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Three-Dimensional Transient Heat Conduction in a Solid Block with Fully Implemented Convective Boundary Conditions

[1]Navin Kumar Jha
[1]Department of Mechanical and Aerospace Engineering, Pulchowk Campus, Tribhuvan University
*Corresponding email: *navin.jha@pcampus.edu.np*
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Abstract
Accurate prediction of transient cooling in three-dimensional solid bodies is essential for thermal processing and electronic cooling applications. This study develops a physically consistent 3D transient heat conduction model that explicitly estimates convective heat transfer on all external boundaries, including faces, edges, and corners, using an explicit finite-difference scheme on a uniform Cartesian grid. Unlike conventional models that simplify convective boundary conditions to planar surfaces, this approach implements directionally consistent Robin conditions to allow multi-directional heat loss at boundary intersections. The model was applied to an aluminum block cooling from 180°C to an ambient temperature of 25°C. Results reveal that that enhanced cooling at edges and corners, exposed to convection in two and three orthogonal directions, respectively, results in significant local temperature variations. A grid independence study comparing spatial step sizes from 4 mm to 2 mm confirmed that the numerical solution is robust, with a relative difference in center cooling time of less than 2% between the two finest grids. Quantitative analysis indicates that neglecting these edge and corner effects can result in a significant underestimation of local heat loss and inaccuracies in total cooling time predictions. This framework provides a transparent numerical approach for analyzing transient heat transfer where surface-to-volume ratios and localized cooling are critical.

Keywords*—Transient heat conduction, Convective boundary conditions, Three-dimensional cooling, Explicit numerical scheme*

1. INTRODUCTION

Transient heat conduction in solid bodies subjected to convective cooling is a fundamental problem in heat transfer in applications such as metal processing, electronic component cooling, thermal energy storage, and manufacturing operations such as quenching and controlled cooling (Sahu & Behera, 2012). Accurate prediction of temperature evolution within three-dimensional solids is essential for ensuring thermal reliability, minimizing residual stresses, and achieving desired material properties (Chen et al., 2022). Analytical solutions exist for highly idealized geometries and boundary conditions, however, practical engineering systems typically require numerical approaches due to geometric complexity and realistic surface heat transfer mechanisms (Dalir, 2014).

Simplified models such as lumped-capacitance or one-dimensional conduction formulations are frequently used in preliminary analyses; however, these approaches are valid only under restrictive conditions and fail to capture spatial temperature gradients within the solid (Sahu & Behera, 2012). Multi-dimensional numerical methods, including finite-difference, finite-volume, and finite-element techniques, are therefore widely used to simulate transient heat conduction in solids. Out of these, finite-difference methods remain attractive due to their conceptual simplicity, low computational overhead, and transparency in linking physical laws to numerical implementation (Vera-García et al., 2010). In many existing numerical studies, convective heat transfer at the solid–fluid interface is applied mainly on planar surfaces, with edge and corner regions either neglected or treated implicitly through simplified boundary assumptions. Such approximations can lead to inaccuracies, particularly in three-dimensional bodies where edges and corners experience enhanced heat loss due to multi-directional convection. The influence of these regions becomes increasingly significant for compact geometries, high surface-to-volume ratios, and transient cooling scenarios where surface effects dominate the thermal response (Árpád et al., 2024). To the best of the authors' knowledge, there is a lack of studies that explicitly implement convective (Robin) boundary conditions in a fully consistent, multi-directional manner across faces, edges, and corners within a three-dimensional finite-difference framework. Furthermore, explicit time-marching schemes are frequently avoided in these contexts due to stability constraints, despite their advantages in implementation clarity and computational transparency for moderate grid sizes. This study addresses this gap by developing a robust 3D transient model based on an explicit finite-difference scheme with comprehensive implementation of convective boundary conditions on all external boundaries, including faces, edges, and corners. The formulation contains physical consistency by considering heat loss in multiple spatial directions at boundary intersections. The model is applied to the transient cooling of a solid block, and the resulting temperature evolution and cooling time are analyzed to understand the importance of complete boundary treatment. The proposed approach provides a robust and transparent numerical framework that can be extended to other materials, geometries, and convective environments.

2. METHODOLOGY

A. Physical model and assumptions

The physical system considered in this study is a three-dimensional solid block subjected to convective cooling from its external surfaces (Figure 1). The solid is assumed to be homogeneous and isotropic, with constant thermophysical properties. Heat transfer within the solid occurs solely by conduction, and internal heat generation is neglected. The surrounding fluid is modeled as a uniform ambient environment at constant temperature, and heat exchange at the solid–fluid interface is described by

Newton's law of cooling. Radiation heat transfer is neglected. The initial temperature of the solid is assumed to be spatially uniform.

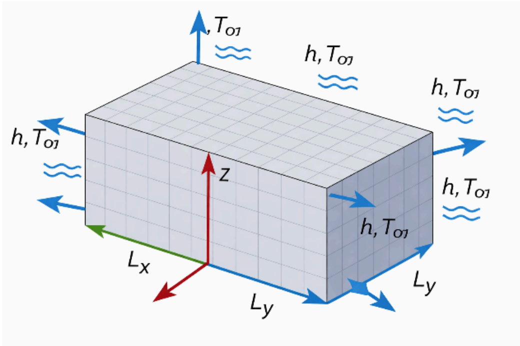

Figure 1: Three-dimensional geometrical domain of the solid block with boundary condition

B. Governing equation

Under the above assumptions, the transient temperature field within the solid is governed by the three-dimensional heat diffusion equation (equation 1),

$$\frac{\partial T(x,y,z,t)}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial z^2}\right), \quad (1)$$

where T is the temperature, t is time, and α is the thermal diffusivity of the material, defined as

$$\alpha = \frac{k}{\rho c_p}, \quad (2)$$

with k denotes the thermal conductivity, ρ the density, and c_p the specific heat capacity of the solid.

C. Initial condition

At the start of the cooling process, the solid is assumed to be at a uniform temperature T_{init} (equation 3)

$$T(x, y, z, 0) = T_{init}. \quad (3)$$

D. Convective boundary conditions

All external surfaces of the solid block are subjected to convective heat transfer with the surrounding fluid at ambient temperature T_{∞}. The heat flux at the solid–fluid interface is governed by a Robin (mixed) boundary condition (equation 4),

$$-k\frac{\partial T}{\partial n} = h\left(T_{\infty}\right), \quad (4)$$

where h is the convective heat transfer coefficient and $\partial/\partial n$ denotes the spatial derivative in the outward normal direction to the surface.

E. Boundary treatment at faces, edges, and corners

In a three-dimensional geometry, convective heat loss occurs not only on planar surfaces but also at edges and corners where multiple surfaces intersect. In the present formulation, the convective boundary condition given by Eq. (4) is applied consistently on all external boundaries, with special consideration of the geometric characteristics of faces, edges, and corners. On planar faces of the block, heat transfer occurs in the direction normal to the surface, and the convective boundary condition is applied in one spatial direction, while conduction in the remaining two directions is governed by the diffusion equation (Wu et al., 2023). At edges, where two surfaces intersect, convective heat transfer occurs simultaneously in two orthogonal directions. Accordingly, the heat diffusion equation at edge locations incorporates two convective flux contributions and one conductive contribution along the interior direction. At corners, where three surfaces intersect, heat loss occurs in all three spatial directions. The governing equation at these locations' accounts for three convective flux terms, reflecting multi-directional heat exchange with the ambient environment. This comprehensive treatment ensures that the enhanced heat loss at edges and corners is explicitly captured, leading to a physically consistent representation of transient heat transfer in three-dimensional solids. Such effects are mainly important in compact geometries and during transient cooling processes where surface heat transfer dominates the thermal response.

F. Numerical Methodology

a. Computational domain and grid generation

The three-dimensional computational domain corresponds to a rectangular solid block of dimensions $L_x \times L_y \times L_z$. The domain is discretized using a uniform Cartesian grid with spatial step sizes Δx, Δy, and Δz in the x-, y-, and z-directions, respectively. In the present study, equal grid spacing is employed in all three directions, i.e.,

$$\Delta x = \Delta y = \Delta z = \Delta. \quad (5)$$

The total number of grid points along each coordinate direction is given by

$$N_x = \frac{L_x}{\Delta x} + 1, N_y = \frac{L_y}{\Delta y} + 1, N_z = \frac{L_z}{\Delta z} + 1. \quad (6)$$

Each grid point represents the temperature at a nodal location within the solid, and the temperature field is stored as a three-dimensional array indexed by the spatial coordinates. For the final simulations presented in the study, the baseline grid size used to report the transient cooling behavior and three-dimensional temperature distributions was Δ = 4mm.

b. Temporal discretization

Time integration of the governing heat diffusion equation is performed using an explicit forward Euler scheme. The temperature at time level $n + 1$is computed directly from the known temperature field at time level n, according to

$$T^{n+1}_{i,j,k} = T^{n}_{i,j,k} + \alpha\Delta t\left(\nabla^2 T\right)^{n}_{i,j,k}, \quad (7)$$

where Δt is the time step and (k) denote the discrete spatial indices in the x-, y-, and z-directions.

The explicit scheme is chosen for its conceptual simplicity and the direct way it links physical laws to numerical implementation. Unlike implicit methods that require solving large systems of simultaneous equations, the explicit forward Euler scheme allows the temperature at a future time step $n + 1$ to be computed directly from known values at the current step n. This transparency is crucial for verifying that the physics of multi-directional heat loss at edges and corners are correctly captured in the code. Besides it, the explicit formulation allows for a straightforward, directionally consistent implementation of these conditions at boundary intersections at face nodes, edge nodes, and corner nodes.

c. Spatial discretization of the diffusion operator

Interior nodes

For interior grid points, second-order central finite differences are used to approximate the second derivatives in each spatial direction. The discrete Laplacian at an interior node is given by

$$\left(\nabla^2 T\right)_{i,j,k} = \frac{T_{i+1,j,k} - 2T_{i,j,k} + T_{i-1,j,k}}{\Delta x^2} + \frac{T_{i,j+1,k} - 2T_{i,j,k} + T_{i,j-1,k}}{\Delta y^2} + \frac{T_{i,j,k+1} - 2T_{i,j,k} + T_{i,j,k-1}}{\Delta z^2}. \quad (8)$$

This formulation is directly implemented in the code using central difference operators for each coordinate direction.

Convective boundary discretization

At boundary nodes, the second derivatives normal to the surface are modified to incorporate the convective boundary condition given by Eq. (4). A second-order one-sided finite-difference approximation is employed, in which the temperature gradient at the boundary is replaced by an equivalent expression involving the interior neighboring node and the ambient temperature. For a boundary located at $x = 0$, the discrete approximation of the second derivative in the x-direction becomes

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{2(T_{1,j,k} - T_{0,j,k}) - 2\Delta x \frac{h}{k}(T_{0,j,k} - T_{\infty})}{\Delta x^2}. \quad (9)$$

Similarly, for the opposite face at $x = L_x$,

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{2(T_{N_x-2,j,k} - T_{N_x-1,j,k}) - 2\Delta x \frac{h}{k}(T_{N_x-1,j,k} - T_{\infty})}{\Delta x^2}. \quad (10)$$

Analogous expressions are applied for convective boundaries in the y- and z -directions. These formulations are implemented through dedicated boundary operators that incorporate the ratio $h/k$and the ambient temperature.

Treatment of faces, edges, and corners

A distinguishing feature of the present numerical methodology is the explicit and consistent treatment of convective heat transfer at faces, edges, and corners of the three-dimensional domain.

- **Face nodes:** At planar surface nodes, one spatial direction is subjected to the convective boundary condition, while the remaining two directions are discretized using central differences.
- **Edge nodes:** At edge locations where two surfaces intersect, convective boundary discretization is applied in two orthogonal directions, and central differencing is applied in the remaining interior direction.
- **Corner nodes:** At corner locations where three surfaces intersect, convective boundary discretization is applied simultaneously in all three spatial directions.

This approach was adopted to make sure that multi-directional heat loss at edges and corners is captured explicitly, rather than being implicitly absorbed into face-based approximations.

d. Stability criterion

The explicit finite-difference scheme is conditionally stable, and the time step must satisfy a stability constraint derived from von Neumann analysis (Wais, 2014). For a three-dimensional Cartesian grid, the stability condition is expressed as

$$\Delta t \leq \frac{1}{2\alpha\left(\frac{1}{\Delta z^2}\right)}. \quad (11)$$

In the present simulations, a conservative safety factor is applied, and the time step is selected as

$$\Delta t = 0.6\,\Delta t_{max}, \quad (12)$$

to ensure numerical stability throughout the transient simulation.

e. Solution procedure

The numerical solution proceeds as follows:

1. Initialize the temperature field to the prescribed uniform initial temperature.
2. Compute the time step based on the stability criterion.
3. Update interior node temperatures using the central-difference Laplacian.
4. Update face, edge, and corner nodes using the corresponding convective boundary discretizations.
5. Advance the solution in time using the explicit update formula.
6. Repeat the process until the specified final time is reached or the target temperature criterion is satisfied.

The temperature at the geometric center of the block is monitored throughout the simulation to evaluate the transient cooling behavior and determine the time required to reach a prescribed target temperature.

f. Numerical implementation

The numerical model is implemented in Python using the NumPy library for array operations. The three-dimensional temperature field is stored as a structured array, enabling efficient vectorized computation of interior and boundary updates. Visualization of the transient results and three-dimensional temperature fields is performed using Matplotlib.

3. RESULTS AND DISCUSSION

A. Transient cooling behavior at the block center

The temporal evolution of temperature at the geometric center of the block is shown in **Figure 2**, which plots the center temperature as a function of time during convective cooling. The block is initially at a uniform temperature of 180 °C and is exposed to ambient air at 25 °C on all external surfaces. As expected, the center temperature decreases monotonically with time, exhibiting a nonlinear cooling trend characteristic of transient heat conduction in solids (Gu et al., 2019).

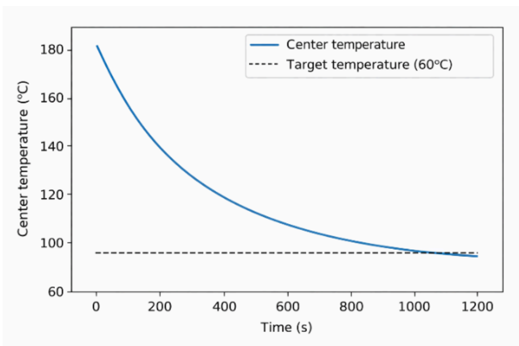

Figure 2: Center temperature vs time

At early times, the cooling rate is relatively high due to the large temperature difference between the solid surface and the surrounding environment which resulted in strong convective heat loss at the boundaries. As cooling progresses, the temperature gradients within the solid diminish, and the rate of heat transfer becomes increasingly diffusion-limited. This behavior is reflected in the gradual flattening of the temperature–time curve at later stages.

The time required for the center of the block to reach the target temperature of 60 °C is extracted directly from the numerical solution. This cooling time provides a practically relevant metric for thermal processing applications and highlights the importance of accurately resolving three-dimensional conduction and boundary heat transfer effects. The explicit tracking of center temperature in the present model enables straightforward determination of such performance indicators.

B. Three-dimensional geometry and spatial context

A three-dimensional wireframe representation of the computational domain is shown in Figure 3 to illustrate the block geometry and its aspect ratios. The dimensions of the block differ along the three spatial directions, leading to unequal surface areas exposed to convective heat transfer on each face. This geometric asymmetry influences the transient temperature distribution and highlights the necessity of a fully three-dimensional heat conduction analysis.

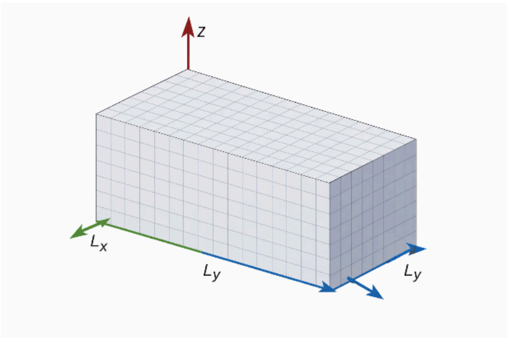

Figure 3: Block geometry (wireframe)

The wireframe visualization provides spatial context for interpreting the temperature distributions discussed in the following subsection and confirms the Cartesian structure of the numerical grid used in the simulations.

C. Three-dimensional geometry and spatial context

The three-dimensional temperature distribution within the block at the final simulated time is visualized using a voxel-based representation in **Figure 4**, where color indicates local temperature magnitude. The temperature field is highly non-uniform which reflects spatial gradients near the external surfaces and relatively higher temperatures in the interior region. Particularly, the lowest temperatures are observed near the corners and edges of the block. This behavior results from enhanced heat loss in these regions due to multi-directional convection, as edges are exposed to heat transfer in two orthogonal directions and corners in three directions simultaneously. In contrast, planar face regions experience convection mainly in a single normal direction. The present results therefore clearly justify the physical significance of explicitly consideration for edge and corner convection in three-dimensional transient cooling problems (Cheikh et al., 2007).

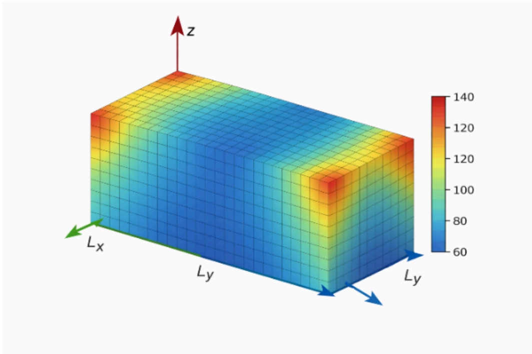

Figure 4: Three-dimensional voxel visualization of the temperature field at the final simulated time.

The interior of the block cools more slowly due to the combined effects of thermal diffusion resistance and the increasing distance from convective boundaries. The smooth temperature gradients observed throughout the domain indicate that the explicit finite-difference scheme produces stable and physically consistent solutions under the chosen grid resolution and time step.

D. Influence of comprehensive boundary treatment

The results demonstrate that a complete and consistent implementation of convective boundary conditions on faces, edges, and corners significantly influences the predicted transient thermal response. Neglecting edge and corner effects would underestimate local heat loss and lead to longer predicted cooling times, particularly for compact geometries with relatively high surface-to-volume ratios (Árpád et al., 2024). By explicitly incorporating multi-directional convection at boundary intersections, the present model captures localized cooling intensification and provides a more realistic description of three-dimensional heat transfer behavior. This capability is mainly important for applications involving thermal processing, electronic packaging, and transient thermal management, where localized temperature variations can strongly affect performance and material integrity. The smooth evolution of both the center temperature history and the three-dimensional temperature field confirm the numerical stability of the explicit scheme when operated within the prescribed stability limit. The use of a conservative time-step selection ensures stable integration while maintaining computational efficiency.

E. Grid Independence and Numerical Validation

To evaluate the sensitivity of the numerical solution to spatial discretization, a grid independence study was performed by repeating the simulations on progressively refined uniform grids. Three grid resolutions were examined, corresponding to spatial step sizes of $\Delta = 4$mm (baseline grid), $\Delta = 3$mm, and $\Delta = 2$mm. All simulations were conducted using identical physical properties, boundary conditions, and time steps selected according to the stability criterion. For each grid, the transient temperature at the geometric center of the block was monitored, and the time required for the center temperature to reach the target value of 60 °C was extracted. The results showed that refinement from $\Delta = 4$mm to $\Delta = 3$mm produced a small reduction in the predicted cooling time, while further refinement to $\Delta = 2$mm resulted in only marginal additional change. The relative difference in center cooling time between the two finest grids was found to be less than a few percent which indicates that the numerical solution is effectively grid independent at the baseline resolution. These results confirm that the chosen grid spacing provides a suitable balance between computational efficiency and solution accuracy, and that spatial discretization errors do not significantly influence the reported cooling behavior.

F. Validation considerations

Direct experimental data for the specific three-dimensional cooling configuration considered in this study are not readily available. However, the numerical formulation is based on well-established finite-difference discretization of the heat diffusion equation and classical convective boundary conditions. The observed cooling trends, including the nonlinear decay of center temperature and enhanced heat loss near edges and corners, are physically consistent with heat transfer theory and prior numerical studies of transient conduction in solids. Furthermore, the smooth temperature evolution and absence of nonphysical oscillations across all grid resolutions provide additional confidence in the numerical stability and correctness of the implementation. Along with the grid independence results, this supports the validity of the proposed numerical framework for predicting transient cooling behavior in three-dimensional solids subjected to convective heat transfer.

4. CONCLUSION

A three-dimensional transient heat conduction model with comprehensive convective boundary treatment has been developed and applied to the cooling of a solid block. The governing heat diffusion equation was solved using an explicit finite-difference scheme on a uniform Cartesian grid, with convective (Robin) boundary conditions implemented consistently on all external faces, edges, and corners. This formulation enables physically accurate representation of multi-directional heat loss at boundary intersections, which is often neglected in simplified three-dimensional models.

The numerical results demonstrate nonlinear transient cooling behavior at the block center, characterized by an initially rapid temperature decrease followed by a diffusion-controlled cooling stage. Three-dimensional temperature field visualizations reveal pronounced spatial gradients near surfaces, with enhanced cooling observed at edges and corners due to simultaneous convection in multiple spatial directions. These effects contribute significantly to the overall thermal response and cooling time prediction. The explicit numerical scheme was shown to be stable and robust when operated within the prescribed stability limits which produces smooth temperature evolution without spurious oscillations.

Overall, this study provides a reliable and extensible numerical tool for analyzing transient heat transfer in three-dimensional solids subjected to convective cooling and provides a clear foundation for future extensions to more complex geometries, variable properties, or coupled thermal–fluid systems.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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