

## Reliability Analysis of Bearing Capacity of Shallow Foundation on $c-\phi$ Soil

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### Abstract

Bearing capacity of shallow foundation is a challenging problem for geotechnical engineers. The difficulty comes from multiple sources of uncertainty. Only the uncertainties in Soil properties; unit weight ( $\gamma$ ), cohesion( $c$ ), and angle of friction ( $\phi$ ) are taken in this study. The Reliability Based Design method is used for the assessment of bearing capacity of shallow foundation by First-and second-order approximation using Taylor's series expansion and Mathcad Computer program which provide systematic analysis of uncertainties and allow the designer how reliable are their designs. The Terzaghi Bearing capacity equation is used for the analysis along with the certain range of variability (CV %) associated with the variables. For the reliability analysis, to become more realistic two real cases of shallow foundation on  $c-\phi$  soil has been taken as a sample case.

**Keywords:** Reliability, Factor of safety, Ultimate Bearing capacity, Mathcad, Coefficient of Variance.

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### 1. Background

The bearing capacity of soils is perhaps the most important of all the topics in soil mechanics and foundation engineering. It is the load carrying capacity of soil. The bearing capacity of a shallow foundation can be defined as the maximum value of the load applied, for which no point of the subsoil reaches failure point. If the bearing capacity of soil at shallow depth is sufficient to safely take the load of the structure, a shallow foundation is provided. Isolated footing, combined footing or strip footing are the option for the shallow foundation. Shallow foundations are designed to ensure that the risk of bearing capacity failure is minimal.

In geotechnical engineering there are mainly two design approaches: Deterministic approach (conventional / ASD method) and Probabilistic approach (reliability base design RBD or LRFD method). Deterministic approach is the conventional one in which the available equation and charts are used to assess the allowable bearing capacity. It is well known that the input parameters for geotechnical calculation are associated with uncertainty. A factor of safety usually three (2.5-3) is commonly applied for the ultimate bearing capacity to account for the uncertainty (Bowles 1996, Murthy 2003, Das 2007). Since the factor of safety chosen in this approach is usually based on experience and judgment, quantitative measure of risk cannot be determined.

Reliability based design approach has become very popular worldwide since last three decade and the trend are towards it increased use. The reliability base method to design shallow foundation is becoming accepted as power tools to assist the designers investigate how reliable their designs are (that means, provide a more consist level of reliability than use of other conventional ASD or deterministic approach). In spite of the fact that it has potential value, reliability theory has not been much used in routine geotechnical practice. This is due to the fact that pre requirements of knowledge in the field of statistics and probability which must designer are not familiar with.

As we built stable structures (Foundation) from long time by increase the section or the surface of the elements constituting this construction by using the empirical total factor of safety. And we are not sure about its reliability and economic construction. So for the reliable and economical design, new concept based on probabilistic approach by covering the coefficient of variance or degree of uncertainty on different design parameters, the reliability analysis concept is developed. Because of the variability and randomness of soil properties, we cannot quantify the risk factor associated with bearing capacity of shallow foundation by using the conventional total factor of safety method only and requires the reliability analysis for economic and reliable design.

## 2. Literature Review

The ultimate bearing capacity theories were given by Terzaghi, (1943; Meyerhof, 1951, 1953, 1963, 1965 and 1967) including methods by Hansen (1961 and 1970) and Vesic (1973) with modification by Bowles (1996) as the basic equation of bearing capacity analysis.

Reliability Analysis of shallow foundations bearing capacity on sand by Ali Alhajami as the thesis of degree of Master of Science in Civil Engineering: In his work, using the lab tests results published Felipe Alberto, the reliability analysis done was performed to evaluate the Terzaghi's theory and evaluate the reliability index of the ASD method that is currently adapted in geotechnical design using Terzaghi's theory for bearing capacity on sand.

Reliability based assessment of shallow foundation was carried out using Mathcad (Ahmad Shlash Alawneh, Osama K. and Ahmed AbdulEzel Al-Mufty (2006). In this work he found that reliability of footing system is more sensitive to the value of angle of friction and its coefficient of variance rather than the other soil properties such as the unit weight or the cohesion. Reliability increases as any of the coefficient of variance of the soil properties decreases. Adopting higher safety factor for cases with higher values of angle of internal friction is recommended as a small variation in the angle of may cause higher variation in ultimate bearing capacity and risk increases.

### 2.1 Variability of soil properties

There are three major sources of uncertainty associated with geotechnical engineering practice: natural heterogeneity, measurement, and transformation uncertainty. Here in this study, only the natural heterogeneity i.e. variability in soil properties is considered. Lumb (1974) stated that most coefficients of variation of soil properties commonly range between 10 and 25% and values out of this range should be avoided or used cautiously. Table 2.1 presents recommended ranges for the coefficient of variation for soil parameters taken from different researchers. Engineering judgment and experience play the basis for selecting the coefficient of variance.

Table 1: Variability of soil properties

Parameters	CV (%)	Note	Reference
$\phi$ , sand	5-15	Recommended 10%	Lee et al. (1983)
$\phi$ , clay	12-56		Lee et al. (1983)
$\tan\phi$ (sand)	5-15		Lumb (1974)
$\phi'$	4		Christian et al. (1994)
$\phi'$ estimated from PI	15-20	clay	Phoon and Kulhway (1996)

$\phi'$ (direct shear test)	7-20	sand, clay	Phoon and Kulhway (1996)
$\phi'$ (triaxial)	10-15	Recommended 30%	Phoon and Kulhway (1996)
Cu(sandy soil)	25-30		Lee et al. (1983)
Cu(clays)	20-50		Lee et al. (1983)
Cu(clays)	20-50		Lumb(1974)
$\square_d$ (Modified Proctor)	1-7	Recommended 5%	Lee et al. (1983)
$\square$	5-10		Lumb (1974)
$\square$	3-7		Duncan (2000)

### 3. Methodology and Materials

The bearing capacity of soil is based on the various variables like load (self-weight and applied load) soil parameters  $c$ ,  $\phi$ ,  $\square$ ; dimension of foundations with certain value of uncertainty. Because of these uncertainty and randomness of these parameter, it is expected that the ultimate bearing capacity ( $q_{ult}$ ) of the foundation will be random variable. By using the values of these random variables ultimate bearing capacity ( $q_{ult}$ ) has been find out with the help of Terzaghi's bearing capacity equation.

Most of the research done on the subject of bearing capacity has used the Terzaghi's equation as it produces very closed value to the actual one and therefore it is the most safe equation compare to other one(Felipe Alberto ,2000).

In order to address this uncertainty and randomness of the bearing capacity variable, the coefficient of variance (CV) is applied for each random variable with the some reference of literature and from some engineering judgments. Table 2.1 presents recommended ranges for the coefficient of variation for soil parameters taken from different researchers.

The probabilistic methods that may be used to determine the distribution of  $q_{ult}$  and consequently the reliability of the chosen factor of safety fall into four categories:

- 1.Point estimation methods (PEM, Rosenbluth method).
- 2.First- and second-order approximation methods usually making use of Taylor's series expansion.
- 3.Monte Carlo methods.
- 4.The exact methods. (e.g.,Cherubini, 1990; Easa,1992).

In this study, Taylor's series is used to expand the distribution function of the ultimate bearing capacity. The method is supported by several authors, (Kapur and Lamberson, 1977; Harr, 1977; Basheer and Najjar, 1998; Duncan, 2000).

The computer program Mathcad is used to facilitate all mathematical and computational effort. Mean ( $\mu_{ult}$ ) and standard deviation ( $\sigma_{ult}$ ) of ultimate bearing capacity are calculated by Mathcad computer program.

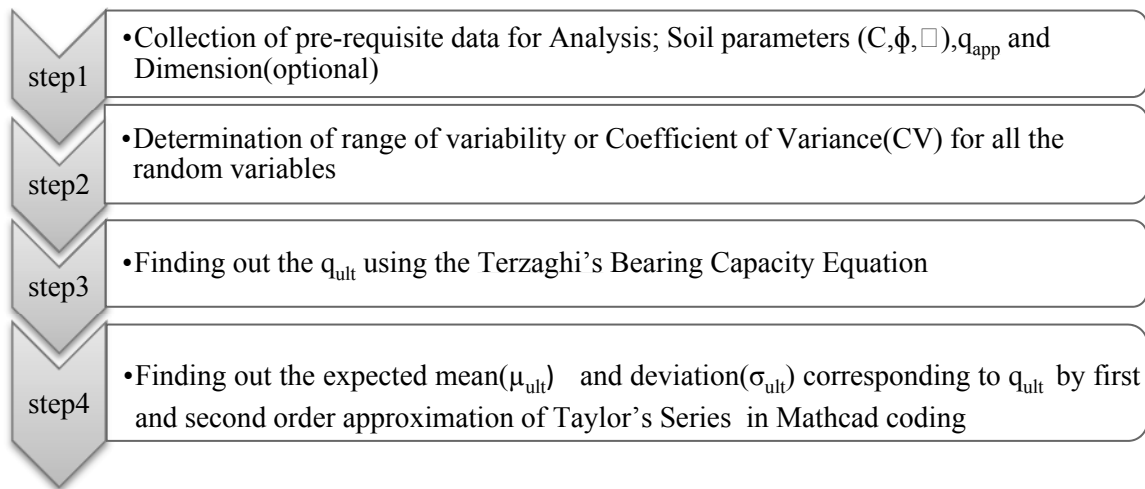
For the Reliability analysis of bearing capacity, it is assumed that probability distribution of ultimate bearing capacity ( $q_{ult}$ ) is a Gaussian Normal one, which is not a very accurate but the level of approximation is accepted. And the probability of failure ( $P_f$ ) is calculated and hence the reliability( $R$ ) and reliability index ( $\beta$ ) both as the normal distributed.

One case of analysis and one for design is considered depending on the input and output parameters. And finally compare the result under different conditions and check the reliability with different value of factor of safety.

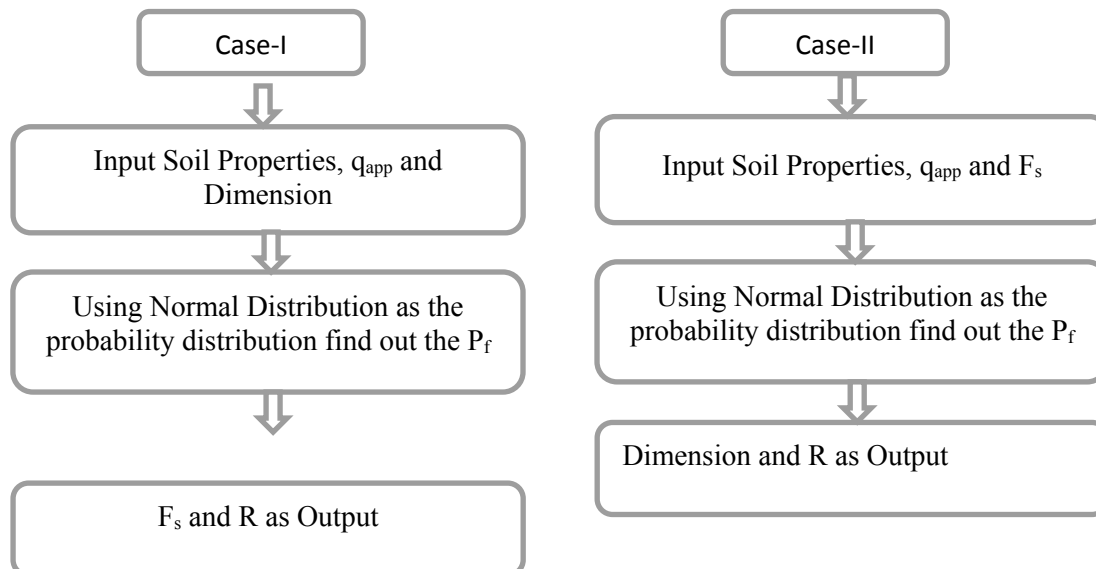
In this study, two real site has been taken as a case study; the shallow foundation of Nuwakot District Hospital, Nuwakot Nepal and the shallow foundation of purposed Ware house of Sipradi Trading Pvt. Ltd, Pakali, Itahari ,Sunsari Nepal for the reliability analysis of bearing capacity.

From the structural Analysis report of the building, the design loads (Q) has been found out for different column footing. From the geotechnical investigation report of the site, the soil parameters have been taken. Then after the reliability of the bearing capacity under the different varied condition has been calculated and the foundation can be designed for the targeted reliability.

**The Methodology for accomplish the objective of the work, can be summarized as:**



Then two cases may be considered according to input output parameters, case-I for analysis and case-II for design.



#### 4. Results and Discussions

For the reliability analysis, two real cases have been taken as the reliability analysis of shallow foundation for c- $\phi$  soil.

For both analysis and design case, due to lack of large number of test data, the coefficient of variance for  $c$ ,  $\phi$  and  $\gamma$  ( $CV_\phi$ ,  $CV_c$ ,  $CV_\gamma$ ) has been adopted from the different research's recommended value as in table 2.3. And based on this recommended value reliability has been checked. However, this level of variability may not occur in these real sites.

$c$ - $\phi$  soil, where the value of  $\phi$  is dominant and value of  $c$  is small. The shallow foundation of Nuwakot District Hospital, Nuwakot Nepal.

Design Load=551 KN (From structural Analysis report)

$C=5$  KN/m<sup>2</sup> (from Geotechnical Investigation Report)

$\phi =27^\circ$  (from Geotechnical Investigation Report)

$\square=18$  KN/m<sup>3</sup> (from Geotechnical Investigation Report)

#### Case-II

$c$ - $\phi$  soil where both  $c$  and  $\phi$  has the significant value. The shallow foundation of purposed Warehouse of Sipradi Trading Pvt. Ltd, Pakali, Itahari, Sunsari Nepal.

Design Load=504 KN (From structural Analysis report)

$C=25$  KN/m<sup>2</sup> (from Geotechnical Investigation Report)

$\phi =24^\circ$  (from Geotechnical Investigation Report)

$\square=20$  KN/m<sup>3</sup> (from Geotechnical Investigation Report)

And adopted range of coefficient of variance  $CV_c=0.2-0.5$ ,  $CV_\phi=0.07-0.20$  and  $CV_\gamma=0.02-0.10$ .

Here by adopting different range of coefficient of variance (CV); lower range ( $CV_c=0.2$ ,  $CV_\phi=0.07$ ,  $CV_\gamma=0.02$ ), medium range ( $CV_c=0.3$ ,  $CV_\phi=0.15$ ,  $CV_\gamma=0.07$ ) and upper range ( $CV_c=0.5$ ,  $CV_\phi=0.2$ ,  $CV_\gamma=0.01$ ) and other factor remaining same the reliability has been analyzed.

Table 2: Output of foundation design with different value of factor of safety for case-I

Given Factor of safety ( $F_s$ )	Dimension ( $B*B$ )	Reliability (R)			Reliability Index ( $\beta$ )		
		Lower range of CV	Medium range of CV	Upper range of CV	Lower range of CV	Medium range of CV	Upper range of CV
1	0.852	0.545	0.597	0.627	0.114	0.246	0.325
1.5	1.031	0.959	0.845	0.815	1.743	1.015	0.896
2	1.18	0.995	0.918	0.881	2.545	1.393	1.178
2.5	1.309	0.997	0.947	0.911	3.018	1.616	1.244
3	1.425	0.998	0.961	0.927	3.328	1.762	1.454
3.5	1.529	0.999	0.969	0.937	3.545	1.864	1.531
4	1.626	0.999	0.974	0.944	3.706	1.94	1.588
4.5	1.7166	0.999	0.977	0.949	3.828	1.998	1.631

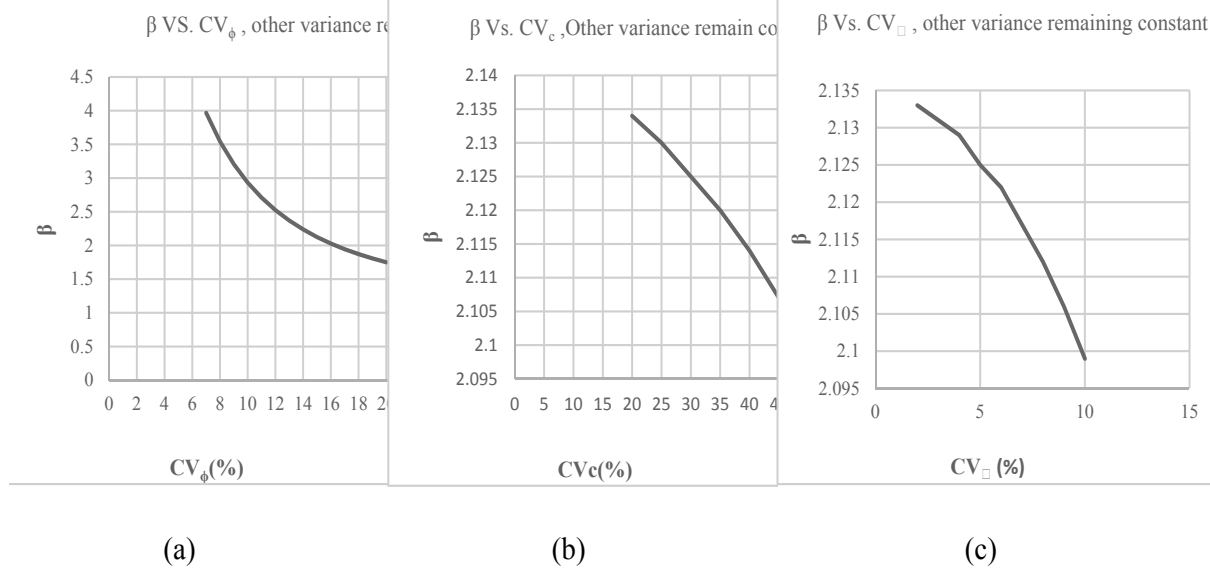


Figure 1: Case-I: Variation of reliability index ( $\beta$ ) with coefficient of variance (CV) for  $\phi$  Figure (a) or Figure (b) and for  $\gamma$  Figure(c).

Table3 Output of foundation design with different value of factor of safety for case-II

Given Factor of safety( $F_s$ )	Dimension ( $B^*B$ )	Reliability (R)			Reliability Index ( $\beta$ )		
		Lower range of CV	Medium range of CV	Upper range of CV	Lower range of CV	Medium range of CV	Upper range of CV
1	0.71	0.526	0.563	0.5781	0.065	0.159	0.197
1.5	1.867	0.967	0.865	0.803	1.842	1.102	0.854
2	0.998	0.997	0.942	0.881	2.731	1.572	1.182
2.5	1.114	0.999	0.968	0.916	3.265	1.853	1.38
3	1.218	0.9999	0.979	0.935	3.621	2.04	1.511
3.5	1.313	0.9999	0.985	0.946	3.876	2.173	1.605

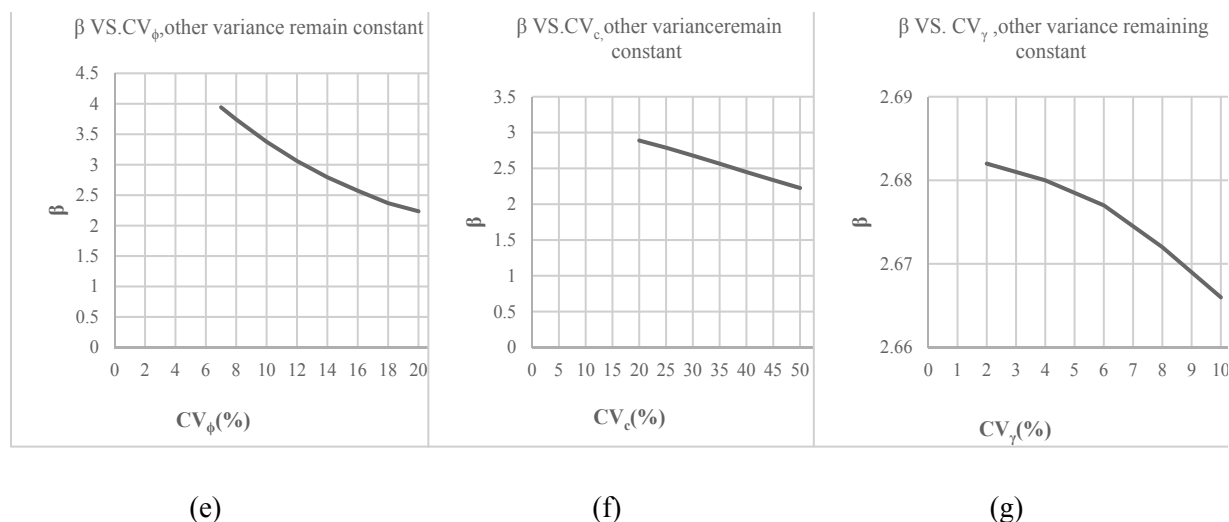


Figure 2: Case-II: Variation of reliability index ( $\beta$ ) with coefficient of variance (CV) for  $\phi$  Figure (d) for c Figure (e) and for  $\gamma$  Figure (f).

In case-I, For the same Factor of safety  $F_s=4.5$ , the reliability( $R$ ) and the reliability index ( $\beta$ ) with different range of CV is 0.999 and 3.828 > 3.8 ( $\beta$  for RC2 as per Euro code) for lower range, 0.977 and 1.998 for medium and 0.949 and 1.63 for upper range. Similarly for case-II for the same Factor of safety  $F_s=3.5$ , the reliability( $R$ ) and the reliability index( $\beta$ ) with Different range of CV is 0.999 and 3.8763.8 ( $\beta$  for RC2 as per Euro code )for lower range ,0.98 and 2.173 for medium range and 0.946 and 1.605 for upper range. So, it is clear that single factor of safety is not always sufficiently reliable and we have to also analyze the problems in probabilistic approach.

And by varying the coefficient of variance for a single variable at a time and remaining other fixed at recommended values, the reliability has been checked. In case-I , the reliability index ( $\beta$ ) has been changed from 1.75 to 3.967 when the coefficient of variance  $CV_\phi$  change from 7% to 20%. And in the case of c and  $\square$ , the variation in  $\beta$  is very small (from 2.099 to 2.134) compared to  $\phi$ . In case-II reliability index ( $\beta$ ) has been changed from 2.236 to 3.943 when the coefficient of variance  $CV_\phi$  change from 7% to 20%,  $\beta$  has been changed from 2.226 to 2.889 when  $CV_c$  change from 20% to 50% and  $\beta$  has been changed from 2.666 to 2.682 when  $CV_\gamma$  change from 2% to 10%. Among these three variables c,  $\phi$  and  $\square$ , the friction angle ( $\phi$ ) has been found most sensitive in case-I while both c and  $\phi$  has significant effect in case-II.

## 5. Conclusions

When we considered the reliability approach in design and analysis, we shall be more confident on our design and analysis as it provides better assessment for the possible involved risk. For a given factor of safety, the reliability of the design may be changed as the coefficient of variance varies. So, it is clear that single factor of safety is not always sufficiently reliable and we have to also analyze the problems in probabilistic approach

## 6. Recommendations

Reliability analysis of bearing capacity in case of shallow foundation has been study by introducing the coefficient of variance for the soil parameters ( $CV_c$ ,  $CV_\phi$  and  $CV_\square$ ) only .For further study coefficient of variance on other variable like variation on magnitude and direction of loading can be done. Reliability analysis of clayey soil or sandy soil can also be done for further study.

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