

## PATTERN-BASED EVACUATION ROUTING PLANNING FOR URBAN AREA

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### Abstract

The rescue of affected people and their property in the urban areas during disaster has always been prominently important. The effective way of the evacuation has been important as well as interesting field of research for decades. The pattern-based evacuation (PBE) model restructures the traffic routing with the assumption of avoiding the crossing conflicts and minimizing the merging conflicts in the intersections. The problem has been formulated as a mix-integer programming problem. This paper focuses on the survey of the PBE model.

**Keywords:** *Evacuation planning, mix-integer programming, disaster management.*

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### 1. Introduction

The population of the world has been facing disastrous problems like earthquake, landslides, floods, fire, roadside accidents and is going to face those problems in the coming days too. The number of natural or manmade disasters and the size of population affected by such events have been rising [25]. Evacuation of people in those situations is one way to saving lives, protecting property and mitigating vulnerability. Evacuation is a process in which affected people and their property are shifted from dangerous places to safer places to mitigate vulnerability during disaster. The word "disaster management" includes preparedness, planning, response and recovery that initiate all aspects of planning for and responding to disasters, including pre, during and post-disaster activities. Preparedness is focused on activities and measures taken in advance to ensure effective response to the impact of hazards, including the issuance of timely and effective early warnings and the temporary evacuation of people and property from disaster zone. Evacuation planning is valuable because evacuations routinely result in travel demand that exceeds the available network capacity and rescues many more lives. Planning measures can take many forms including the construction of shelters, efficient transportation, implementation of an emergency communication, supply of necessary logistics, creation of back-up life-line services. Response is the provision of assistance or intervention during or immediately after a disaster to meet the life preservation and basic subsistence needs of those affected people. Decisions and actions taken after a disaster with a view to restoring or improving the pre-disaster living conditions of the stricken community, while encouraging and facilitating necessary adjustments to reduce disaster risk in the recovery phase.

Many cities of the world are not only suffering from disasters but also from traffic congestion. The congestion happens due to population density, unplanned city, narrow streets, over crowded vehicles and intersections within street sections due to lack of fly over or fly under within intersections. It is also great task that sending the people as early as possible out of the dangerous zone in to a safe zone during disaster due to above mentioned causes. It seems that the systematic traffic routing planning should be developed in order to mitigate vulnerability during disaster. The arising large-scale

evacuation planning problems have established an emerging research frontier not only from the mathematical point of view but also for the betterment and security of society from the perspectives of life and property[23]. The techniques that are currently employed could be broadly based on operations research methodology and simulation approaches.

The paper focuses disaster planning concentrating on traffic routing that is pattern-based evacuation PBE routing planning in urban area which deals with the form of dynamic network flow. The PBE model will restructure the traffic routing so that the evacuees leave the evacuation area as safe as possible and as early as possible within the considered time horizon [24]. Bretschneider and Kimms[4] assume that every person leaves the evacuation zone by own vehicles. The PBE model is the mixed inter programming problem which minimizes the weighted sum of flows entering the arcs that lead to the super sink where the weights are increasing with time. The pattern-based evacuation model is all specially network structured problems that can be solved optimally. We review the state-of-art of the pattern-based evacuation planning for urban area.

The paper is organized follows: Section 2 gives the prescriptive evacuation routing planning, Section 3 provides the mathematical model formulation, Section 4 provides solutions procedure and Section 5 describes the concluding remarks.

## **2. Prescriptive Evacuation Routine Planning**

Ford and Fulkerson [10] introduce dynamic network flow problem. The maximum dynamic flow problem is to determine the maximum amount of flow from source to sink within discrete time horizon  $T$  that means sending maximum evacuees from disaster zone in to safe zone within evacuation time. It can be used to describe an elementary evacuation problem that is formulated as the linear programming problem. Those models describe network structured real-life decision making problems including evacuation and traffic route assignment systems. The approach of dynamic network flows over time namely maximum dynamic flows, earliest arrival flows or quickest flows can be applied to disaster planning in the urban areas. A maximum flow of evacuees is obtained converting a maximal dynamic flow into time expanded static flow network that is given by Ford and Fulkerson [11]. The dynamic flow problem sends maximum flow, based on temporally repeated flows computed by minimum cost flow problem, of evacuees from the disaster zone to safety zone see the reference of authors[8,13,28]. The maximum dynamic flow problem can be solved by taking temporarily repeated flow technique. In this approach first a minimum cost flow algorithm is applied to the original static network. Then the optimal flow is decomposed in to chain flows and finally each chain flow is repeated from time 0 to time horizon  $T$  (total time on the chain). Aronson [1] and Hoppe and Trados[14] present a state-of-the-art survey of dynamic network flows. The most frequently considered dynamic network flow problems are the maximum flow problem, the lexicographical maximum flow problem, the quickest flow problem, the earliest arrival flow problem and the minimum cost flow problem. Gale [12] extends the maximal dynamic flow problem into earliest arrival flow problem which is called maximum universal flow. The earliest arrival flow problem translated into the evacuation context means that every considered time period, the maximal amount of evacuees enters the safe area. Aronson [1] and Hoppe and Trados[14] give surveys of discrete dynamic network flows where the time horizon is subdivided into time periods of equal length. Hoppe and Trados[14] present algorithms for dynamic network flow problems.

Barret et.al [2] develop dynamic traffic management model for hurricane evacuation. Hamachar and Tjandra[13] give the mathematical modeling of evacuation problems. They present variations of discrete time dynamic network flow problems to model evacuation problems for building evacuation. The lane reversal strategy is incorporated in the following researches of evacuation planning: Cova and Johnson [7] present a network flow model identifying optimal lane-based evacuation within a

complex urban road network. The model is based on an integer extension of the minimum cost flow problem routing plan a lane-based model that minimizes the total travel distance while forbidding intersection crossing conflicts and bounding the total number of merging conflicts. The number of crossing conflicts within an intersection can be computed by:

$$\alpha(\alpha - 2) + \alpha \sum_{n=0}^{\alpha-3} n\{-n + (\alpha - 1)\},$$

Where  $\alpha$  is the number of lanes and  $\alpha \geq 3$ .

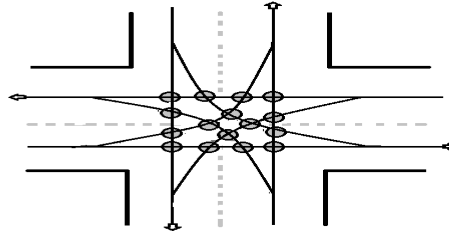


Fig 1 Crossing conflicts within intersection

But this model is not dynamic one. To valid the results "Microscopic Traffic Simulation Software-Paramices" is used. This idea has recently been expanded by Rebennack et al [20]. Rebennack et al [20] and Kim and Shekhar [16] prove that the problem of contra flow network for evacuation planning is NP-complete and present a heuristic to solve the problem for large scale instances.

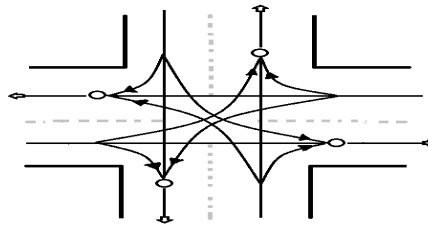


Fig 2 Merging conflicts within intersection

Chiu and Zheng [6] present an "one-destination cell-transmission-based linear programming model". The approach is extended for the multi-commodity case of same authors in [5]. Skutella [21] gives an introduction to network flows over time concentrating on continuous flows over time. Stepanov and MacGregor-Smith [22] consider a route assignment model for an evacuation. This is an integer programming model with multiple objectives; minimizing the total evacuation time, the total distance and traffic congestion. Xie and Turnquist [26] develop a bi-level program based on a cell transmission model formulation that prohibits crossing conflicts within intersections and allows lane reversal on streets. Kimms and Massen [18] develop a cell transmission based model that includes lane reversal and suggest [17] a heuristic approach to solve the model for large-scale networks. Bretschneider and Kimms [4] present the detailed overview of evacuation approaches a pattern-based dynamic network flow model that restructures the traffic routing for the case of an evacuation.

### 3. Mathematical Model Formulation

#### 3.1 Mixed -Integer Urban Evacuation Model

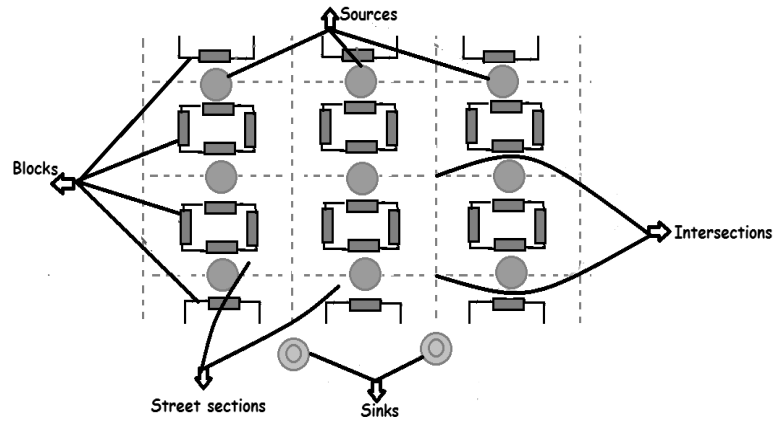


Fig 3 A street network representation with blocks, sources, sinks, street sections and intersections

Kimms et.al [3] develops the idea of a network flow model formulation by defining an appropriate network and formulates a mix integer programming within intersections. The network that exists between the disaster zone and the safety zone to be considered as graph network with nodes (multiple sources, multiple sinks and intersections within street sections) and arcs (usable lanes within street sections). Super source  $S$  and super sink  $D$  are introduced to handle multiple sources and multiple sinks. The super source node is connected to the source nodes, having zero travel time and infinite capacity and the super sink node is connected to all sinks with zero travel time and infinite capacity.

Let,  $G = (N, A)$  be a static directed network where  $N = \text{set of nodes} = \{\{S, 0\}\} \cup S \cup N_j \cup D \cup \{D, 0\}$  and  $A = \text{set of arcs} = \{A_S \cup A_C \cup A_J \cup A_D \cup A_{D^*}\}$ . The other notations are mentioned below:

$J = \text{Set of intersections}$ ,  $S = \text{Set of sources}$ ,  $D = \text{Set of sinks}$ ,  $P = J \times J = \text{Set of all ordered pairs of intersections}$ .

$N_j = \text{Set of nodes in the intersections}$ ,  $A_J = \text{Set of arcs in the intersections}$ ,  $A_D = \text{Set of arcs joining } (j, \ell, i, 0) \text{ and } (i, 0)$ , for all  $(i, 0) \in D$ ,  $A_{D^*} = \text{Set of arcs from every sink in } D \text{ to the super sink } \{D, 0\} \text{ in the intersections}$ ,

$P = S \cup D$ ,  $A_C = \text{Set of arcs that connect sources and sinks}$ ,  $A_J^{sf} = \text{Set of straight forward arcs that are all arcs that do not directly turn left or right}$ ,  $A_J^l = \text{Set of left turn arcs within an intersection}$ ,  $\{S, 0\} = \text{Super source}$ ,  $\{D, 0\} = \text{Super sink}$ ,  $L(j, \ell, i, h) = \text{The number of lanes in the street section between two intersections } i \text{ and } j$ ,  $\lambda(j, \ell, i, h) = \text{The number of usable lanes in the street section where } \forall (j, \ell, i, h) \in A_C \cup A_J \cup A_D$ .  $T = \text{Time units}$ ,

$\mathcal{T} = \{0, 1, 2, \dots, T\}$ ,  $o(i, j) = \text{The number of vehicles which start at the corresponding source } (i, j) \in S$ ,  $\tau(j, \ell, i, h) \in N = \text{The travel time on arc } (j, \ell, i, h) \in A$ ,  $\tau(j, \ell, i, h) = 0 \forall (j, \ell, i, h) \in A_S \cup A_D$ .

$C^{in}(j, \ell, i, h, p) = \text{The number of vehicles which can enter a lane per time unit where } (j, \ell, i, h) \in A_C \cup A_J \cup A_D$ .

$c(j, \ell, i, h, p) = \text{The maximum number of vehicles on a single lane (per time unit) for the street where}$

$(j, \ell, i, h) \in A_C \cup A_J \cup A_D$ . The arcs in the set  $A_S \cup A_{D^*}$  have unlimited capacity.  $x(j, \ell, i, h, t)$  = The inflow of vehicles from node  $(j, \ell)$  to node  $(i, h)$  in time  $t + (\tau_{j, \ell, i, h}), \forall t \in \mathcal{T}$ . Let  $y(j, \ell, i, h)$  be binary variable such that  $y(j, \ell, i, h) = 1$  when  $(j, \ell, i, h) \in A_J^{sf} \cup A_J^l$  and  $y(j, \ell, i, h) = 0$ , otherwise.

The evacuation model is given as:

### Objective Function

The weighted sum of flows on the arcs to the super sink  $(D, 0)$  where the weights increase in time, is to be minimized. Mathematically,

$$\text{Minimize } \sum_{t \in \mathcal{T}} \sum_{(j, l, D, 0) \in A_{D^*}} t \cdot x(j, l, D, 0, t) \quad (1)$$

With flow enforcement constraints, flow conservation constraint, inflow capacity constraint, street capacity constraint, lane consistency constraints, coupling constraints, prohibition of crossing conflicts constraints and avoidance of un-normal flow behavior constraints.

### 3.2 Pattern-Based Urban Evacuation Model

Kimms et.al.[4] develop pattern-based evacuation model that is based on dynamic network flow problem with time expanded detailed network which is the extension of evacuation model that is discussed in section (3.1) also called mixed - integer urban evacuation model. Furthermore, Kimms et.al.[4] assume that evacuees leave evacuation zone by own vehicles i.e. it is auto based evacuation model. Patterns are generated for arcs within intersections and for arcs corresponding to street sections between two intersections. Patterns are considered non-dominated patterns to solve the PBE model. A pattern is said to be non-dominated if it is not included in any other pattern. In other words, if it is not possible to add one lane without violating a constraint. Patterns determine the capacities of street section, the inflow capacity and the number of usable lanes. Exactly one pattern has to be chosen for each section.

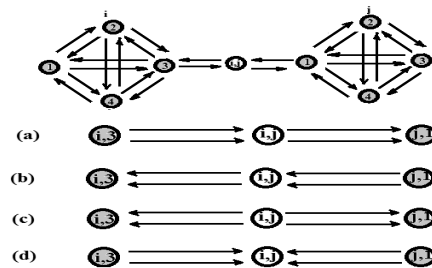


Fig 4 All non-dominated patterns with two usable lanes on both street segments but (d) is not considered

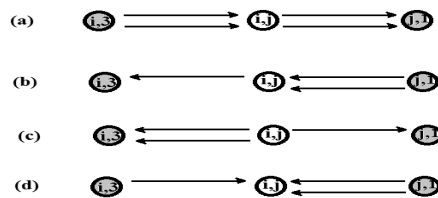


Fig 5 Patterns (b), (c) and (d) are dominated by pattern (a)

This evacuation model minimizes the weighted sum of flow entering the arcs that point to the super-destination (safe zone) within time horizon  $T$ . The minimization of the objective model is that the evacuees should leave the evacuation zone as early as possible within the time horizon  $T$ . The goal is

to restructure the traffic routing for the case of an evacuation through the readjustment of every street to one-way street, street can be divided into two sections and sections can lead in opposing directions, prohibition of crossing conflicts and limitation of number of merging lanes within intersections. The notations for this model denote as follows:

Let,  $G = (N, A)$  be a static directed network where  $N = \text{set of nodes} = \{\{S, 0\}\} \cup S \cup N_J \cup D \cup \{D, 0\}$  and

$A = \text{set of arcs} = \{A_S \cup A_C \cup A_J \cup A_D \cup A_{D^*}\}$ . The other notations are mentioned below:

$J = \text{Set of intersections}, S = \text{Set of sources}, D = \text{Set of sinks}, P = J \times J = \text{Set of all ordered pairs of intersections.}$

$N_J = \text{Set of nodes in the intersections}, A_J = \text{Set of arcs in the intersections}, A_D = \text{Set of arcs joining } (j, \ell, i, 0) \text{ and } (i, 0), \forall (i, 0) \in D, A_{D^*} = \text{Set of arcs from every sink in } D \text{ to the super sink } \{D, 0\} \text{ in the intersections,}$

$P = S \cup D, A_C = \text{Set of arcs that connect sources and sinks}, A_J^{sf} = \text{Set of straight forward arcs that are all arcs that do not directly turn left or right}, A_J^l = \text{Set of left turn arcs within an intersection}, \{S, 0\} = \text{Super source}, \{D, 0\} = \text{Super sink}, L(j, \ell, i, h) = \text{The number of lanes in the street section between two intersections } i \text{ and } j. \lambda(j, \ell, i, h) = \text{The number of usable lanes in the street section where } \forall (j, \ell, i, h) \in A_C \cup A_J \cup A_D. T = \text{Time units,}$

$\mathcal{T} = \{0, 1, 2, \dots, T\}. P_J = \text{Set of all feasible patterns of intersections}, P_C = \text{Set of all feasible patterns associated with street sections between two intersections}, G_J = \text{Distinguished pattern for all } i \in J, G_C = \text{Different types of patterns.}$

$\forall (i, j) \in P \text{ where } P = S \cup D. P_C = \bigcup_{g=1}^{G_C} P_C(g) = \text{Set of patterns for arcs that connect two intersections.}$

$P_J = \bigcup_{g=1}^{G_J} P_J(g) = \text{Set of patterns for arcs within intersections. } o(i, j) = \text{The number of evacuees that are assigned to source } (i, j) \in S, \tau(j, \ell, i, h) \in N = \text{The travel time on arc } (j, \ell, i, h) \in A, \tau(j, \ell, i, h) = 0 \forall (j, \ell, i, h) \in A_S \cup A_D. C^{in}(j, \ell, i, h, p) = \text{The inflow capacity of the arc } (j, \ell, i, h) \in A_C \text{ in pattern } p \in P_C. c(j, \ell, i, h, p) = \text{Total flow of arc } (j, \ell, i, h) \text{ in pattern } p \in P_C. x(j, \ell, i, h, t) = \text{The inflow of vehicles from node } (j, \ell) \text{ to node } (i, h) \text{ in time } t + \tau(j, \ell, i, h), \forall t \in \mathcal{T}. \text{Let } y_c((i, j, p)) \text{ be binary variable such that } y_c((i, j, p)) = 1 \text{ when } p \in P_C, (i, j) \in P \text{ and } y_c((i, j, p)) = 0 \text{ otherwise. Similarly } y_j((j, p)) \text{ be binary variable such that } y_j((j, p)) = 1 \text{ when } p \in P_J, j \in J \text{ and } y_j((j, p)) = 0, \text{ otherwise.}$

### Objective Function

The weighted sum of flows entering the arc that lead to the super-sink is to be minimized, where:  $\mathcal{T} \rightarrow \mathcal{R}$  is increasing with time. Mathematically,

$$\text{Minimize } \sum_{t \in \mathcal{T}} \sum_{(d, i) \in N_D} \gamma(t). x(d, i, D, 0, t) \quad (2)$$

Network flow constraints

Flow enforcement: All vehicles of evacuees have to leave the corresponding sources and reach the sink within the considered time horizon T.

$$\sum_{t \in \mathcal{T}} x(S, 0, j, i, t) = O(j, i), \text{ for all } (i, j) \in S \quad (3)$$

$$\sum_{t \in \mathcal{T}} \sum_{(d_1, d_2) \in D} x(d_1, d_2, D, 0, t) = \sum_{(i, j) \in S} O(j, i) \quad (4)$$

Flow conservation constraint: A flow that leaves a node in time period t must have reached that node before (i.e. no holdover time).

$$\sum_{(i,h) \in N: (i,h,j,l) \in A} x(i, h, j, l, t - \tau(i, h, j, l)) = \sum_{(i,h) \in N: (i,h,j,l) \in A} x(i, h, j, l, t) \quad (5)$$

for all  $(j, l) \in N(S, 0), (D, 0): t \in \tau$

Traffic routing constraints: Patterns determine the parameters inflow and total capacity of the corresponding street sections.

$$\sum_{p \in P_{J(\text{type}(i))}} y_J(i, p) = 1, \text{ for all } i \in J: \alpha_i \geq 2 \quad (6)$$

$$\sum_{p \in P_{C(\text{type}(i,j))}} y_C(i, j, p) = 1 \text{ for all } (i, j) \in P \quad (7)$$

Inflow capacity constraints: The restriction of the inflow for every point in time depends on the chosen pattern for every street segment within an intersection (with  $\alpha \geq 2$ ) and for every street section between two intersections.

$$X(i, h, l, t) \leq \sum_{p \in P_{J(\text{type}(i))}} c^{in}(i, l, i, h, p) y_J(i, p) \quad (8)$$

for all  $(i, l, i, h, p) y_J(i, p)$

$$X(i, l, i, j, t) \leq \sum_{p \in P_{C(\text{type}(i,j))}} c^{in}(i, l, i, j, p) y_C(i, j, p) \quad (9)$$

for all  $(i, l, i, j) \in A_C$

with  $(i, l) \in N_j: t \in \tau$

Total street capacity constraints: The restriction of the total flow for every point in time depends on the chosen pattern for every street segment within an intersection (with  $\alpha \geq 2$ ) and for every street section between two intersections.

$$\sum_{\bar{t} \in \{t - \tau(i, l, i, h) + 1, \dots, t\}} x(i, l, j, \bar{t}) \leq \sum_{p \in P_{J(\text{type}(i))}} C(i, l, i, h, p) y_J(i, p) \quad (10)$$

for all  $(i, l, i, h) \in A_j, t \in \tau$

$$\sum_{\bar{t} \in \{t - \tau(i, l, i, h) + 1, \dots, t\}} x(i, l, j, \bar{t}) \leq \sum_{p \in P_{C(\text{type}(i,j))}} C(i, l, i, j, p) y_C(i, j, p) \quad (11)$$

for all  $(i, l, i, j) \in A_C, (i, l) \in N_j, t \in \tau$

$$\sum_{\bar{t} \in \{t - \tau(i, l, i, h) + 1, \dots, t\}} x(i, j, i, l, \bar{t}) \leq \sum_{p \in P_{C(\text{type}(i,j))}} C(i, j, i, l, p) y_C(i, j, p) \quad (12)$$

for all  $(i, j, i, j) \in A_C, (i, l) \in N_j, t \in \tau$

## 4. Solution Procedure

### 4.1 Heuristic

The analytic solution for evacuation model does not exist yet however Kimms et.al.[4] has derived computational solution by using 900 instances and applying software AMPL using CPLEX 10.0. A computer running Windows XP with an AMD Athlon(tm) 64 ×2 Dual Core Processor 4600 + (2.41 GHz), 1.96 GB RAM is used. They are considering several networks in "lattice structure" with intersections choosing  $\alpha_i = 4$  entrances. The sinks, the number of evacuees and the number of lanes are varied. The considered time periods are  $\mathcal{T} = \{1, 2, 3 \dots 150\}$

i.e.  $T = 150$ . It is seen that the problem instance can be infeasible if the maximal number of periods  $T$  is too small, in general  $T = 150$  is used. To achieve a nearly optimal solution in an appropriate amount of time, a relaxed version of this model is solved. Then the solution of the  $\lambda$ - variables is adjusted such that it is feasible in terms of the basic evacuation model and finally the model is solved with the adjusted  $\lambda$ - variables being fixed. Relaxed based approaches is applied which restructures this model

without considering "Coupling constraints", "Prohibition of crossing conflicts I, II, III" and without bi-variable  $y$ . The relaxation-based approach is structured as follows:

Solve the relaxation of the evacuation model. Execute the adjustment heuristic: Adjustment of the  $\lambda$ -variables such that they are feasible with respect to the evacuation model. Solve the evacuation model with fixed  $\lambda$ -variables achieved by the adjustment heuristic: compute the inflow variables  $x$  and the value of the objective function.

## 4.2 Two-Stage Heuristic

### 4.2.1 First –Staged Heuristic

In this stage (step 1) the network is reduced that used in the one-stage pattern-based evacuation model. Intersections are considered as single node. The goal is to find patterns and flows of arcs that are associated with street sections between intersections and intersection and source-node or destination node.

Mathematical notations:

$J$  = Set of intersections,  $\alpha_i$  = The number of entrances/exits of intersection  $i$ , for all  $i \in J$ ,  $S$  = Set of sources,

$D$  = Set of sinks,  $P = J \times J$  = Set of all ordered pairs of intersections.  $N_j$  = Set of nodes in the intersections,  $A_j$  = Set of arcs in the intersections,  $A_D$  = Set of arcs joining  $(j, \ell, i, 0)$  and  $(i, 0)$ ,  $\forall (i, 0) \in D$ ,  $A_{D^*}$  = Set of arcs from every sink in  $D$  to the super sink  $\{D, 0\}$  in the intersections,  $P = \text{SU } D$ ,  $A_C$  = Set of arcs that connect sources and sinks,  $A_j^{sf}$  = Set of straight forward arcs that are all arcs that do not directly turn left or right,  $A_j^l$  = Set of left turn arcs within an intersection,  $\{S, 0\}$  = Super source,  $\{D, 0\}$  = Super sink,  $L(j, \ell, i, h)$  = The number of lanes in the street section between two intersections  $i$  and  $j$ .  $\lambda(j, \ell, i, h)$  = The number of usable lanes in the street section where  $\forall (j, \ell, i, h) \in A_C \cup A_j \cup A_D$ .  $T$  = Time units,  $\mathcal{T} = \{0, 1, 2, \dots, T\}$ .  $P_j$  = Set of all feasible patterns of intersections,  $P_C$  = Set of all feasible patterns associated with street sections between two intersections,

$G_j$  = Distinguished pattern for all  $i \in J$ ,  $G_C$  = Different types of patterns for every  $(i, j) \in P$  where  $P = \text{SU } D$ .

$P_C = \bigcup_{g=1}^{G_C} P_C(g)$  = Set of patterns for arcs that connect two intersections.  $P_j = \bigcup_{g=1}^{G_j} P_j(g)$  = Set of patterns for arcs within intersections.  $0(i, j)$  = The number of evacuees that are assigned to source  $(i, j) \in S$ ,  $\tau(j, \ell, i, h) \in N$  = The travel time on arc  $(j, \ell, i, h) \in A$ ,  $\tau(j, \ell, i, h) = 0$ ,  $\forall (j, \ell, i, h) \in A_S \cup A_D$ .  $C^{in}(j, \ell, i, h, p)$  = The inflow capacity of the arc  $(j, \ell, i, h) \in A_C$  in pattern  $p \in P_C$ .  $c(j, \ell, i, h, p)$  = Total flow of arc  $(j, \ell, i, h)$  in pattern  $p \in P_C$ .  $x(j, \ell, i, h, t)$  = The inflow of vehicles from node  $(j, \ell)$  to node  $(i, h)$  in time  $t + \tau(j, \ell, i, h)$ , for all  $t \in \mathcal{T}$ . Let  $y_c((i, j, p))$  be binary variable such that  $y_c((i, j, p)) = 1$  when  $p \in P_C, (i, j) \in P$  and  $y_c((i, j, p)) = 0$  otherwise. Similarly  $y_j((j, p))$  be binary variable such that  $y_j((j, p)) = 1$  when  $p \in P_j, j \in J$  and  $y_j((j, p)) = 0$  otherwise.

### Objective Function

The weighted sum of flows entering the arc that lead to the super-sink is to be minimized, where:  $\mathcal{T} \rightarrow \mathcal{R}$  is increasing with time. Mathematically,

$$\text{Minimize } \sum_{t \in \mathcal{T}} \sum_{(d, i) \in N_D} \gamma(t) \cdot x^l(d, i, D, 0, t) \quad (13)$$

Network flow constraints:

Flow enforcement:



$$\sum_{t \in \tau} x^l(S, 0, j, i, t) = O(j, i), \text{ for all } (i, j) \in S \quad (14)$$

$$\sum_{t \in \tau} \sum_{(d,i) \in N_D} x^l(d, i, D, 0, t) = \sum_{(i,j) \in S} O(j, i) \quad (15)$$

Flow conservation constraint:

$$\sum_{(i,h) \in N^l: (i,h,j,l) \in A^l} x^l(i, h, j, l, t - \tau(i, h, j, l)) = \sum_{(i,h) \in N^l: (i,h,j,l) \in A^l} x^l(i, h, j, l, t) \quad (16)$$

for all  $(j, l) \in N^l(S, 0), (D, 0): t \in \tau$

Traffic routing constraints:

$$\sum_{P \in P_C^l(\text{type}(i,j))} y_c^l(i, j, p) = 1, \text{ for all } (i, j) \in P \quad (17)$$

Capacity constraints:

$$x^l(i, 0, i, j, t) \leq \sum_{P \in P_C(\text{type}(i,j))} c^{inl}(i, 0, i, j, p) y_c^l(i, j, p), \quad (18)$$

for all  $(i, 0, i, j) \in A_C^l, t \in \tau$

Inflow capacity constraints:

$$x^l(i, j, i, 0, t) \leq \sum_{P \in P_C(\text{type}(i,j))} c^{inl}(i, j, i, 0, p) y_c^l(i, j, p), \quad (19)$$

for all  $(i, j, i, 0) \in A_C^l, t \in \tau$

Total street capacity constraints:

$$\sum_{\bar{t} \in \{t - \tau(i, 0, i, j) + 1, \dots, t\}} x^l(i, 0, i, j, \bar{t}) \leq \sum_{p \in P_C(\text{type}(i,j))} c^l(i, 0, i, j, p) y_c^l(i, j, p) \quad (20)$$

for all  $(i, 0, i, j) \in A_C^l, t \in \tau$

$$\sum_{\bar{t} \in \{t - \tau(i, j, i, 0) + 1, \dots, t\}} x^l(i, j, i, 0, \bar{t}) \leq \sum_{p \in P_C(\text{type}(i,j))} c^l(i, j, i, 0, p) y_c^l(i, j, p) \quad (21)$$

for all  $(i, j, i, 0) \in A_C^l, t \in \tau$

#### 4.2.2 Second-Staged Heuristic

In this stage (step 2) directions within intersections are included that is not considered in first stage. Crossing and merging conflicts within intersections are restricted. The result of step 1 of decision variables.

$y_c^l(i, j, p)$  are translated to the model of step 2. Parameters are generated by them in step 2 such that the directions and capacities for the street-segments between intersections, sources and destinations are fixed in step 2. The patterns of street segments between two intersections are fixed which are already selected in step 1. That is just the patterns for the intersections have to be chosen in step 2. All feasible patterns  $P_j$  have to be generated before the optimization. The mathematical formulation of step 2 is similar as the one-stage pattern-based evacuation model which has already mentioned in section 3.2. Only the traffic routing constraint given as

$$y_c^l(i, j, p), \forall (i, j) \in P, p \in P_C(\text{type}(i, j))$$

are decision variables in the entire model. Exactly one pattern is chosen for arcs that connects intersections, sources and destinations.

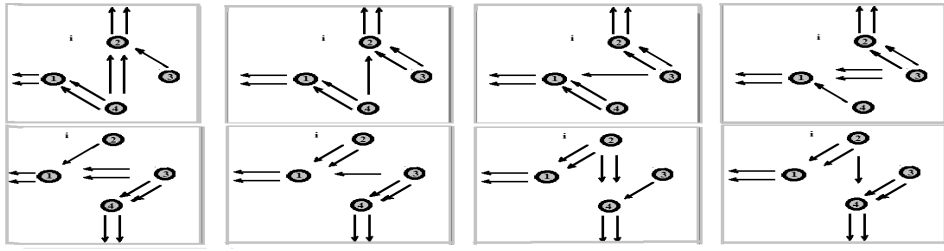


Fig 6 Patterns with two entrances and two exits (neighboring)

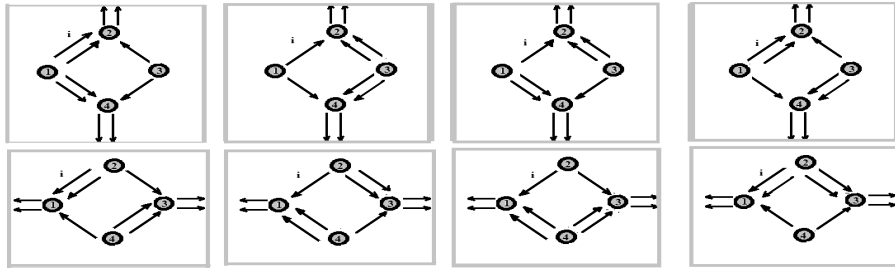


Fig 7 Patterns with two entrances and two exits (alternating)

Khimet.al.andRebennack et.al [16, 20] show that certain dynamic network flow problems with lane reversal which are similar to the PBE model are NP-complete that is this model is NP-complete. Analytic solution may be possible for small-scale evacuation problems but may be possible only approximate solution for large-scale evacuation problems. However, Kimms et.al [4] have derived computational solution by applying software AMPL using CPLEX 10.0. A computer running Windows XP with an AMD Athlon(tm) 64×2Dual Core Processor 4600 + (2.41 GHz), 1.96 GB RAM is used. The considered 1180 instances were based on grid network of different sizes with different exit-patterns and number of inhabitants. The results of the presented approach of 1180 tested instances are near the results of the corresponding relaxations of the one-stage evacuation model of these instances [4].

## 5. Concluding Remarks

In this survey, we intended to integrate various mathematical formulation related to pattern-based evacuation (PBE) model with various heuristic and relaxed approaches. This model is extended form of the evacuation model routing plan within a complex urban road network regarding crossing and merging conflicts within intersection that was presented by Cova and Johnson [7] without lane-reversal. Effectiveness of lane-based model depends on coordination among emergency personal, coordination from evacuees. This model provides systematic traffic routing planning within intersections that avoids crossing and merging conflicts which restructures the street network. It is auto-based model that incorporates those evacuees having own vehicles but not other models like bus-based, transit-based, multi-commodity-based that indicates its limitations and its existence. Smart cities based on this model may be effective to shift people from dangerous places to safer places in order to reduce the health and life vulnerability of affected people as quickly as possible. This model may also apply in the busy road and traffic congestion because of its salient features regarding systematic routing planning that helps to reduce energy consumption and time. It may seem to be the best choice to make smart city to improve civilians' safety and to reduce time and energy consumption. It may be better model to rescue the people by evacuating a certain area than other models. Kimms et.al. [4] have derived computational solution by applying software AMPL using

CPLEX 10.0 for one-stage pattern-based, one-stage and two-stage heuristics approaches as well as relaxed-based approaches. The results of the presented approach of 1180 tested instances are near the results of the corresponding relaxations of the one-stage evacuation model of these instances. The solution of the problem does not exist yet analytically for large scale evacuation problems.

Further research is necessary to get analytic solution through algorithms. Case study and further research will be carried out for implementation of this model for smart city. More advanced software has to be developed in order to get the better approximate solution than Kimms et.al.[4].

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