

Measures of the Sectoral Interdependence in the Input- Output Framework to Identify the "Key" Sectors of an Economy for Future Economic Growth

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INTRODUCTION

Today all the countries of the world are paying more and more attention in collecting the statistics, which are required for the preparation of Input-output Models.

One of the most important uses of an input-output table is that it lays bare the interdependence between various sectors of the economy. The aspect of interdependence arising through technological interconnections between the various sectors of the economy has led to explore the notion of the "Key Sectors."¹ The basic idea is that some sectors are in a favourable position to induce the expansion of other sectors and sometimes even help in the initiation of new industries. Thus a pattern of pressures and incentives can be worked out by investing in those sectors initially which have higher technological linkages.² The purpose of this exercise is to select the important sectors of the economy which would accelerate the growth process through inducements.

In this paper an attempt has been made to analyse the different methods available for identifying the key sectors of the economy for the future economic growth.

By reviewing the literature on linkages and identification of key sectors, we were able to find following methods:

- i) The backward and forward linkages as suggested by Chenery and Watanbe (1958).
- ii) The Yan and Ames Interrelatedness Index (1965).
- iii) The average output Multiplier.
- iv) Rasmussen's backward and forward linkages.

THE BACKWARD AND FORWARD LINKAGES AS SUGGESTED BY CHENERY AND WATANBE

This method³ uses the ratio of purchased inputs to total output to measure the intensity of backward linkage (B_j). Similarly the ratio of intermediate demand to total output (F_i) has been treated as a measure of the intensity of forward linkage:

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That is: Total use by Sectors of input

$$B_j = \frac{\text{purchased from other sectors}}{\text{Total output of sector } j}$$

$$F_i = \frac{\text{Total intermediate use of commodity } i}{\text{Total supply of commodity } i}$$

Obviously therefore:

$$B_j = \frac{\sum_{i=1}^n X_{ij}}{x_j}, \quad \text{and } F_i = \frac{\sum_{j=1}^n X_{ij}}{x_i}$$

where X_{ij} is the transaction matrix of the input-output modal.

It is thus clear that B_j measures the extent to which an industry absorbs inputs from the system of industries in its fabrication, while the F_i shows dependence of system of industries on a particular industry supplies its output to the system of industries. These linkages, it has been suggested, could be used as developmental tool to induce investment activity and to serve other growth objectives as income and employment generation.

The methodology considered by Cheney³ and Watanbe uses only the ratio of intermediate to aggregate demand for a given product. Hence there is every chance of the underestimating the values of B_j and F_i due to aggregation. If there is only one huge transaction, and if other values are zero, then as per this index that sector is well inter connected so these linkages may be high due to high material intensity and in some cases the possibility of wasteful use of resources is not ruled out.

Moreover, the method while measuring inter-industry relatedness takes into consideration only the direct purchase and sales. Thus the technique only measures the direct interconnections between the sectors. However, to see how important a sector is in an economy, it is not enough to examine the direct interrelations between the various sectors.

THE YAN AND AMES INTERRELATEDNESS INDEX

A sector may sell to or buy directly from a few sectors yet its customers and suppliers may be connected with many other sectors of the economy. That is to say that the sector might bear profound influence on the economy through its indirect relations with other sectors. It is, therefore, desirable to consider all direct and indirect relations of a sector with others to decide upon the importance of each sector.

Yan and Ames⁴ associated with each technical coefficient matrix, A , and order matrix OYA , and use the latter to define an "interrelatedness function." This functions in its aggregated version is expressed by a single number the interrelatedness index IYA , for a row or a column. The authors explain some potential applications of their method which, they believe can be a convenient tool for verifying a range of hypotheses concerning interindustrial diversity and specialization.

Yan and Ames explain their index by taking a 4x4 matrix A.

$$A = \begin{bmatrix} 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0.3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Each non-zero element a_{ij} indicates a direct output-input relations between sector i and j, this is called direct relation of first order. A zero element on the other hand indicates the absence of direct output-input relation between a pair of sectors.

The second order relatedness can be found by calculation A^2 for matrix A defined above;

$$A^2 = \begin{bmatrix} 0 & 0 & 0.09 & 0 \\ 0 & 0 & 0 & 0.09 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The A^3 reveals the third order relatedness

$$A^3 = \begin{bmatrix} 0 & 0 & 0 & .027 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Consequently A^4 will be composed of all zeros as well as all higher round matrices will be null.

Denoting the first, second and third order relatedness by number 1, 2 and 3 respectively, an order matrix O^{YA} corresponding to the matrix A may now be defined as;

$$O^{YA} = \begin{bmatrix} X & 1 & 2 & 3 \\ X & X & 1 & 2 \\ X & X & X & 1 \\ X & X & X & X \end{bmatrix}$$

The symbol X in the matrix has been used in place of Zero to avoid indetermination in further calculations.

Interrelatedness of a sector can then be evaluated as follows:

For the output side; sum up the reciprocal of the entries in a row and divide by 4 (because the order of A is 4 by 4):

$$\begin{aligned} \text{Row I} &= \frac{1}{4} \left[\frac{1}{x} + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} \right] \\ &= \frac{1}{4} [0+1+ .50 + .33] = 0.47 \text{ here } \frac{1}{x} = 0 \end{aligned}$$

$$\text{Row II} = \frac{1}{4} \left[\frac{1}{x} + \frac{1}{x} + \frac{1}{1} + \frac{1}{2} \right] = .38$$

$$\text{Row III} = \frac{1}{4} \left[\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{1} \right] = 0.25$$

$$\text{Row IV} = \frac{1}{4} \left[\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right] = \text{ZERO}$$

The closer the row average is to one, the more important is a sector as a supplier of intermediate output. Thus as per above calculations sector I is the most important supplier in the economy. All sectors rely directly or indirectly on it for their inputs.

Similarly the column average of reciprocals may be calculated:

$$\text{Col I} : \frac{1}{4} \left[\frac{1}{x} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right] = \text{Zero}$$

$$\text{Col II} : \frac{1}{4} \left[\frac{1}{1} + \frac{1}{x} + \frac{1}{x} + \frac{1}{x} \right] = 0.25$$

$$\text{Col III} : \frac{1}{4} \left[\frac{1}{2} + \frac{1}{1} + \frac{1}{x} + \frac{1}{x} \right] = 0.38$$

$$\text{Col IV} : \frac{1}{4} \left[\frac{1}{3} + \frac{1}{2} + \frac{1}{1} + \frac{1}{x} \right] = 0.47$$

As seen in this case the index behave quite smoothly. There are precisely three connections of first order, two of second, and one of third, which are appropriately recorded by the order matrix, O^{YA} . However, a more detailed analysis of this method will reveal some objections.

First of all why this formula is limited to the first "meeting" between i's and j's? Why ignore those connections which are not "first"? It may happen, for instance, that a link of a higher order, while being practically insignificant (in terms of its quantity) is still taken into account by O^{YA} because it is the "first", whereas another link, more important in magnitude, is ignored, just because it is the "second" one.

The next question arises: why did Yan and Ames use the reciprocals of the "order number" in their formula?

A critique of the Yan and Ames measured by Blin and Murphy⁵ seems to be fully justified. If one wants to take into account the appropriate magnitude of interindustry connections, it appears to be more reasonable to consider simply the power series A, A^2, A^3 which leads to the Leontief inverse. The later is a better approximation of the importance of particular sector in $I - 0$ structures.

THE OUTPUT MULTIPLIER

Within an input-output frame it is possible to derive a set of multipliers that describe summary measures of the total repercussions in terms of adjustments in output, employment, and income generated by a given change in the final demand vector.

The Leontief inverse⁶ in the open input-output modal is the unique device to capture all the direct and indirect effects of a unit increase in final demand. The elements in each of the column in the inverse give the direct and indirect requirements from the various sectors in the row to produce a unit increase in the final demand of the sectors at the top.

The concept of output multipliers in the input-output frame can be understood in the following way:

The direct plus indirect output

$$\begin{array}{l} \text{Multiplier per unit} \\ \text{change in the Final Demand} \end{array} = \sum_{j=1}^n z_{ij}$$

$$\text{or } I^{OM} = [i (I-A)^{-1}]$$

where i = unit sector row

and $(I-A)^{-1}$ = The Leontief inverse matrix

In a 2 x 2 case the I^{OM} can be defined as

$$\begin{aligned} &= [1 \quad 1] \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \\ &= [z_{11}+z_{21}, \quad z_{12} + z_{22}] \end{aligned}$$

The higher the value of the multiplier of a sector more important it is in the economy.

From the Leontief's inverse one can also calculate the income, employment and wages multipliers.

RASMUSSEN'S BACKWARD AND FORWARD LINKAGES

A more refined way of computing the backward and forward linkages with the help of $(I-A)^{-1}$ matrix is suggested by Rasmussen.⁷ Indices of sectoral linkages based on inverse matrix is developed by Rasmussen in a study of structural change. As explained earlier by using the inverse of an I-o Matrix, these indices take into account the direct as well as the indirect effects of an increase in autonomous expenditure. These linkages are properly weighted and, therefore, more correctly depict the importance of strategic sectors of the economy.

To simplify the Rasmussen's linkages consider Z as an inverse of $(I-A)$ with Z_i 's as its typical elements as we have assumed in the output multiplier method. Z_i 's can be interpreted as the increase in the output in industry i for per unit increase in the final demand for the product of industry j . An index of backward linkage that Rasmussen calls the power of dispersion is defined as follows:

$$V_j = \frac{\frac{1}{n} \sum_{i=1}^n Z_{ij}}{\frac{1}{n} \sum_{i=1}^n \sum_{j=1}^n Z_{ij}} \quad (J = 1, 2 \dots n)$$

Where Z_{ij} is the sum of the column elements and is to be interpreted as the total increase in the output from the whole system of industries needed to cope with an increase in the final demand to be product of industry j by one unit. Similarly an index of forward linkage (that Rasmussen called the index of sensitivity of dispersion is defined as:

$$V_i = \frac{\frac{1}{n} \sum_{j=1}^n Z_{ij}}{\frac{1}{n} \sum_{e=1}^n \sum_{j=1}^n Z_{ij}} \quad (J = 1, 2 \dots n)$$

It is worth noting that these measures pre-dated ideas about the role of linkages in industrial development strategy, and were simply regarded as useful summary measures of the structural interdependence of an economy. Since the above mentioned two indices do not tell the whole story because average are sensitive to extreme values and may also give misleading results to overcome this difficulty by defining a measure of variability represented by both of the following indices of the coefficient of variation.

$$V_j = \frac{\sqrt{\frac{1}{n-1} \sum_{i=1}^n (Z_{ij} - \frac{1}{n} \sum_{i=1}^n Z_{ij})^2}}{\frac{1}{n} \sum_{i=1}^n Z_{ij}} \quad (J = 1, 2 \dots n)$$

$$V_i = \frac{\sqrt{\frac{1}{n-1} \sum_{j=1}^n (Z_{ij} - \frac{1}{n} \sum_{j=1}^n Z_{ij})^2}}{\frac{1}{n} \sum_{e=1}^n Z_{ij}} \quad (J = 1, 2 \dots n)$$

A high V_j means that a particular industry draws unilaterally on the system of industrial, and a low V_j means that an industry draws evenly from the other sectors. The V_i can be interpreted in the same way.

Key sectors are defined as those in which both V_j and V_i exceed unity. A key sector is therefore a sector which generates above - average input requirements from other sectors, and whose output is widely used by other sectors. Once key sectors identified, it is suggested that these sectors be given priority in investment allocation and in industrial promotion strategy. It is believed that if resources can be concentrated on these key sectors, output and employment in the country or region will grow more rapidly than if these resources were allocated in some alternative way.

FOOTNOTES

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