic learning environment. The students may play an active role as learners by making sense of what they are doing and learning. They do not simply follow the teacher's instruction or simply listen to the teacher, but actively construct knowledge of their own from the learning context (Applefield et al., 2001).

Transitional phase in teaching learning. A transitional phase is the one in which change occurs and it is the phase in teaching and learning in which both teacher and students experience some sort of epistemic dilemma and then a movement from lower to higher cognition, perception, experience, and knowledge. The differential of this transition can be experienced in terms of higher level of achievement and a sense of greater consciousness of the phenomena. For this transition to occur, both teacher and students should have foundational knowledge, experience, and skills to move ahead with new challenges, opportunities, and threats.

The differential in perception, beliefs, knowledge, and cognition connects prior experiential world to the new experiential world. It also relates to Vygotsky's (1978) Zone of Proximal Development (ZPD) and Steffe's (1991) Zone of Potential Construction (ZPC). These zones are conceived as the zones of mathematics teaching and learning (Norton & D'Ambrosio, 2008). ZPD purports social constructivism of Vygotsky and ZPC purports radical constructivism of von Glasersfeld and Steffe. According to Norton and D'Ambrosio (2008), these competing epistemic views of teaching and learning mathematics stimulated arguments over the mind as a social or an individual entity. The physical mind as an entity that resides in the brain can be individually located whereas the social mind resides in the collective brain of individual members of a society. Accordingly, they emphasize learning either as an individual construction of knowledge (radical constructivism) or as a social construction of shared knowledge (social constructivism). The transition of teaching and learning either as part of ZPD or ZPC can be viewed as a progressive move from lower cognitive and epistemic status to a more sophisticated status. One's belief about this connection either as ZPD or ZPC or others can guide the entire teaching and learning process

accordingly. My belief narrative made some sense of teacher and students' roles and responsibilities while dealing with the transition and transformation. However, I should have a clearer vision and explanation of such beliefs that can have a transformative effect on my current and future teaching of mathematics in general and GTs with GSP in particular.

Advancement of pedagogy. Advancement of one's own pedagogy depends on the progressive movement towards more sophisticated content knowledge, interest towards sophisticated ideas, commitment to higher-level cognitive and epistemic goals, choice of a more flexible instructional method, and adaptive process of teaching and learning of mathematics. These elements also signify greater teaching and learning outcomes in terms of quality of experiences. Again, we can link this sort of advancement of pedagogy with ZPD and ZPC. Going further, we can think of new zones of pedagogical extensions, elaborations, and connections to new horizons of knowledge contexts and applications. We can think of this possibility as a Zone of Pedagogical Extension (ZPE). Here, the extension of pedagogy can be conceived as new ways of teaching, learning, assessing, and connecting to new contexts and applications.

The idea of ZPE may be enhanced through a greater scope of pedagogical knowledge in terms of pedagogical content knowledge, pedagogical context knowledge, pedagogical cultural knowledge, pedagogical technological knowledge, pedagogical psychological knowledge, and pedagogical philosophical knowledge. Pedagogical sensibility in terms of student-teacher mathematical caring relations as discussed by Hackenberg (2010) can be a part of ZPE, because mathematical caring relations may extend pedagogical possibilities in a more subtle way. Again, I admit that this sort of beliefs were not explicit in my belief narrative. One can, nevertheless, trace a feeble cloud of such beliefs in the above stated narrative.

As a future teacher and teacher educator, I should have a clearer view of how one's pedagogy gets changed with context and further motivation. Also, there might be other elements to consider in relation to pedagogical advancement such as the role of a critical friend and critical students. The pedagogical advancement should also have elements of social justice issues, empowerment of students and teacher, and advancement of learning of mathematics. The belief narrative could be more meaningful if it had some elements of critical and constructive feedback loop.

Epilogue

The interpretation of belief narrative within the eleven key categories may not be sufficient, though they may reveal some necessary parts of such beliefs. The belief itself is very sophisticated in nature and it is very complex to understand in terms of its characters as revealed by the believer. In many cases, beliefs have many hidden parts in the forms of sub-conscious beliefs. To me, it is like an iceberg that appears a little to the world through our language game and a large part remains hidden within our conscious and sub-conscious activities. Now, we can look at other dimensions of beliefs besides those key categories discussed earlier. One such critical dimension is teaching mathematics as a complex adaptive system.

We can compare beliefs about teaching GTs by using GSP with a complex adaptive system. A complex adaptive system consists of many interrelated parts that interact with an environment and undergo necessary changes. There is no real boundary between the system itself and its environment (Chan, 2001). How can we compare beliefs about the teaching of GTs with the complex adaptive system? For this, we have to look at the essentials of the complex adaptive system - distributed control, connectivity, co-evolution, dependence on conditions, emergent order, and state of paradox.

Overall, the belief narratives may have some of these components and may lack many of these characteristics of complex adaptive system. The narrative also lacks dynamic interrelation of belief, knowledge, and practice in an explicit way. The dynamic interplay among belief, knowledge, and practice can be the agenda of study at the next level of research (Belbase, 2012a) that focuses on the complexity of beliefs and interrelation with other psychological and epistemic domains. In this study, I attempted to invent a local theory on "Reflexive Abstraction of My Beliefs about Teaching GTs with GSP" through self-interview. In the next level, I will extend this study to invent a local theory of preservice mathematics teachers' beliefs about teaching GTs with GSP through task-based interviews.

References

Allett, N., Keightley, E., & Pickering, M. (2011, October). Using self-interviews to research memory. *Realities Toolkit#6*. Retrieved from http://www.socialsciences.manchester.ac.uk/morgancentre/realities/toolkits/self-interviews/16-toolkit-using-self-interviews.pdf

Applefield, M. J., Huber, R., & Moallem, M. (2001). Constructivism in theory and practice: Toward a better understanding. *The High School Journal*, *84*(2), 35–69.

Atkins, S., Lewin, S., Smith, H., Engel, M., Fretheim, A., & Volmink, J. (2008). Conducting a meta-ethnography of qualitative literature: Lessons learnt. *BMC Medical Research Methodology*, *8*(21), 8-21. doi:10.1186/1471-2288-8-21

Belbase, S. (2006). *My journey of learning and teaching mathematics: A portrayal of pedagogical metamorphosis* (MPhil dissertation, Kathmandu University School of Education, Nepal). Retrieved from http: //www.eric.ed.gov/PDFS/ED502252.pdf

Belbase, S., Luitel, B. C., & Taylor, P. C. (2008). Autoethnography: A method of research and teaching for transformative education. *Journal of Education and Research*, *1*(1), 86-95.

Belbase, S. (2012a). *Belief, knowledge, and practice: A trichotomy of mathematics teacher education.* Retrieved from http: //www.eric.ed.gov/PDFS/ED530017.pdf

Belbase, S. (2012b). *Epistemic sensibility: The third dimension of virtue epistemology*. Retrieved from http: //www.eric.ed.gov/PDFS/ED538379.pdf

Belbase, S. (2013). *Constructivism and grounded theory: Where they meet?* Manuscript submitted for publication.

Boufoy-Bastick, B. (2004). Auto-interviewing, auto-ethnography and critical incident methodology for eliciting a self-conceptualized worldview. *FORUM: QUALITATIVE SOCIAL RESEARCH (FQS)*, *5*(1), Art. 37.

Cavin, R. M. (2007). *Developing technological pedagogical content knowledge in preservice teachers through microteaching lesson study* (Doctoral dissertation). Florida State University, USA.

Chan, S. (2001). *Complex adaptive systems*. Retrieved from http://web.mit.edu/esd.83/www/notebook/Complex%20Adaptive%20Systems.pdf

Charmaz, K. (2005). Grounded theory in the 21st century: Applications for advancing social justice studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (3rd ed., pp. 507-535). Thousand Oaks, CA: SAGE Publications, Inc.

Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. Thousand Oaks, CA: SAGE.

Corbin, J., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: SAGE.

Crawley, S. L. (2012). Autoethnography as a feminist self-interview. In J. F. Gubrium, J. A. Holstein, A. B. Marvasti, & K. D. McKinney (Eds.), *The SAGE handbook of interview research: The complexity of the craft* (2nd ed., pp. 143-161). Thousand Oaks, CA: SAGE.

Denzin, N. K., & Lincoln, Y. S. (Eds.). (2005). *The SAGE handbook of qualitative research* (3rd ed.). Thousand Oaks, CA: SAGE.

Ernest, P. (1988). *The impact of beliefs on the teaching of mathematics*. Paper presented at the 6th International Congress of Mathematics Education, Budapest, Hungary.

Fenstermacher, G. (1979). A philosophical consideration of recent research on teacher effectiveness. In L. S. Shulman (Ed.), *Review of research in education* (Vol. 6, pp. 157-185). Itasca, IL: Peacock.

Fenstermacher, G. (1994). The place of practical arguments in the education of teachers. In V. Richardson (Ed.), *Teacher change and the staff development process: A case in reading instruction* (pp. 23-42). New York, NY: Teachers College Press.

Glaser, B. (1992). *Emergence versus forcing: Basics of grounded theory analysis*. Mill Valley, CA: Sociology Press.

Glaser, B. (1978). *Theoretical sensitivity*. Mill Valley, CA: The Sociology Press.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* New York, NY: Aldine de Gruyter.

Green, T. (1971). *The activities of teaching*. New York, NY: McGraw-Hill.

Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education*, *41*(3), 236-273.

Johnson, N. (2009). The role of self and emotion within qualitative sensitive research: A reflective account. *ENQUIRE, 4*, November. Retrieved from http://www.nottingham.ac.uk/sociology/prospective/postgraduate/enquire/enquire-pdfs/4th-johnson.pdf

Kaiser, G., Schwarz, B., & Krackowitz, S. (2007). The role of beliefs on future teacher's professional knowledge. *The Montana Mathematics Enthusiast* (Monograph 3), 99-116.

Kosnik, C., & Beck, C. (2003). The internship component of a teacher education program: Opportunities for learning. *The Teacher Educator*, *39*(1), 18-34.

Lundeberg, M. A., & Levin, B. B. (2003). Promoting the development of preservice teachers' beliefs through cases, action research, problem-based learning, and technology. In J.

Raths & A. C. McAninch (Eds.), *Teacher beliefs and classroom performance: The impact of teacher education* (pp. 23-42). Greenwich, CT: Information Age Publishing.

Ma, Y., Lai, G., Shouldiams, D. C., & Prejean, L. (2008). Teacher's belief changes in a technology-enhanced pedagogical laboratory. *Journal of Educational Technology Development and Exchange, 1*(1), 13-28.

Maturana, H. R., & Varela, F. J. (1980). *Autopoiesis and cognition: The realization of the living*. Boston, MA: Reidel Publishing Company.

Norton, A., & D'Ambrosio, B. S. (2008). ZPC and ZDP: Zones of teaching and learning. *Journal for Research in Mathematics Education, 39*(3), 220-246.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332.

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. F. (1989). Teachers' pedagogical content beliefs in mathematics. *Cognition and Instruction, 6*(l), 1-40.

Raths, J. (2001). Teachers' beliefs and teaching beliefs. *Early Childhood Research and Practice, 3*(1), article 3. Retrieved from http://ecrp.uiuc.edu/v3n1/raths.html

Raths, J., & McAninch, A. C. (Eds.). (2003). *Teacher beliefs and classroom performance: The impact of teacher education*. Greenwich, CT: Information Age Publishing.

Richardson, V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (Ed.), *The handbook of research in teacher education* (pp. 102-119). New York: Macmillan.

Richardson, V. (2003). Preservice teachers' beliefs. In J. Raths & A. C. McAninch (Eds.), *Teacher beliefs and classroom performance: The impact of teacher education* (pp. 1-22). Greenwich, CT: Information Age Publishing.

Steffe, L. P. (1991). The constructivist teaching experiment: Illustrations and implications. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 177-194). The Netherlands: Kluwer Academic Publishers.

Steffe, L. P. (1995). Alternative epistemologies: An educator's perspective. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 489-524). Hillsdale, NJ: Lawrence Erlbaum Associates.

Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application.* New York, NY: Praeger.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213-226.

Strauss, A. (1987). *Qualitative analysis for social scientists*. New York, NY: Cambridge University Press.

Strauss, A. L., & Corbin, J. M. (1990). *Basic of qualitative research: Grounded theory procedures and techniques.* Newbury Park, CA: SAGE Publishing.

Strauss, A. L., & Corbin, J. M. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: SAGE Publications, Inc.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York, NY: Macmillan.

von Glasersfeld, E. (1978). Radical constructivism and Piaget's concept of knowledge. In F. B. Murray (Ed.), *The impact of Piagetian theory* (pp. 109-122). Baltimore, MD: University Park Press.

von Glasersfeld, E. (1994). La construcción del conocimiento [The construction of knowledge]. In D. F. Schnitman (Ed.), *Nuevos paradigmas, cultura y subjetividad* [*New paradigms, culture and subjectivity*] (pp. 115–128). Buenos Aires: Paidos.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.