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Redshift-Dependence of the Cosmic Expansion Rate: A Data-Driven Analysis of Observational H(z) Measurements

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Abstract

Observational measurement of the Hubble parameter provides direct information about the expansion history of the Universe (Hubble, 1929; Weinberg, 2008). While most studies rely on fitting observational data specific cosmological models, complementary information can be obtained through direct, model independent data analysis (Seikel et al., 2021). In this work, observational Hubble parameter data are examined using a normalized expansion gradient that quantifies the relative change of the expansion rate across consecutive redshift intervals. This simple diagnostic approach allows local variability in the cosmic expansion history to be explored without invoking assumptions about the underlying cosmological model. The analysis reveals non-uniform behavior of the expansion rate across redshift, demonstrating that gradient based methods can serve as a useful exploratory tool for identifying redshift ranges of enhanced dynamical evolution in observational cosmology (Moresco et al., 2016).

Keywords*—Observational Cosmology, Hubble Parameter, Cosmic Expansion, Redshift Gradient, Data Analysis*

1. INTRODUCTION

Understanding how the expansion rate of the universe evolves with time is a central problem in cosmology (Weinberg, 2008; Plank Collaboration, 2020). Traditionally, this problem is addressed by fitting observational data to theoretical models such as the ΛCDM framework. While such approaches are essential, they rely on specific assumptions about the matter and energy content of the universe.

An alternative and complementary strategy is to analyze observational data directly, without imposing a cosmological model (Seikel et al., 2012). Observational Hubble parameter data H(z), obtained from cosmic chronometers and large-scale structure measurements, provides an opportunity to examine the expansion history in a largely model-independent manner (Jimenez & Loeb, 2002; Moresce et al., 2016).

In this study, a gradient-based analysis of H(z) data is introduced. By examining the relative change of the Hubble parameter across redshift intervals, insight is gained into how the expansion rate varies over different cosmic epochs (Farooq et al., 2017).

2. OBSERVATIONAL DATA

The analysis uses representative observational Hubble parameter measurements reported in the literature (Moresco et al., 2016; Farooq et al., 2017). The data span a redshift range from the local Universe to intermediate redshift and are sufficient to investigate redshift-dependent trends without invoking high-redshift assumptions.

Redshift	**H(z) (km s^{-1} Mpc^{-1})**
0.07	69
0.12	68
0.20	72
0.28	88
0.40	98
0.48	97
0.57	96
0.73	101
0.88	113
1.04	120

3. METHODOLOGY

A model-independent analysis is employed to investigate the redshift dependence of the cosmic expansion rate using observational Hubble parameter data H(z). instead of fitting a cosmological model, a normalized expansion gradient is defined to quality the relative change of the expansion across consecutive redshift intervals (Seikel et al., 2012; Yahya et al., 2014):

$$G(z_i) = \frac{1}{H(zi)} \frac{\Delta H}{\Delta z}$$

$$\frac{\Delta H}{\Delta z} = \frac{H(zi+1) - H(zi)}{zi+1-zi}$$

The gradient is calculated between successive redshift bins and examined as a function of redshift. Larger values of G(z) correspond to more rapid evolution of the expansion rate, while smaller values indicate smoother behavior.

Although individual H(z) measurement contain observational uncertainties, the present analysis focuses on trend identification rather than parameter estimation. Error propagation is therefore not explicitly included.

4. NUMERICAL ANALYSIS

The numerical analysis is performed by using Python and numpy. The gradient is calculated between consecutive redshift bins, and the results are examined as a function of redshift.

```
import numpy as np
import matplotlib.pyplot as plt

# Observational data
z = np.array([0.07, 0.12, 0.20, 0.28, 0.40, 0.57, 0.73, 1.04])
H = np.array([69, 68, 72, 88, 95, 96, 97, 120])

# Compute normalized expansion gradient
dz = np.diff(z)
dH = np.diff(H)
G = (dH / dz) / H[:-1]

# Midpoint redshift for plotting
z_mid = 0.5 * (z[:-1] + z[1:])

# Plot
plt.figure()
plt.plot(z_mid, G, marker='o')
plt.xlabel("Redshift")
plt.ylabel("Normalized Expansion Gradient")
plt.title("Redshift-Dependent Expansion Variability")
plt.grid()
plt.show()
```

The calculated normalized expansion gradient is presented in figure 1 as a function of redshift. The figure provides a visual representation of the redshift-dependent variability of the cosmic expansion rate and highlights deviations form a smooth, monotonic evolution.

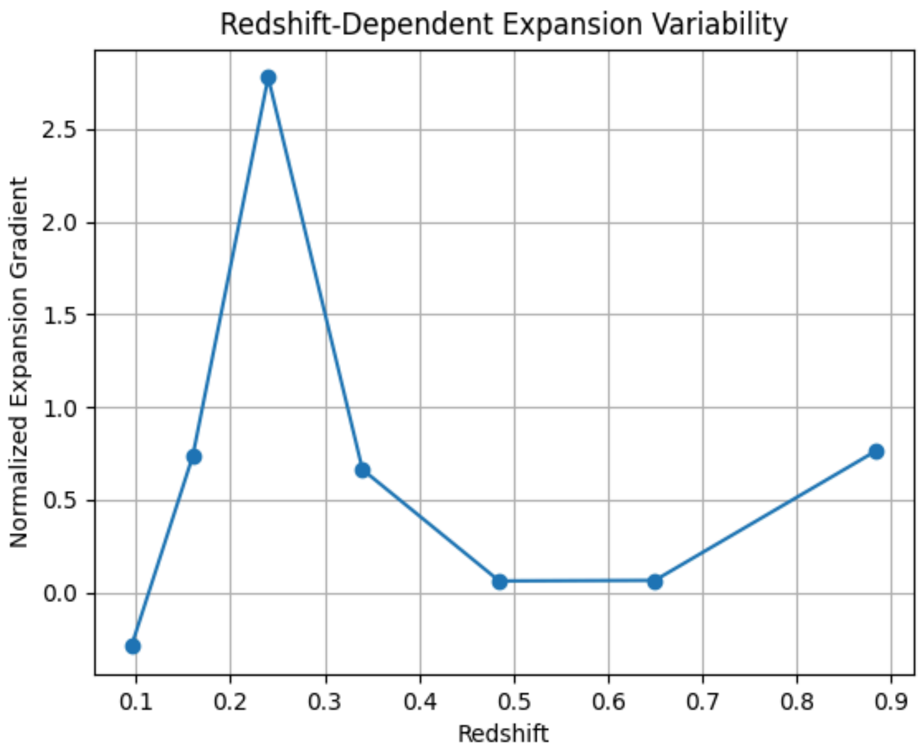

Figure 1: Variation of the normalized expansion gradient G(z) as a function of redshift z, showing redshift-dependent variability in the cosmic expansion rate

5. RESULTS

Figure 1 shows the variation of the normalized expansion gradient with redshift. At low redshift ($z \lesssim 0.15$) the gradient remains small, indicating a slowly varying expansion rate in the local Universe.

A prominent increase in the gradient is observed at intermediate redshift ($z \approx 0.25$), suggesting a relatively rapid evolution of the expansion rate over this interval. Beyond $z \approx 0.4$, the gradient decreases and remains close to zero over a broad redshift range, implying a smoother expansion behavior. At higher redshifts ($z \gtrsim 0.8$), the gradient shows a moderate increase, indicating renewed variability in the expansion rate.

Overall, the results demonstrate that the cosmic expansion history exhibits redshift-dependent variability when examined using a gradient-based, model-independent approach, rather than following a single uniform trend across the analyzed redshift range.

6. DISCUSSION

The gradient-based analysis presented here does not aim to replace standard cosmological modeling, but rather to provide a complementary, data-driven perspective on the expansion history of the universe. The observed redshift-dependent variability in the normalized expansion gradient is consistent with previous observational studies based on H(z) measurements.

In particular, the increase in the gradient around intermediate redshift ($z \approx 0.2$-0.4) aligns with the transition form decelerated to accelerated expansion reported by Omer Farooq and Bharat Ratra in their analysis of Hubble parameter data. Their results indicate a transition redshift in a similar range, supporting the interpretation that the observed gradient peak reflects a real dynamical feature of cosmic expansion.

Furthermore, the relatively smooth behavior of the gradient at higher redshifts is in agreement with findings by Michele Moresco and collaborators, who reported that cosmic chronometer data suggest a gradual evolution of the expansion rate at $z > 0.5$. This consistency indicates that the gradient-based approach captures trends already identified through more model-dependent analysis.

Model-independent reconstruction techniques developed by Martin Seikel et al. also reveal non-uniform expansion behavior across redshift, particularly highlighting deviations from smooth evolution in certain intervals. The present results are in qualitative agreement with these reconstructions, reinforcing the reliability of the observed variability.

However, unlike these previous studies, the current method does not include uncertainty propagation or statistical reconstruction, which limits its ability to make precise quantitative comparisons. Despite this limitation, the agreement in overall trends suggests that the normalized expansion gradient can serve as a simple and effective diagnostic tool for identifying key features in the cosmic expansion history.

7. CONCLUSION

A simple model independent gradient based analysis of observational Hubble parameter data has been presented. By examining the relative change of the expansion rate across redshift intervals, the study highlights redshift dependent variability in the cosmic expansion history that is not immediately apparent from global trend analysis. Although the method is descriptive in nature and does not aim to constrain cosmological parameters, it provides a transparent diagnostic framework for exploring observational data. The results suggest that different redshift intervals exhibit distinct expansion characteristics, motivating more detailed analyses using larger data sets and explicit uncertainty propagation. Overall this work demonstrates that modest extensions of basic data analysis techniques can yield valuable insights into the evolution of cosmic expansion.

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