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A Note on Growth Models: Harrod-Domar and Neo-Classical

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INTRODUCTION

The publication of John Maynard Keynes' General Theory of Employment, Interest, and Money in 1936 was followed by a spate of theories relating to the requirements for steady-state growth. These theories, essentially elaborations and refinements of the Keynesian system, were directed toward problems of relatively more developed countries. When they were conceived, the problem of "secular stagnation" or chronic unemployment was still very much in people's mind. However, they were sufficiently general to be useful in dealing with problems of inflationary pressure during Second World War. This same generality makes them valid, with appropriate modifications of the underlying empirical assumptions, in underdeveloped countries as well. They do not, however, tell us much about how to launch a process of development where it does not exist.

R.F. Harrod published his famous article, An Essay in Dynamic Theory, in 1939. In his article, Harrod concentrates upon the explanation of secular trends. He insists that it is precisely this explanation of trends that is the distinguishing characteristic of Dynamic Economics. Harrod's theory is directed toward an explanation of the secular causes of unemployment and inflation, and of the factors determining the optimum and the actual rate of capital accumulation.

The literature on economic growth contains frequent references to the "Harrod - Domar" model. Evsey D. Domar presented a model, seemingly similar in form to Harrod's, that he discovered independently and at about the same time. In fact, however, the implications of the Domar's model are not identical with those of Harrod's theory. Domar identifies additional or supplementary requirements for steady-state growth. Harrod was concerned with conditions that would keep enterpreneurs content with their investment plans so that they would repeat in each period the investment decisions of earlier periods. Domar was concerned with the income growth required for full utilization of a growing capital stock, with full employment and stable prices. There is no priori reason for these two sets of requirements to coincide.

"There is wide agreement about the major goals of economic policy: high employment, stable prices, and rapid growth. There is less agreement that these goals are mutually compatible or, among those who regard them as incompatible, about the terms at which they can and should be substituted for one another. There is least agreement about the role that various instruments of policy can and should play in achieving the several goals", according to M. Friedman.

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Daniel Hamberg demonstrates mathematically what is really apparent intuitively: the Harrod and Domar models become identical if, in Harrod's terms, $G_w = G_n$. That is, if the economy is on the full-employment, full-capacity path and this is what entrepreneurs like, then both Harrod's and Domar's conditions will coincide if income and investment continue to grow at the same constant rate, equal to G_n.

Robert M. Solow published "A Contribution to the Theory of Economic Growth" in February, 1956. He wrote: "A 'crucial' assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect. I wish to argue that something like this is true of the Harrod - Domar model of Economic growth."

The characteristic of the simple Harrod - Domar model of economic growth is that even for the long run the economic system is at best balanced on a "knife-edge" of equilibrium growth. When the economy deviates slightly from the natural growth rate the result will be either growing unemployment or prolonged inflation, since the system has no built-in equilibrating forces. But this fundamental opposition of warranted and natural rates of growth turns out in the end to flow from the crucial assumption that production takes place under conditions of fixed proportions. This assumption of fixed proportions in the combination of capital and labour has been the main object of criticism from the advocates of the neo-classical growth model like Solow, Swan, Meade, and Samuelson.

According to P. Burrows and T. Hitiris, "It is the assumption about the variability of the capital-labour ratio which constitutes the basic difference between the Harrod-Domar and neo-classical models. In other respects they are the same: the expenditure sector is similar, with its assumed constant savings ratio, and the rate of growth of the labour force is exogenous to both models. But while the condition to be satisfied for steady state growth proves to be the same in both models (as we shall see) the emphasis in the neo-classical case is more on HOW the system will move from a disequilibrium (unsteady state) situation to a steady state."

The objectives of this paper are to present the basic Harrod-Domar and the simple neo-classical one-sector, non-monetary growth models without technical progress; to analyze them, to draw shortcomings of each of them, to compare with and contrast to one another, and to endeavour to evaluate their importance for practical purposes.

Indeed, there are a number of different sophisticated growth models relating to the requirements for a steady-state growth. A fully articulated model of growth requires to specify the effect of changes in the capital-stock on per capita output and the effect of changes in the capital-labour ratio, the savings ratio, the rate of monetary growth, the distribution of income, and the progress of technology embodied in each factor of production. "Unfortunately, such a model does not exist at present", Ott, Ott and Yoo opine, and "which model should be chosen to explain real world experience cannot be answered without more thorough and careful study of the various models." But we can get the basic essential ideas from the study of the simple growth models. Therefore, this paper just attempts to explore two above mentioned growth models even though they hardly represent the present state of the theory of growth.

THE HARROD-DOMAR GROWTH MODEL

The basic Harrod-Domar growth model stems from the analysis of capital accumulation, in the absence of technical progress. The simplifying assumptions of this model are:

(1) There is a single homogeneous good used both for consumption and as a perfectly 'malleable' stock of capital;

(2) Labour is the only input other than capital used in production; and it is homogeneous;

(3) There is no possibility of substituting labour for capital in production. The relative capital or labour intensity of the production process is fixed;

(4) The (proportional) rate of growth of the labour force is exogenously fixed by non-economic, demographic, forces. The labour force is assumed to grow over time at a constant rate, n.

(5) The existing labour force is offered and fully taken up for employment by the demand from the product market with perfect elasticity at a constant real wage rate;

(6) The proportion of income saved does not vary as income changes;

(7) There is no monetary sector and the price-level is constant, and

(8) There is no technical progress.

The Harrod-Domar growth model is derived in the following way:

Let Y = output or real income produced in the period t;

K = stock of capital used in period t;

N = labour force employed in the period t;

u = W/Y, a constant labour - output ratio;

v = K/Y, a constant capital - output ratio;

p = index of price - level;

w = money wage - rate;

t = index of time;

C = consumption in the period t;

S = savings in the period t;

I = investment in the period t;

s = average or marginal propensity to save.

Since the equilibrium growth rate $\dot{Y}_e$ is the rate which is associated with full-employment equilibrium in the economy, $\dot{I}_e$ then requires that the change in the capital-stock dK/dt = I be equal to savings S and that demand for labour be equal to the supply of labour simultaneously:

$$\dot{Y}e = \frac{1}{Y}\frac{dY}{dt} \qquad \dots \quad \dots \text{(1a)}$$

$$dK/dt = I = S \qquad \dots \quad \dots \text{(1b)} \qquad \dots \text{(1)}$$

$$N^d = N_o\ e^{nt} \qquad \dots \quad \dots \text{(1c)}$$

where N^d is the demand for labour, and $N_o\ e^{nt}$ the supply of labour growing at the rate n.

The conditions for equilibrium growth can be shown in terms of the aggregate demand and supply curves. Since Eq. (1) represents a series of continuous equilibria over time, the locus of equilibria must constitute a long-run equilibrium path.

The equation for the equilibrium growth path can be expressed by defining aggregate supply in terms of the labour force and the average physical productivity of labour.

We have, $Y = \frac{Y}{N}.\ N$

Taking logarithms of the both sides, $\log Y = \log \frac{Y}{N} + \log N.$

Differentiating with respect to time t, $\frac{1}{Y}\frac{dY}{dt} = \frac{1}{Y/N}\frac{d(Y/N)}{dt} + \frac{1}{N}.\frac{dN}{dt}$

$$\dot{Y} = (\dot{\frac{Y}{N}}) + \dot{N} \qquad \dots \text{(2)}$$

In words, the rate of economic growth is equal to the growth rate of labour-productivity plus the growth rate of the labour force.

If the relative shares of capital and labour are assumed to be constant over time, then the rate of change in the price level is equal to the rate of change in the money wage-rate less the rate of growth of labour productivity (Please refer to Eqn. 7).

By assumption (3), we have

$$K = v.Y \quad \ldots \quad \ldots \quad (3a)$$

$$N = u.Y \quad \ldots \quad \ldots \quad (3b)$$

...(3)

Since the factor proportions are assumed to be fixed for all levels of output, we have:

$$\frac{Y}{K} = \frac{dY}{dK} = \frac{1}{v} \quad \ldots \quad \ldots \quad (4a)$$

$$\frac{Y}{N} = \frac{dY}{dN} = \frac{1}{u} \quad \ldots \quad \ldots \quad (4b)$$

...(4)

From Eq. (4a) we have, $dY = \frac{1}{v}\, dK$ ... ... (4a*)

That is, changes in the output supplied at any price level (assumed constant in Harrod-Domar model), and therefore the extent of the shift to the right of the aggregate supply curve, are determined by the amount of out-out produced by a unit of capital and by the increase in the capital-stock, the level of investment.

$$I = \frac{dK}{dt} = v.\frac{dY}{dt} \quad \ldots \quad (5)$$

and we have $\frac{Y}{N} = \frac{1}{u} = \frac{dY}{dN} = \frac{w}{p}$ ... (6)

Taking logarithms of Eq. (6), we obtain, $\log\left(\frac{Y}{N}\right) = \log w - \log p$

Differentiating both sides with respect to time, we get,

$$\frac{1}{Y/N}\,\frac{d(Y/N)}{dt} = \frac{1}{w}\,\frac{dw}{dt} - \frac{1}{p}\,\frac{dp}{dt}$$

$$\left(\frac{\dot{Y}}{N}\right) = \dot{w} - \dot{p} \quad \ldots \quad (7)$$

From Eqns. (2) and (7), we obtain,

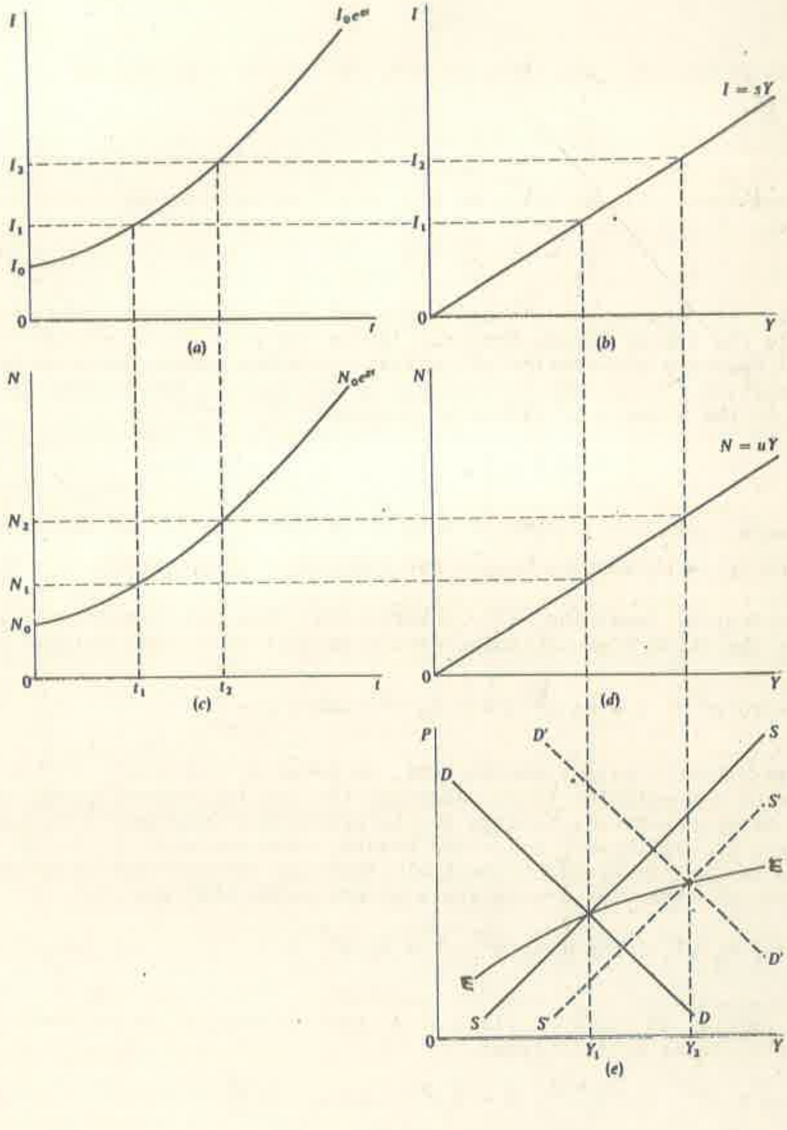
$$\dot{p} = \dot{w} - \dot{Y} + \dot{N} \quad \ldots \quad (8)$$

Eq. (8) together with the full-employment condition given in Eq. (1) represents the equation for the equilibrium growth path depicted by the EE curve in Figure I panel (e).

In formulating the flow equilibrium conditions of the product market, Harrod-Domar model assumes that there are no lags in demand or in output, that the saving function is of proportional to real income:

Figure I (a-e)

Effect of growth in labour force and investment on equilibrium growth.



$$\frac{S}{Y} = \frac{dS}{dY} = s \qquad \ldots (9)$$

and that investment and saving are equal also in ex-ante sense: I = S
... (10)

From Eqns. (1b), (9), and (10), we get, the flow equilibrium condition:

$$\frac{dK}{dt} = s\,Y \qquad \ldots (11)$$

Again from Eqns. (5) and (11), we get, the flow equilibrium condition:

$$\dot{Y} = \frac{1}{Y}\frac{dY}{dt} = \frac{s}{v} \qquad \ldots (11)$$

While Eqn. (2) states the condition for the full-employment growth with regard to the labour force, Eqn. (11) gives the equilibrium growth rate the full capacity utilization of capital. For the labour force to be fully employed, its growth rate n must be the same as the economic growth rate $\dot{Y}$, in the absence of technical progress.

$$n = \dot{Y} = \frac{s}{v} \qquad \ldots (12)$$

In Harrod's terms, the warranted rate of growth ($g_n = \frac{s}{v}$) and the natural of growth ($g_n = n$) must be equal, for a steady - state growth.

If the essential condition (12) is satisfied, then the steady-state growth paths of the three time-variables in the product and factor markets are:

$$Y = Yo\,e^{gt},\quad K = Ko\,e^{gt},\ N = N_o\,e^{gt} \text{ where } g = \frac{s}{v} = n. \qquad \ldots (13)$$

The fixed labour - output coefficient, u, makes no appearance in the formulation of the solution (13). However, the initial values Y_o, K_o, and N_o must be appropriately related by the production function, i.e. $K_o = v\,Y_o$ and $N_o = u\,Y_o$, and both u and v are needed. For instance, if N_o is considered to be the 'independent' initial condition (exogenously determined labour force), then the steady-state growth paths (13) are

$$Y = \frac{1}{u} N_o\,e^{gt},\ K = \frac{v}{u} N_o\,e^{gt},\ N = N_o\,e^{gt} \qquad \ldots (13a)$$

If we interpret the full-capacity condition in such a way that the increment in capital is fully utilized at a constant capital-output ratio, v, K can be replaced by I in (13a).

Thus, $Y = Y_o e^{gt}$, $I = I_o e^{gt}$, $N = N_o e^{gt}$, where $I = \frac{dk}{dt}$ ... (13b)

From Eqns. (9) and (10), we have the flow equilibrium condition:

$$\dot{I} = sY \qquad \ldots (11a)$$

The equilibrium growth path in the product and factor markets given by Eqns. (13a) and the flow equilibrium condition for investment and saving (11a) are depicted in the Figure I. They are related to the equilibrium path of $\dot{Y}$ and $\dot{p}$ represented by Eqns. (2) and (7), as depicted in panel (e) of Figure I.

Since $g_w = g_n$, the aggregate supply curve shifts out to S'S' by an amount determined by the slope u of Eqn. (3b) in panel (d), as depicted in panel (c) of Figure I. The marginal propensity to consume, (1-s), implicitly guarantees that the aggregate demand curve shifts up to D'D' so that the equilibrium level of the price level and output always lies on the equilibrium growth path EE.

Thus, the Harrod-Domar condition for a steady-state growth can be simply stated:

$$g = \frac{s}{v} = n$$

In words, the actual, warranted, and natural rates of growth must be equal for a steady-state growth: $G_a = G_w = G_n$ ... (14)

But, unfortunately, the three parameters namely s, v, and n, which enter the Harrod-Domar condition are determined independently of each other. There is no guarantee that they will coincide each other. According to Harrod and Domar, there is no adjustment - mechanism which operates to make those three parameters equal to produce a steady - state growth. This is the famous Harrod-Domar instability, which led them to conclude that in general full-employment steady-state growth is not possible.

R.F. Harrod in his An Essay in Dynamic-Theory, states, "Suppose an increase in the propensity to save, which means that the values of s are increased for all levels of income. This necessarily involves, ceteris paribus, a higher rate of warranted growth. But if the actual growth was previously equal to the warranted growth, the immediate effect is to raise the warranted rate above the actual rate. This state of affairs sets up a depressing influence which will drag the actual rate progressively farther below the warranted rate. In this as in other cases, the movement of a dynamic determinant has an opposite effect on the warranted path of growth to that which it has on its actual path. ... A departure from equilibrium, instead of being self-righting, will be self-aggravating. Gw represents a moving equilibrium, but a highly unstable."

Harrod's main argument depends on the acceleration principle which states that the level of investment planned is related to the size of the change in income (output) rather than to the level of output. For if $G_a > G_w$, what this really means is that the rate of increase in total spending is greater than is necessary to call forth the current rate of investment, and consequently investment will increase. Harrod argues that the greater investment brought about will raise G_a still further above G_w, in the next period and a cumulative movement away from equilibrium will set in. Thus according to R.F. Harrod: "Around the line of advance which, if adhered to, would alone give satisfaction, centrifugal forces are at work, causing the system to depart further and further from the required line of advance."

With the introduction of G_n, Harrod develops a theory of "stag - flation". If G_w exceeds G_n (as it may happen when population growth tapers off, or the rate of improvement in technique or discovery of new resources tapers off), G_a also tend to lie below G_w, and the economy will be chronically depressed. After all, G_a can exceed G_n only in the recovery phase of the business cycle. Conversely, in a rapidly expanding economy (in which population growth, or technological progress, geographic expansion is at a high level) there will be a chronic excess of G_n over G_w, and also of G_a over G_w, and thus a chronic, perpetual tendency for inflationary boom to develop.

CRITICISM OF THE HARROD-DOMAR GROWTH MODEL

The advocates of Unemployment Equilibrium Growth Model argue that the presence of unemployment, even of continuously increasing amounts of unemployment, is not incompatible with equilibrium steady-state growth. As Harrod wrote, it is true that the system cannot advance more quickly than the natural rate allows. There is, however, an asymmetry that it is possible for employment to grow less fast than the labour force. Growth at the warranted rate s/v is therefore compatible with equilibrium if $s/v \leq n$. Provided the above inequality is satisfied, it is possible to regard Harrod's warranted rate of growth s/v as an equilibrium steady-state growth rate. It is what Kahn (1959) called a Bastard Golden Age, as opposed to a true Golden Age in which s/v/ = n.

The advocates of Classical Growth Model content that the rate of growth of the population or of the labour force, n, may be a variable over time that responds to economic factors rather than a constant. The equality of s/v and n may then be achieved not by 'chance' but by the adjustment of n.

The advocates of Neo-classical Growth Model attack the rigid Harrod-Domar assumption of fixed proportions in the combination of capital and labour. They assume that the capital-output ratio, v, is adjustable, instead of being fixed, and this provides a way in which s/v and n may be brought into equality.

The equality of s/v and n may, alternatively, be made possible by flexibility in s. Various assumptions may give rise to flexibility in s; one which is prominent in the growth models is the assumption of differences in the propensities to save of wage-earners, and of profit-earners, according to what is the distribution of income between them.

THE NEO-CLASSICAL GROWTH MODEL

In contrast to the Harrod-Domar model of "fixed coefficients", the neo-classical growth model assumes the production function to be homogeneous of degree one and "well-behaved". This amounts to assuming that there is no scarce non-augmentable resource like land. The scarce-land case would lead to decreasing returns to scale in capital and land, and the model would become more classical.

We confine ourselves only to a real sector model without technical progress in this paper.

Let the technological possibilities be represented by a production function:

$$Y = F(K, N) \qquad \dots (1)$$

Here, Y is to be understood as net output after adjusting allowance for the depreciation, in the period t.

The neo-classical growth model accepts all the other Harrod-Domar assumptions except that of fixed proportions. Therefore, this model is given by the following equations:

Production function: $Y = F(K, N)$ ... (1)

Acceleration principle: $I = \frac{dK}{dt}$ ... (2)

Constant marginal propensity to save: $S = s\,Y$ ... (3)

Product market equilibrium condition: $I = S = \frac{dK}{dt} = s.F(K,N)$ ... (4)

Harrod-Domar's natural rate of growth n: $N = N_o e^{nt}$ ... (5)

In Eqn. (4), N stands for total employment demanded; and in Eqn.(5) N stands for available labour force supplied in the labour market. By equating them, we assume that full-employment of the available labour force is perpetually maintained. Thus,

$$\frac{dK}{dt} = s.F(K, N_o e^{nt}) \qquad \dots (6)$$

The basic differential equation (6) determines the time-path of capital accumulation that must be followed if all available labour force is to be employed.

To find if there is always a capital accumulation path consistent with any rate of growth of the labour force, n, let us proceed the analysis of growth model in terms of PER CAPITA variables, in other words, efficiency units.

Let us introduce a new variable $k = \frac{K}{N}$, the capital - labour ratio. ... (7)

Taking logarithms of both sides of the Eqn. (7), we obtain,

$$\log k = \log K - \log N \qquad \dots (8)$$

Differentiating (8), with respect to time, both sides of Eqn. (8), we obtain,

$$\frac{1}{k}.\frac{dk}{dt} = \frac{1}{K}.\frac{dk}{dt} = \frac{1}{N}\frac{dN}{dt} \qquad \dots (9)$$

Since the production function (1) is homogeneous of degree one,

$$Y = N.F(\frac{K}{N}, 1) = N.F(k, 1) \qquad \ldots (10)$$

From Eqns. (4), and (10), we get, $\frac{dK}{dt} = s.N.F(k, 1)$

$$\frac{1}{K}\frac{dK}{dt} = s.\frac{N}{K}F(k, 1) = \frac{s}{k}.F(k, 1) \qquad \ldots (11)$$

Taking logarithms of both sides of Eqn. (5), we get,

$$\log N = \log N_o + nt$$

Differentiating both sides with respect to time, we get

$$\frac{1}{N}.\frac{dN}{dt} = n \qquad \ldots (12)$$

Making substitutions from Eqns. (11) and (12) to Eqn. (9), we obtain:

$$\dot{k} = \frac{s}{k}.F(k, 1) - n \qquad \ldots (13)$$

Equation (13) is a differential equation involving the capital - labour ratio only. It represents the total product curve as varying amounts of k of capital are employed with one unit of labour. Alternatively, it gives per capita output as a function of per capita capital - stock. It states that the rate of change of the capital - labour ratio is the difference of two terms, one representing the increment of capital and the other the increment of labour.

Given an initial value $k = k_o$, the solution of the Eqn. (13) results in the steady equilibrium path of the capital - labour ratio k^* over time, i.e. that k which makes $\dot{k}$ equal to zero. When $\dot{k} = 0$, the capital - labour ratio is a constant, and the capital - stock must be expanding at the same rate as the labour force, namely n, for a steady - state growth.

Thus, the Eqn. (13) reduces to: $\frac{s}{k^*}.F(k^*, 1) = n \qquad \ldots (14)$

In other words, the warranted rate of growth $G_w = \frac{s}{v}$, where $v = \frac{k^*}{F(k^*,1)}$, warranted by the appropriate real rate of return to capital, equals the natural rate of growth n.

Equilibrium output per capita, $y^* = F(k^*, 1) = \frac{n}{s} . k^* \qquad \ldots (15)$

To depict the relationship between consumption and investment within the context of the neo-classical model, we refer to Eqns. (4) and (15) and get the following result:

$$\frac{I}{N} = s.F(k, 1)$$

$$s.F(k^*, 1) = nk^* \qquad \ldots (16)$$

The neo-classical growth model in PER CAPITA is illustrated in Figure II, where capital per capita is on the horizontal axis and output per capita on the vertical axis. The convex upward curve $\frac{Y}{N}$ represents the per capita production function Eqn. (10). The $\frac{I}{N}$ curve is derived from the $\frac{Y}{N}$ curve given the constant savings ratio $s=\frac{S}{Y}$, which in the Figure II is equal to $\frac{Ck^*}{Bk^*}$. The $\frac{I}{N}$ curve, of course, is the amount of capital created per capita, the first term on the R.H.S. of Eqn.

$$\frac{d(\frac{K}{N})}{dt} = s.F(\frac{K}{N}, 1) - (\frac{K}{N})\, n \qquad \text{... (13a)}$$

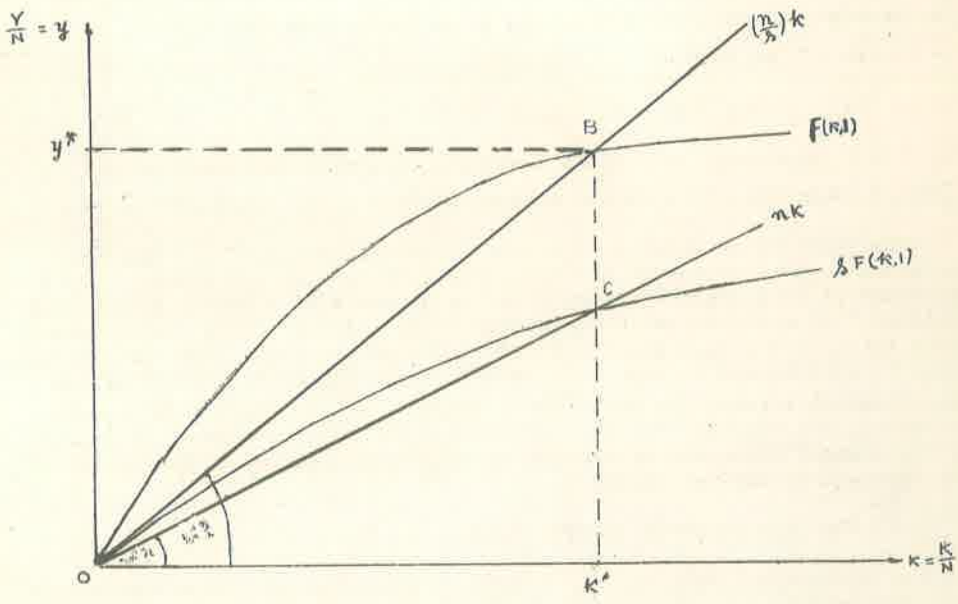
a variant of Eqn. (13) on page 70).

Finally, the $n\frac{K}{N}$ curve, with a slope equal to n, is the amount by which $\frac{K}{N}$ is reduced by the increase in the labour force, the second term in Eqn. (13a), and can alternatively be viewed as the amount of capital creation required to keep $\frac{K}{N}$ constant.

The above relationships of the variables are depicted graphically in Figure II.

Figure II

Equilibrium capital - labour & per capita saving



As shown in the Figure II, the neo-classical system tends to converge to the equilibrium point B if the values of y and k are different from y* and k*, respectively. At the point of intersection B,

$$F(k^*, 1) = \frac{n}{s} \cdot k^*, \text{ where } \dot{k} = 0.$$

If the capital - labour ratio k* should ever be established, it will be maintained, and capital and labour grow henceforward in proportion. By the assumption of constant returns to scale, output will also grow at the same rate n, and per capita output will be constant. If $k \neq k^*$, then $\dot{k} \neq 0$. If $k > k^*$, $\dot{k} < 0$, and from Eqn. (13), $F(k, 1) < \frac{n}{s} k$, the curve $F(k, 1)$ is below the line $\frac{n}{s}k$, and thus k will decrease toward k*. Conversely, if initially $k < k^*$, $\dot{k} > 0$, k will increase toward k*. Thus the equilibrium value k* of capital - labour ratio is stable. If the initial capital - stock is below the equilibrium level, capital and output will grow at a faster rate than the labour force till the equilibrium capital - labour ratio is restored. Conversely, if the initial capital - labour is above the equilibrium level, capital and output will grow more slowly than the labour force. The growth of output is always intermediate between those of labour and capital. The time path of capital and output is not exactly exponential as assumed by Harrod-Domar model except asymptotically.

The point C of intersection of the curve $sF(k, 1)$ and the line nk gives per capita investment, in equilibrium. The solution for k at the point C given by Eqn. (16) must be the same as the solution for k at the point B given by Eqn. (15). The line nk must be flatter than the line $\frac{n}{s} \cdot k$, because the marginal propensity to saves is less than unity. In the Figure II, we have,

$$0y^* = k^* C + CB \qquad \ldots (17)$$

where k* C represents per capita investment $\frac{I}{N}$ and CB the level of per capita consumption, in a closed economy without public sector.

The equilibrium point B represents a constellation of all unknown variables such as to ensure full-employment of capital and labour. The existence of such a point arises from the presence of a smooth production function. It contrasts with the Harrod-Domar model of fixed coefficients, where there is a single point instead of the smooth curve like F(k,1), and there is no presumption that this point lies on the curve $\frac{n}{s}$ k. What the neo-classical argument in favour of a steady - state growth thus amounts to is that any tendency for the capital - stock to grow more or less rapidly than labour force can be avoided by selecting a method of production of the appropriate capital intensity.

AN EXAMPLE OF NEO-CLASSICAL GROWTH MODEL

To investigate empirically if there is always a capital accumulation path consistent with any rate of growth of the labour force, let us take the Cobb-Douglas production function: $Y = K^a . N^b$, $a+b = 1$. ... (1a)

The Eqn. (6) of the Section reduces to: $\frac{dK}{dt} = sK^a (N_o e^{nt})^b$ ... (6a)

The solution of the differential equation (6a) is the following:

$$K_t = (K_o^b - \frac{s}{n} N_o^b + \frac{s}{n} N_o^b e^{nbt})^{\frac{1}{b}}, \quad \text{... (18)}$$

where $b = 1-a$, $0 < a < 1$ & K_o = initial capital-stock.

As t becomes large, K_t grows essentially like $(\frac{s}{n})^{1/b} N_o e^{nt}$, namely at the same rate of growth as the labour force. The equilibrium value of capital - labour ratio is $k^* = K_o/N_o = (\frac{s}{n})^{1/b}$. ... (19)

Reasonably enough, this equilibrium capital - labour ratio is larger, the higher the savings ratio and the lower the rate of increase of the labour force. The time-path of real output can be obtained from the Eqn. (1a). Obviously Y must behave asymptotically like K and N, that is, grow at relative rate N. Per capita output, $\frac{Y}{N}$, tends to the value $(\frac{s}{n})^{a/b}$.

Hence, the capital - output ratio, $\frac{K}{Y} = \frac{K/N}{Y/N} = \frac{(s/n)^{1/b}}{(S/n)^{a/b}} = (\frac{s}{n})^{\frac{1-a}{b}} = \frac{s}{n}$.

Thus, in the long - run, the warranted rate of growth, $s/v = n$, the natural rate of growth, not "by a fluke" but as a consequence of demand - supply adjustments.

T.W. SWAN VERSION OF NEO - CLASSICAL GROWTH MODEL

Taking logarithms of both sides of Eqn. (1a), we get,

$$\log Y = a.\log K + b.\log N$$

Differentiating both sides with respect to time, we get,

$$\frac{1}{Y}.\frac{dY}{dt} = a\frac{1}{K}\frac{dK}{dt} + b.\frac{1}{N}\frac{dN}{dt}$$

Using Eqns. (3), (4), and (12) of Section III in the immediately preceding equation, we obtain:

$$\frac{1}{Y}.\frac{dY}{dt} = a.s.\frac{Y}{K} + b.n \quad \text{... (20)}$$

T.W. Swan (1956) states, "Effective demand is so regulated (via the rate of interest or otherwise) that all savings are profitably invested, productive capacity is fully utilized, and the level of employment can never be increased merely by raising the level of spending. The forces of perfect competition drive the rate of profit or interest r and the (real) wage rate w into equality with the marginal productivities of capital and labour derived from the production function."

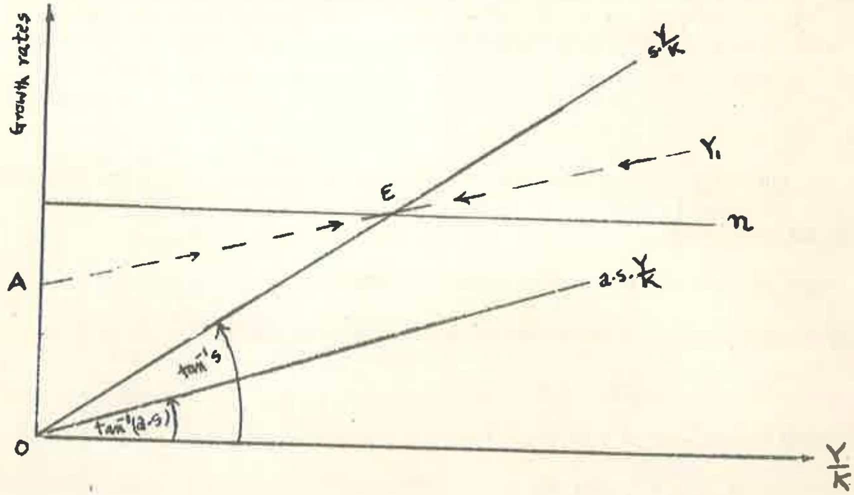
Differentiating partially both sides of Eqn. (1a) with respect to K and N, separately, we obtain:

$$r = \frac{dY}{dK} = a.\ K^{a-1}.N^{b} = a.\ \frac{Y}{K} \qquad \ldots (21)$$

$$w = \frac{dY}{dN} = b.K^{a}\ N^{b-1} = b.\ \frac{Y}{N} \qquad \ldots (22)$$

Thus, the profit rate is proportional to output-capital ratio, $\frac{Y}{K}$; and the wage - rate is proportional to per capita output $\frac{Y}{N}$. The relative shares of total profits and total wages in real income are constants, given by the production elasticities a & b. The above relations of the variables are depicted graphically in Figure III.

Figure III



In the above Figure III, the rate of growth of capital, $s.\frac{Y}{K}$, is shown as a function of the output-capital ratio, $\frac{Y}{K}$, by a straight line passing through the origin with a slope equal to the savings ratio, s. Swan calls it "the growth line of capital." The resulting contribution of capital to the growth of output, $a.s.\frac{Y}{K}$, is another line through the origin, of slope, a.s, and is called "the contribution line of capital." The constant rate of growth of the labour force, n, is represented by the horizontal "growth line of labour."

In the Figure III, the distance OA on the vertical axis is equal to b.n, the contribution of the labour to the growth of output. Adding the contributions of the labour and capital, we get "the growth line of output", AY_1.

With constant returns to scale, $a + b = 1$, these three growth lines of capital, labour, and output must intersect at the same point E, where growth in each case equals n, the Harrod's natural rate of growth. The growth line of output lies between those of capital and labour, and divides the vertical distance between them in the ratio a:b.

To the left of the intersection point E, growth line of output is above that of capital, i.e., output is growing faster than capital, so the output - capital ratio is rising -- moving rightward. Anywhere right of E, growth line of output is below that of capital, capital is growing faster than output, so the movement of the output-capital ratio is leftward. The economy is stable only at the equilibrium point E. At any other point the economy is always in motion towards E, as shown by the arrows on the line AY_1. A point such as E is a stable equilibrium point because the growth line of capital cuts that of labour (and so the growth line of output) from below. If over a certain range the saving ratio s were a decreasing function of $\frac{Y}{K}$, the growth line of capital might cut the growth line of labour from above, and this second intersection would be an unstable equilibrium point (the arrows would be directed away from it on either side).

CRITICAL EVALUATION OF NEO-CLASSICAL GROWTH MODEL OVER HARROD-DOMAR MODEL

Both the Harrod-Domar and the neo-classical growth models make many abstractions from reality as the list of assumptions in Section I indicates; while relaxing the assumption (3), the neo-classical model adds the assumption that factor prices are perfectly flexible. Even with capital and labour being technically substitutable, factor price rigidities, such as the downward stickiness of the (real) wage rate or the (real) rate of interest, will prevent the automatic adjustment of the capital - labour ratio through the substitution of one factor for the other at the margin.

The neo-classical approach to the problem of steady-state growth is based on the assumption of an indefinitely large number of production processes, shading off smoothly from one to another, so that the capital-output ratio, v, varies continuously and there is a greater chance that the warranted rate of growth $G_w = \frac{s}{v}$, equal the natural rate of growth $G_n = n$. The question is: does this chance become a near - certainty for a large number of production processes ? Probably NOT !

The neo-classical growth model, with a Cobb-Douglas production function, is stable in the sense of equilibrium dynamics. The same can be obtained at least for certain other production functions like C.E.S. production function and for a classical saving function. But nothing is said about stability in a disequilibrium sense. There is no mechanism in the model itself to ensure that the 'right' output-capital ratio is achieved. A steady-state growth is consistent with the neo-classical model in the sense that, once the 'right' output-capital ratio is achieved, then it remains constant on the steady-state equilibrium path. The 'right' output-capital ratio is that specified by the familiar Harrod-Domar condition, $\frac{s}{v} = n$. The limitation of the neo-classical approach is that it leaves wide open the question: what happens when the output-capital ratio at any time is NOT that appropriate to a steady-state growth ? A partial answer, no more, may be found in the working of the neo-classical model itself.

In its basic form, the neo-classical growth model rests on the assumption that it is always possible and consistent with equilibrium that investment should be undertaken of an amount equal to full-employment savings. Neo-classical writers generally have in mind some financial error-adjustment mechanism that makes use of the rate of interest and/or an investment function. They assume that the capital-stock initially is of that amount appropriate to a steady-state growth, so that the rate of interest, that makes investment equal to full-employment savings in the short period, is also the rate of interest required for the steady-state growth. The interest-rate may adjust to this level either (1) by the operation of Say's Law: supply creates its own demand, in an economy without money (or one in which the demand for money is not interest-elastic); or (2) because the price-level can always be made to adjust in such a way as to produce the appropriate interest-rate through its effect on the level of real money balances according to Eisner and Kahn ; or (3) by the actions of the monetary authorities according to Meade. The Keynesian objections are well known: a sticky monetary system or an interest-inelastic investment function may cause the neo-classical error-adjustment mechanism to collapse. Investment of full-employment savings may not be obtained at any level of interest-rate. Harrod argues that the interest-rate, fixed by the sticky monetary system, leads entrepreneurs to select a capital-output ratio different from the one required for the steady-state growth.

Even given a monetarily determined interest-rate, it has been argued by Tobin, Kaldor and Solow that there is a "solution" for the steady-state growth. In equilibrium, the own-rate of interest (i.e. the return on capital in units of capital-stock, q) on capital, ρ , must be equal to the money-rate of interest, r, in terms of output. Thus,

$$r = \rho + \frac{dp}{dt}\Big/_{p} \text{ where } \rho = \frac{dF(K,N)}{dK} = \frac{q}{p}$$

If the price level is constant, $\frac{dp}{dt} = 0$, then $r = \rho$.

If the price level is rising, $\frac{dp}{dt} > 0$, then $r < \rho$.

The logical cogency of the above argument is beyond reproach, provided the appropriate expectations can be generated and maintained. It argues that there exists some expectation of a rate of increase in money prices, p, which will allow ρ to be as small as we please. But it is not clear that, in the absence of government intervention in the form of monetary and/or fiscal policy, this way out of the Keynesian difficulty is feasible.

Considering the speed of error-adjustment, the neo-classical growth model asserts that the warranted rate of growth will adjust to the natural rate, but it does not indicate if the adjustment process would be fast enough to be acceptable for practical purposes. Sato has shown with numerical results that the error-adjustment process in the neo-classical system takes place only at an extremely slow rate. Rynzo Sato in 1964 remarks, "For most practical purposes the neo-classical system of variable proportions by no means rules out the use of fixed proportions in the combination of capital and labour which is a characteristic of the Harrod-Domar model. The insight of the Harrod-Domar model may still serve as a guiding landmark in theoretical growth analysis, provided that the production side is accounted for in a manner similar to the neo-classical growth model."

Harrod anticipates the criticism that his model "probably gives too much importance to the acceleration principle", and he suggests that the criticism could be met by introducing the "deepening" factor, d, "the value of new capital installations during the unit period, expressed for convenience as a fraction of income, involved in the lengthening of the production process." He envisages a neo-classical type error-adjustment mechanism via the "deepening" factor, d, which "may have a positive value because the rate of interest is falling." So Harrod is aware of the arguments on which the neo-classical growth model is based.

He considered them but rejected the neo-classical approach on orthodox Keynesian grounds. According to T.W. Swan, "He argues that natural market forces cannot be expected to achieve the desired results, but does not despair that Keynesian policies may be successful."

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BOOK REVIEW

Mukherjee, S.B. (1988), Population Growth And Urbanization in South And South-East Asia, Sterling Publisher, New Delhi, pp. vii + 166, Price Rs. 125.00 I.C.

Professor S.B. Mukherjee has initiated a number of studies on the related problems of demographic changes, urbanization and economic development. The present book is monograph of the findings from the research undertaken by the author. After going through this book it must be acknowledged that this book is commendable for a long and proud list of his other works. In this book he has drawn a vivid picture of an ironical situation that the rich countries are growing in wealth and the poor countries are growing in population. According to him, existing population problem of South and South-East Asia caused by high birth-rates and low death-rates is unique. The reason is that the countries of Europe and America, the so-called richer countries of the world, did not, face similar problem during their underdevelopment stage.

The phenomenon of population growth in South and South-East Asia has alarming implications for the future of Asia as well as to the world. This problem has been explained in the first chapter of the book. According to the author, the problem of accelerated growth of population, caused by declined death-rates and near stable birth-rates, requires the continued economic development so that some favourable results can be brought about. Not that there is no economic development in South and South-East Asia. The tragedy is that, the rate of development is so slow that the disparity between South and South-East Asia on one hand and Europe and North America on the other is greater today than 40 years back. The disparity may be wider in the future. In Europe and North America industrialization and urbanization proceeded hand in hand. But, here in South and South-East Asia the pattern of urbanization is not marked by industrialization. In the light of this situation the subsequent chapters have been developed.

As is clear from the name of the book, the geographic coverage of the study includes South and South-East Asia. They include Eastern South Asia and Middle South Asia. Of the 19 countries of these areas 12 have been included in the study. Of these 6 are SAARC countries: Bangladesh, Bhutan, India, Nepal, Pakistan and Sri Lanka; five of them are ASEAN countries: Burma, Indonesia, Thailand, Philippines and Malaysia. The twelfth country under study is Afghanistan.

In the second chapter, a bird's-eye view of the countries under study has been given. The selected countries greatly differ among themselves in area, population and the level of development. But, until recently they were similarly characterized by a high birth-rate, a high death-rate, a low life expedancy at birth, a low level of literacy, low per capita income, low rate of savings and investment and a weak industrial base. Area, population, employment situation, major crops, major products etc. of the selected countries are explained in Chapter 2. ASEAN countries are in better position than the SAARC countries in every respect.

The third chapter provides a very interesting broad demographic profile of the selected countries. To make it more interesting, data have been taken from the Demographic Year Book, Statistical Year Book and other publications of United Nations. And they have been supplemented by information obtained from other specialized studies. It is interesting to note that the density of the population in the study area is 124 per square kilometers, while that for the world as a whole is only 34. Among the countries under study Bangladesh, Afghanistan and Pakistan have higher birth-rates and those for Sri Lanka and Thailand are lower. Regarding death-rates, Sri Lanka has minimum and Nepal has maximum. Not only this, Sri Lanka tops the table of the life expectancy at birth with 69 years. Afghanistan has 37 years only. Both child and infant mortality rates are higher in Afghanistan and lower in Malaysia and Sri Lanka.

On coming to the fourth chapter we find a pretty well explained picture of socio-economic correlates of population growth. Population growth depends upon four factors: births, deaths, immigration and emigration. They are influenced by and correlated to social and economic factors but not in uniform manner. Not only this, social, cultural and economic factors influence and are influenced by demographic variables. To examine the inter-relations and interdependence of social, economic, cultural and demographic variables 9 variables have been selected for 23 countries. The 9 variables are: birth-rate, death rate, total fertility rate, life expectancy, per capita GDP, proportion of GDP generated in industry, proportion of labour force in industry, proportion of population living in urban areas and literacy-rate. The inter-relations and interdependence among these variables are shown with the help of the statistical tools of correlation analysis and regression analysis. To test the significance of correlation coefficient the t-test has been used and to test the overall significance of regressions, F-test has been used. The correlation coefficients among the variables and corresponding t-values are given in a matrix form. In this way, using the mathematical and statistical tools the author has given a vivid picture of socio-economic correlates of population growth.

The fifth chapter is furnished with age composition in the study area. There is comparison of age distributions in the study area and the advanced countries. India has been taken in particular due to the availability of data for a reasonable period of time. There is use of diagrams, pyramids and Lorenz curves along with Gini coefficients of dissimilarity.

Sixth and seven chapters deal with the urbanization aspect. In developing countries, the rate of growth of urban population is now 3 times the rate in the developed countries. The urban population is growing several times faster than in the rural areas, through natural growth and migration from the rural area. A detailed study in this respect is found in chapter six; and the chapter seven explains the underlying cause of the growth of big cities in India.

The eighth chapter examines the population policies in the study area. The author does not agree to the arguments of the so-called anti-Malthusians. He mentions different points showing that anti-Malthusian arguments are not applicable in the context of present day situation of the study area. The author has highlighted the population policy of China. Finally, he comes to the point that only socio-economic development is not the best contraceptive. Over and above this, vigorous efforts for control of fertility are called for. South and South-East Asia have a lot to learn from China in this respect.

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