The Economic Journal of Nepal, Vol. 13, No. 4, Issue 52, October-December 1990 © DOE-TU

Social Welfare Measurement : Some Theorems and A New Index

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The paper seeks to make good certain deficiencies in common partial and overall welfare measures and to develop an overall welfare index which takes into consideration distribution on both the sides of the poverty line.

INTRODUCTION

Quite a few years in the recent past have witnessed efforts at operationalisation of the social welfare function in order to evolve some empirical quantitative measures. These measures can be fruitfully employed for comparison of welfare/illfare content of income distribution vectors belonging to different points of time or different groups/ regions. Viewed in this way, measures relating to poverty and inequality are also social welfare measures. But they are either partial or negative or partial and negative both.

However, while overall welfare measures developed by Sen (1974) and Kakwani (1980) are homogeneous of degree one in individual incomes, poverty and inequality measures developed by a host of people are homogeneous of degree zero. Inequality measures are also normalized for population size, exception being Theil's entropy measure (1967). The same is true of usual poverty measure which makes explicit use of absolute poverty line along with truncated income vector.

As regards Sen's index of poverty, Takayama (1979) finds it focussing on absolute aspects of poverty and 'less geared to relativities than it needs be', Blackorby and Donaldson (1980) find it to be a relative poverty index which they define as one whose value is unchanged when all incomes and the poverty line itself are multiplied by a positive scalar. They defined absolute poverty index which is normalized neither for poverty line nor for population size. Since usual welfare measures are normalized for size of population, it makes sense to do so even while partial measures are under consideration. We propose to do it in what follows. So far we do not have a positive partial measure which we develop here.

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This is a revised version of a paper presented in the Indian Econometric Silver Jubilee Conference held at I.S.I. Bangalore.

Still further, in shifting our focus to lower segment of the income distribution vector we forget to improve our overall welfare measure. In fact, we desire an overall welfare index which respects poverty line. In other words, while we focus our attention on poor section we do not lose sight of the nonpoor section. While Takayama (1979) considers their existence and develops a negative measure, we propose to formulate a measure which considers the whole distribution alongwith poverty line in a manner that relative positions of the two sections come into bold relief. Though, our proposal is tentative, it is better than the ones suggested earlier.

After the preliminaries in Section 1, we discuss Sen's and Kakwani's measures of welfare (which we contrast with ours) in Section 2 alongwith our net social welfare measure which takes poverty line into consideration. In Section 3 we develop indices for illfare of the poor and welfare of the nonpoor as also those for poverty and prosperity which are defined in absolute sense. In Section 4 we discuss the problem of comparing and combining prosperity and poverty measures so developed, alongwith the implications. In the last Section, we propose a welfare index which considers both the sections of society - below and above poverty line. We also provide empirical correlates of indices provided by Sen and Kakwani alongwith the one proposed here.

PRELIMINARIES

Instead of bringing the concept of individual utilities, Sen and his associates consider welfare to be a straightforward function of individual incomes Y_1 through Y_N:

$$W(Y) = W(Y_1, Y_2, \ldots\ldots\ldots\ldots, Y_N) \qquad (1)$$

which is symmetric, non-decreasing and quasi-concave (Sen, 1973: 52-53). This function is non-individualistic as it by-passes individual preferences and assumes some paternalistic form. Sen argues that it is desirable too (Sen, 1973: 50-51). In this sense, most social welfare functions are in fact social welfare evaluation functions.

In giving the operational content, Sen often goes for normalized linearly weighted aggregation of individual incomes, where weights are supposed to take due care of general form of the welfare function. Thus,

$$W(Y) = \Sigma w_i \cdot Y_i \qquad (2)$$

where

$$w_i = w_i(Y_1, Y_2, \ldots\ldots\ldots, Y_N)^3. \qquad (2.1)$$

It may be noted that Sen has also considered other schemes of linear aggregation but we would stick to this which is the simplest. Further, in derivation of poverty and prosperity indices, Y_i will be replaced by income gaps g_p or income excesses h_r as the case may be.

While it is possible to choose any ordering of incomes, we are using non-decreasing one:

$$0 \leq Y_1 \leq Y_2 \leq \ldots \leq Y_N \qquad (3)$$

where N is the size of population of society. However, as we require truncation of this series into two segments belonging to the poor and the non-poor (prosperous), we would re-number them from the minimum income ($Y_{min} = Y_1$) to poverty line level Y* and from poverty line level Y* to the maximum income ($Y_{max} = Y_N$) in the latter case using a prime. That is,

$$0 \leq Y_1 \leq Y_2 \leq \ldots \leq Y_p \leq \ldots \leq Y_n < Y^* \leq Y'_1 \leq Y'_2 \leq \ldots \leq Y'_r \leq \ldots \leq Y'_m \qquad (3.1)$$

where n is the number of the poor and m, the number of the non-poor. We would also require defining:

$$g_p = Y^* - Y_p, \qquad p = 1, \ldots, n \qquad (4.1)$$

$$h_r = Y'_r - Y^*, \qquad r = 1, \ldots, m \qquad (4.2)$$

Thus, we would be considering only positive gaps or excesses. In some schemes, say in Takayaman, it could be alright to assume positive gaps (excesses) equal to their respective values and non-positive gaps (deficits) equal to zero. Then, really we would be using censored income distributions. One can also conceive of the case, where there is inequality in one segment but not in the other and can work out measures.

$$0 \leq Y_1 \leq Y_2 \leq Y_p \leq \ldots \leq Y_n < \underbrace{Y^* = Y^* = Y^* \ldots = Y^*}_{\text{(m-times)}} \qquad (3.2)$$

$$0 \leq \underbrace{Y^* = Y^* = Y^* \ldots = Y^*}_{\text{(n-times)}} < Y'_1 \leq Y'_2 \leq \ldots \leq Y' \leq \ldots \leq Y'_m \qquad (3.3)$$

However, our measures P of poverty, R of prosperity and S of overall welfare would be found to be based on following functions:

$$P = P(Y_1, Y_2, \ldots, Y_n, Y^*) \qquad (5.1)$$

$$R = R(Y_{n+1}, Y_{n+2}, \ldots, Y_N, Y^*) \qquad (5.2)$$

$$S = S(Y_1, Y_2, \ldots, Y_n, Y_{n+1}, \ldots, Y_N, Y^*) \qquad (5.3)$$

and whereas P and R functions would be found to be absolute indices, S would be a 'relative' index in Blackorby-Donaldson sense.

Finally, let us note that there are two types of axioms that have been discussed by people working in this field. One set is of those axioms that should be passed by a good index and the other set is of those axioms that have to be used in the derivation of the index. Two important test axioms are known as monotonicity axiom and transfer axiom. These axioms are slightly different than those stated in the earlier literature. Let us state them:

Monotonicity Axiom

Other things remaining the same, an increase in mean income results in increase in a positive welfare measure and decrease in a negative welfare measure.

Transfer Axiom

Other things remaining the same, a (pure) transfer from a richer person to a poorer person results in increase in a positive welfare measure and decrease in a negative welfare measure.

It may be noted that the first axiom relates to distribution-neutral growth and the second axiom relates to growth-neutral social justice. However, in addition to these two, another axiom called focus axiom is used as a test axiom in partial measures. It can be spelt out as below:

Focus Axiom

So long there is no change-absolute or relative in the income distribution of the segment in question, the value of partial measure should not change irrespective of happens in the other segment.

Sen mentions this axiom in a later writing (1981) only as a test axiom. However, this is clearly used in derivation too as the part of income vector not belonging to the segment in question is altogether ignored except the number of non-poverty incomes.

SEN'S AND KAKWANI'S WELFARE MEASURES

For deriving the social welfare measure of income distribution from the social welfare function, we can employ the following set of axioms in order to make use of (2). These axioms are more straightforward than Sen's and a little different as well.

Axioms

1. The sum of weights must be equal to 1:

$$\Sigma w_i = 1 \tag{6.1}$$

2. Everyone prefers (Y, i) to (Y, j) when $Y_i > Y_j$.

3. When everyone prefers (Y, i) to (Y, j), $w_j > w_i$.

4. Everyone prefers Y_{i+1} to Y_i and Y_{j+1} to Y_j, then

$$w_i - w_{i+1} = w_j - w_{j+1} \quad (6.2)$$

For the first three axioms, mostly there is no objection, but on Axiom 4, people have reservation and offer other schemes. Nevertheless, these four axioms lead to the following theorem:

Theorem 1: The social welfare measure of the income distribution Y under Axioms 1 through 4 is given by,

$$W\ (Y) = \mu\ (1 - G). \quad (T.1)$$

Proof: Axioms 3 and 4 hold true only when

$$w_i = \alpha\ (N + 1 - 1) \quad (7.1)$$

which on summation and use of Axiom 1 yields,

$$w_i = 2\ (N + 1 - i) /\ N(N + 1)$$

as α turns out to be $[2/N\ (N + 1)]$. This can be approximated as:

$$= \frac{2}{N^2}\ (N + 1 - i) \quad (7.2)$$

when N is large. Substituting (7.2) in (2) for W (Y), we obtain:

$$W\ (Y) = \frac{2}{N^2}\ \Sigma\ (N + 1 - i)\ Y_i. \quad (7.3)$$

Simplifying (7.3) and making use of the definition of Gini coefficient of inequality, we prove the theorem.

The result passes both the tests. We can note that W = 0 only when all incomes are zero or distribution is completely unequal, leading to G = 1 which is possible when N is large.

It would be worthwhile to discuss, although briefly the two propositions by Kakwani. In the first contribution, Kakwani (1980) derives:

$$W\ (Y) = \frac{\mu}{1 + G} \quad (8.1)$$

by using continuous distribution function but more significantly modifying the Axiom 4 as:

$$\frac{w_i - w_{i+1}}{Y_i} = \frac{w_j - w_{j+1}}{Y_j}$$

In another contribution (1985), he considers another aggregation scheme and introduces explicitly the concept of loss of welfare suffered by a person due to envy. The degree of envy defined by k is however uniform for all persons and depends on the judgement of the one who does the exercise. The result is stated as:

$$W\ (Y) = \mu\ (1 - kG),\quad k \geq 0 \tag{8.2}$$

While (8.1) is more sensitive to growth always, (T. 1) is more sensitive to growth so long $G < 0.5$. But what is more important is that whereas in (T.1) W can be 0 when $G = 1$, it will equal to only $\mu/2$ in (8.1). We can have Sen's result (T.1) only when $k = 1$ which means that the loss suffered due to envy by an individual is exactly equal to the difference between his income and the income of anyone richer selected for comparison. But (8.2) gives $W = \mu$ when either $k = 0$ or $G = 0$ or both. In Sen's measure the question of envy or guilt, apathy, antipathy or sympathy, superiority or inferiority is not considered in individualistic sense but it is judged in some paternalistic manner. Sen's scheme fixes k equal to 1. Even in Kakwani's case it depends on the judgement of the analyst who assigns equal k for everyone. However, if $k > \frac{1}{G}$, W will turn out to be negative.

We can check, however, Kakwani's measures pass both the test axioms we have set up as does Sen's. However, in what follows we prefer to choose Sen's framework as it is simple and sufficiently egalitarian. Moreover when k is uniform for all persons, naturally it is to be decided in some paternalistic manner.

There can also be a view that so long as one does not enjoy something over and above a certain basic minimum, one has no welfare at all. Instead, he suffers from illfare. If this is the attitude we adopt, individual's welfare is equal to zero not when income is zero but when income is equal to Y*. In fact, those who have income less than Y* do not enjoy welfare but suffer from illfare. Then, we have:

Theorem 2: The net social welfare measure N of the income distribution Y under the Axiom 1 through 4 is:

$$N\ (Y) = \mu\ (1 - G) - Y^* \tag{T.2}$$

Proof: We ~~need~~ modify the aggregation rule (2) as below:

$$N\ (Y) = \Sigma\ w_i\ (Y_i - Y^*) \tag{2.2}$$

but use the same weighting pattern as given in (7.2). Then, we have:

$$N\ (Y) = \frac{2}{N^2}\ \Sigma\ (N + 1 - i)\ \ (Y_i - Y^*) \tag{9}$$

which on simplification yields the desired result.

The result of the Theorem 2 not only passes both the test axioms, it has more egalitarian bias and focus on poverty. While Sen's proposal and Kakwani's first proposal could never be negative, Kakwani's second proposal and the proposal put forward here could yield negative value to social welfare measure. Kakwani's W could be negative when k was chosen to be greater than 1/G, whatever the value of G. Thus, if G=1/3 and $k > 3$, $N < 0$. But in our proposal $N < 0$ if $G > 1 - Y^*/u$. Thus if $\mu = \frac{3}{2} Y^*$, our measure would yield a negative number in case $G > 1/3$. Sensitivity to Y* may not be considered something great in itself. But in this format, N looks closer to poverty measures as proposed by Sen and his followers.

INDICES FOR ILLFARE OF THE POOR AND WELFARE OF THE NON-POOR

Sen (1976, 1981) derives a poverty index for the poor segment of the society. This is essentially a negative partial welfare measure but derived under the focus axiom which makes the measure independent of what happens in the other segment of the society except the size of non-poor population. However, in what we propose here people can move from one segment to the other, depending upon the change in income. Here we first derive the illfare measure for the poor segment and multiply it by the proportion of the poor in the society and define it as poverty measure.

We need not modify any of the axioms for smaller incomes mean larger gaps. Thus, whereas in positive welfare measures larger weights we attach to smaller incomes (excesses), in negative welfare measures, we have to attach larger weights to larger gaps. The theorem may formally be stated as:

Theorem 3: The illfare measure for the segment of the people with incomes lower than Y*, is given by

$$I = Y^* - \mu_p (1 - G_p) \qquad (T.3)$$

where μ_p is the mean income of the poor and G_p is the Gini measure of inequality in incomes of the poor.

Proof: We need remember that now p varies from 1 to n and, therefore, weight w_p is, for large n, given by

$$w_p = \frac{2(n+1-p)}{n^2}.$$

The measure can be defined as a weighted linear aggregate of individual g_p in the following form:

$$I = \Sigma w_p \cdot g_p \qquad (2.3)$$

which on proper substitution becomes:

$$I = \frac{2}{n^2} \sum_{p=1}^{n} (n + i - p) (Y* - Y_p) \qquad (10.2)$$

Using (10.2) and the proper definition of Gini coefficient of inequality among the poor G_p, we can prove the theorem for large n.

Since the poor constitute only a segment of the society, it is proper to conceive poverty index P as product of proportion of the poor in the society H_p and the illfare index of the poor I. That is,

$$P = H_p [Y* - \mu_p (1 - G_p)] \qquad (T.3.1)$$

The poverty index P passes the two tests of monotonicity and transfer axioms. In addition, it passes test of INCIDENCE AXIOM which may be stated as: Given other things, increase in incidence increases the poverty measure.

The expression (T.3.1) is equivalent to Sen's measure as derived in Anand (1983, p. 120) but for a multiplicative factor Y*. Unlike Sen's measure its upper limit is not 1. However, so is the case with other welfare measures. It is of the genre proposed as absolute poverty indices (Blackorby and Donaldson 1980: 1057-8). It is better to stick to this form so that it remains comparable to the standard welfare indices.

From (T.3.1), we find that the poor suffer from illfare because of (i) incomes being lower than Y* and (ii) inequality in the income distribution among the poor. When $G_p = 0$, $p = H_p (Y* - \mu_p)$. But as G_p increases P increases and when $G_p = 1$, P gets augmented to $H_p.Y*$ no matter what the last person receives. Moreover P will never be zero so long there is some H_p whatever the value of G_p and μ_p for $\mu_p < Y*$ and $G_p < 1$. But if H_p is zero, P is zero. The maximum P is of course equal to Y* if each number of the society has income zero.

This tempts us to develop a prosperity index for the non-poor segment of the society. In our conception, only those who enjoy incomes in excess of Y* enjoy positive welfare. In its derivation, test axioms remain the same as for poverty measure but for the substitution of word prosperity for poverty. The theorem may formally be stated as:

<u>Theorem 4:</u> For large number of the non-poor, the measure of positive welfare enjoyed by the non-poor is to be given by

$$J = \mu_r (1 - G_r) - Y* \qquad (T.4)$$

where μ_r is the mean income of the non-poor and G_r, the Gini coefficient of concentration of incomes among the non-poor.

Proof: Lower incomes and therefore lower income excesses should receive higher weights and, therefore,

$$w_r = \frac{2\,(m + 1 - r)}{m^2} \qquad (11.1)$$

which can be derived in the similar fashion as done in earlier sections.

If we define the positive welfare for the non-poor segment as:

$$J = \Sigma\, w_r \cdot h_r \qquad (2.4)$$

and substitute (11.1) in (2.4) and make use of the definition of Gini coefficient of inequality for the non-poor, we prove the theorem stated above.

Again, since the non-poor constitute only a segment of the society, the prosperity index R can be defined as:

$$R = H_r\, [\mu_r\, (1 - G_r) - Y^*] \qquad (T.4.1)$$

where H_r is the proportion of non-poor people in the society.

While (T.4) passes the two tests of monotonicity and transfer axioms only, (T.4.1) also passes that of incidence. One can note that prosperity gets mitigated because of presence of inequality. And R may too turn out to be negative when G_r turns out to be greater than $(1 - Y^*/\mu_r)$. This is however little realised. If everybody enjoys income equal to μ_r, then R is also equal to $\mu_r - Y^*$.

COMBINING PARTIAL MEASURES

Although there was express need for developing separate welfare measures for the two parts of the society, yet it is necessary to realise that the two measures added together overestimate the overall social welfare. The simple reason is that there exists a between-segment inequality which is lost sight of in the development of separate measures under the focus axiom, that is the assumption of independence. If we remove within-inequalities altogether, R_{max} and P_{min} would be given by

$$R_{max} = H_r\, [\mu_r - Y^*] \qquad (12.1)$$

$$P_{min} = H_p\, [Y^* - \mu_p] \qquad (12.2)$$

and their positive welfare sum would appear as:

$$R_{max} - P_{min} = \mu - Y^* \qquad (12.3)$$

But this is far from truth. Social welfare S_b in such a case would be given by

$$S_b = \mu\ (1 - G_b) - Y^* = \mu - Y^* - H_p\ (\mu - \mu_p) \qquad (12.4)$$

where G_b is the Gini coefficient of inequality between the two classes. It is lower than that given in (12.3). Thus, mitigating the gap between the two segments must constitute an important element of social justice in growth. In a situation, where rich should have grown richer and the poor could not become less poor, the overall welfare might have reduced.

Such neat expressions do not emerge for the situation in which group inequalities do exist. However, we should compare

$$R - P = \mu - Y^* - [H_r.\mu_r.G_r + H_p.\mu_p.G_p] \qquad (12.5)$$

and

$$N = \mu - Y^* - [H_r^2.\mu_r.G_r + H_p^2.\mu_p.G_p + H_p.H_r\ (\mu_r - \mu_p)] \qquad (12.6)$$

where N is expanded by making use of decomposition of G in terms of G_p, G_r and G_b. This comparison shows that R - P would not be equal to N unless, (which means $\mu_r = 3\ \mu_p$ if G_p and G_r are around 0.5 and $\mu_r = 2\ \mu_p$ if G_p and G_r are around 0.33),

$$\frac{\mu_r}{\mu_p} = \frac{1 + G_p}{1 - G_r} \qquad (12.7)$$

Before we end this section, we should note the elasticities for three measures, whose values crucially depend upon the values of other parameters involved. Yet, on the whole, they are more sensitive to growth when respective inequality coefficients are less than 0.5, which they are:

Table 1
Elasticities of Poverty, Prosperty and Welfare Measures

Factor / Index	Incidence	Mean Income Growth	Inequality Coefficient
Poverty	1, + ve	$\frac{-\mu_p\ (1-G_p)}{Y^*-\mu_p(1-G_p)}$, >1,-ve	$\frac{\mu_p.G_p}{Y^*-\mu_p(1-G_p)}$, > 1, + ve
Prosperity	1, + ve	$\frac{\mu_r(1-G_r)}{\mu_r(1-G_r) - Y^*}$, >1,+ve	$\frac{-\mu_r.G_r}{\mu_r(1-G_r) - Y^*}$, > 1,-ve
Welfare	------	$\frac{\mu(1-G)}{\mu(1-G) - Y^*}$, > 1,+ve	$\frac{*\ -\mu.G}{\mu(1-G) - Y^*}$, > 1,-ve

TOWARDS AN OVERALL WELFARE INDEX

Now, let us think of a situation in which the prosperity index has risen as also has the poverty index. How does one evaluate this situation in overall context ? An improvement or a deterioration ? A Rawls would say deterioration while a Bentham may find an improvement. A Pareto would say 'don't know'. But there continues to be a need where we should somehow compare the different components affecting the welfare of the society. The obvious three candidates for consideration are P, R, and G_b.

Under a simple axiomatic structure which demands that an increase in R, a decrease in P and a decrease in G_b should *per se* lead to an increase in the overall social welfare S, we can formulate the following index:

$$S = \frac{R}{P} - G_b \qquad (13.1)$$

which is unit-free. The same can be expanded as:

$$S = Q_p + \frac{H_r\,[\mu_r\,(1 - G_r) - Y^*]}{H_p\,[Y^* - \mu_p\,(1 - G_p)]} - H_p \qquad (13.2)$$

which increases when H_r increases (H_p decreases), μ_r increases, μ_p increases, G_r decreases, or G_p decreases.

There may be people holding that though (13.1) respects poverty line, it considers only distribution aspects and ignores growth aspect altogether. In this case, if every income rises by the same factor and poverty line is also raised by the same factor, welfare shall remain the same. If people find it appealing that growth *per se* raises the welfare of the people irrespective of whether poverty line too is raised by the same factor then the index (13.1) may be modified as:

$$S^* = \mu\,(\frac{R}{P} - G_b) \qquad (13.3)$$

which can be expanded as below:

$$S^* = \mu\,\frac{H_r\,[\mu_r\,(1 - G_r) - Y^*]}{H_p\,[Y^* - \mu_p\,(1 - G_p)]} - H_p\,(\mu - \mu_p) \qquad (13.4)$$

When applied to the data (see Appendix) obtained from a survey conducted in Varanasi City in 1987, our S* provides the same ranking of Hindu, Muslim and Total population than do Sen's and Kakwani's indices while S* provides a different ranking. See Table 1. However, the question we have attempted to answer is larger than whether a new index provides the same or different ranking as do older indices, in a given specific situation.

Table 2
Alternative Welfare Indices

	Unit	Hindu	Muslim	All
Sen	Rs.	2350.29	1767.34	2044.23
Kakwani	Rs.	4234.36	1992.05	3430.14
Chaubey (1)	Number	3.73	2.41	2.10
Chaubey (2)	Rs.	26312.24	4255.75	11781.44

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Appendix

Basic Parameters of Distribution of Income Over Households of Varanasi

Indices \ Population	Hindu	Muslim	Total
μ	Rs. 7059.52	Rs. 2662.58	Rs. 5610.21
μ_p	Rs. 1464.95	Rs. 1536.18	Rs. 1491.74
μ_r	Rs. 10313.64	Rs. 3490.37	Rs. 8936.80
H_p	0.367751	0.6282894	0.4470821
H_r	0.6322489	0.3717105	0.5529179
Q_p	0.0763134	0.4265894	0.1187972
Q_r	0.9236865	0.5734207	0.8812627
G_p	0.1465094	0.1190299	0.1547522
G_r	0.6361758	0.1825389	0.6137468
G_b	0.2914375	0.2017102	0.32835