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A Note on the Approaches to Play With Figures

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INTRODUCTION

Background

Mathematics is the base of all other sciences, and arithmetic, the science of numbers, is the base of mathematics. Numbers are also divided into two parts: (a) Real numbers, and (b) Imaginary numbers. Real numbers comprise of all rational and irrational numbers and rational numbers include all integers and fractions. Positive integers are the numbers usually used in counting. These numbers consist of whole numbers which are formed by the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and any combinations of them. The set of simple four operations viz. addition, subtraction, multiplication, and division is the base of arithmetic. By the help of calculators we can find out all these values correctly, but we may not always have carried a calculator with us. So we have to solve the problems by hand and there remains still some doubt about the correctness of results. Thus, to tackle with such difficulties in calculations, an attempt has been made to formulate some techniques for checking the results of calculations.

Objectives

The principal objective of this paper is to design some of the tricks of division by the first one dozen natural numbers. Furthermore, an effort has been made to present some techniques to check other operations also. The utility of the techniques mentioned here is that it saves our time and labour, and also confirms the accuracy of the results.

In the following sections, a list of the basic formulations of all rules (techniques) has been presented. Formal proof and a numerical example for each technique has also been provided. Finally, the article is concluded.

BASIC FORMULATIONS

Addition

If we have to add some numbers and check the results, there is no such an exciting method. The only way is to add those numbers separately with different combinations, taking each number once and only once, and finally add all these values. If our result is correct, the final results must be same.

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Subtraction

If we have to subtract a whole number from another, the most easy method to check the result is to add the result with the number that was subtracted. If we get the same number from which the number was subtracted, then the result is correct, otherwise not.

Multiplication

Multiplication is a little bit more difficult task to accomplish than addition or subtraction. If we multiply a number (say first number) with another number (say second number) and get a result, then to check whether our result is correct or not, we can perform the following steps:

(a) Take the sum of all digits of the first number. Divide it by 9 and get the remainder.

(b) Repeat step (a) for the second number also and get the remainder.

(c) Multiply the two remainders obtained from (a) and (b), divide this number by 9 and get the remainder.

(d) Take the sum of all the digits of the multiplication result and also divide it by 9 and get the remainder.

If our multiplication result is correct, then the remainders obtained from (c) and (d) must be equal. We should note here that even if the multiplication is not correct, sometimes (c) and (d) may be equal. An easier technique to get the remainder when we divide a number by 9 is discussed under the sub-section of division by 9.

Division

Division is a tedious business in the execution among the four operations of arithmetic. When we have to do a division, we are given with a divisor and a divident, and required to find out the quotient and the remainder. The relationship among them can be shown as:

Divisor) Divident (Quotient

....

Remainder

To check the result multiply the divisor and the quotient, add the remainder and compare it with the divident. They must be equal for our solution to be correct.

When we have to simplify some fractions, then we have to think about a number that will divide both the numerator and denominator. Most of the time we have to try with every possible number. It not only consumes our time but is also a tiring job. Here an effort is made to present some tricks that will ease our problem. Some rules have been presented to see whether a number is divisible by any of the first one dozen natural numbers or not.

Following are the basic definitions of the terms used in this paper:

(a) Number: It is a finite integer (whole) number.

(b) Divisible: Here it means exactly divisible, i.e. the remainder is zero.

(c) Natural Numbers: These are the positive integers, which are most frequently used in counting.

(d) Any number can be written as $\ldots + 1000X_4 + 100X_3 + 10X_2 + X_1$, where X_1, X_2, X_3, X_4, ... are the digits of unit's place, hundredth place, thousandth place, ... respectively.

0. Division of and by zero

Zero (0) is divisible by any number other than zero, and the result is 0. When we divide any number (except 0) by 0, the result will be infinity, i.e. we will not get any finite value. Division of 0 by 0 is not defined.

1. Division by 1

Any number is divisible by 1 and the result will be the same given number.

2. Division by 2

Any number ending with an even number is divisible by 2.

3. Division by 3

Any number is divisible by 3 if the sum of all the digits of the given number ($\ldots + X_4 + X_3 + X_2 + X_1$) is divisible by 3.

4. Division by 4

Any number is divisible by 4 if the number with the last two digits (in the same order) is divisible by 4.

5. Division by 5

Any number is divisible by 5 if the number terminates with 0 or 5.

6. Division by 6

Any number is divisible by 6 if the number is even and the sum of all its digits is divisible by 3.

7. Division by 7

Any number is divisible by 7 if the sum of first digit, three times the second digit, two times the third digit, six times the fourth digit, four times the fifth digit, five times the sixth digit, and so on is divisible by 7. (The ordinary trial and error method is much easier than this method !)

8. Division by 8

Any number is divisible by 8 if the number with the last three digits is divisible by 8.

9. Division by 9

Any number is divisible by 9 if the sum (of the sum ...) of all the digits of the given number is divisible by 9 (or it is exactly 9).

10. Division by 10

Any number that ends up with 0 is divisible by 10.

11. Division by 11

Any number is divisible by 11 if the difference of the sums of alternate digits is divisible by 11 (or zero).

12. Division by 12

Any number is divisible by 12 if the number with the last two digits is divisible by 4, and if the sum of all the digits of the given number is divisible by 3.

EMPIRICAL EVIDENCES

In this section the formal proof followed by an example of what has been mentioned in the preceding section is given.

Addition

If we have to add the numbers a, b, c, d, ..., then

$$a + b + c + d + \ldots$$
$$= (a + b) + (c + d) + \ldots$$
$$= (a + c) + (b + d) + \ldots \text{, and so on.}$$

Subtraction

Suppose we have to subtract any number b from a and get the result c, then c = a - b. Then transposing b to the left hand side, we get c+ b = a.

Example:

$$\begin{array}{r} 231409 \\ -124598 \\ \hline 106811 \end{array} \qquad \begin{array}{r} 106811 \\ +124598 \\ \hline 231409 \end{array}$$

Multiplication

Let the two numbers that we are going to multiply be

$$\ldots + 100\ X_3 + 10\ X_2 + X_1$$

and $\ldots + 100\ Y_3 + 10\ Y_2 + Y_1$

Where X_1, X_2, X_3, ... are the digits of unit's place, tenth place, hundredth place, ... and so on, and the same for Y_1, Y_2, Y_3, ... too.

The multiplication of these numbers will be

$$\begin{array}{r} \ldots + 100\ X_3 + 10\ X_2 + X_1 \\ \times \ldots + 100\ Y_3 + 10\ Y_2 + Y_1 \\ \hline \end{array}$$

$$= \ldots + 10000\ X_3Y_3 + 1000\ X_2Y_3 + 100\ X_1Y_3$$

$$\ldots + 1000\ X_3Y_2 + 100\ X_2Y_2 + 10\ X_1Y_2$$

$$\ldots + 100\ X_3Y_1 + 10\ X_2Y_1 + X_1Y_1$$

$$= \ldots + 10000\ (X_3Y_3 + \ldots) + 1000\ (X_2Y_3 + X_3Y_2 + \ldots)$$

$$+ 100\ (X_1Y_3 + X_2Y_2 + X_3Y_1) + 10\ (X_1Y_2 + X_2Y_1)$$

$$+ X_1Y_1.$$

$$= [\ \ldots + 9999\ (X_3Y_3 + \ldots) + 999\ (X_2Y_3 + X_3Y_2 + \ldots)$$

$$+ 99\ (X_1Y_3 + X_2Y_2 + X_3Y_1) + 9\ (X_1Y_2 + X_2Y_1)\ ]$$

$$+ [\ \ldots + (X_3Y_3 + \ldots) + (X_2Y_3 + X_3Y_2 + \ldots)$$

$$+ (X_1Y_3 + X_2Y_2 + X_3Y_1) + (X_1Y_2 + X_2Y_1)$$

$$+ X_1Y_1\ ]$$

The numbers within the first big parenthesis is exactly divisible by 9 and the remainder (if any) will be in the second big parenthesis. The remainder will be in the expression

$$\ldots + (X_3Y_3 + \ldots) + (X_2Y_3 + X_3Y_2 + \ldots)$$
$$+ (X_1Y_3 + X_2Y_2 + X_3Y_1) + (X_1Y_2 + X_2Y_1)$$
$$+ X_1Y_1.$$

$$= \ldots + X_3Y_3 + X_2Y_3 + X_1Y_3$$
$$\ldots + X_3Y_2 + X_2Y_2 + X_1Y_2$$
$$\ldots + X_3Y_1 + X_2Y_1 + X_1Y_1$$

$$= \ldots + Y_3(\ldots + X_3 + X_2 + X_1) + Y_2(\ldots + X_3 + X_2 + X_1)$$
$$+ Y_1(\ldots + X_3 + X_2 + X_1)$$

$$= (\ldots + Y_3 + Y_2 + Y_1)(\ldots + X_3 + X_2 + X_1).$$

Using the above mentioned method recursively, we get the proof of our statement.

More about the division of any number by 9 will be proved later on.

Example	Check
13478	(a) 1+3+4+7+8 = 23
	Remainder = 5
x 2579	(b) 2+5+7+9 = 23
	Remainder = 5
34759762	(c) 5 x 5 = 25
	Remainder = 7
	(d) 3+4+7+5+9+7+6+2 = 43
	Remainder = 7

Since (c) and (d) are equal, then this may be correct answer (actually it is). If the values in (c) and (d) are not equal, we can confidently say that the product-result is not correct.

Division

1. Division by 1

We do not have any problem with the division of any number by 1, because the result will be the same given number.

$a / 1 = a.$

2. Division by 2

Any number can be written as

$... + 1000\ X_4\ 100\ X_3 + 10\ X_2 + X_1.$

All the other terms of the above expression, except the last one, are divisible by 2 because all of them are the integer multiples of 10 which is also always divisible by 2. So if X_1 is divisible by 2, then the given number is divisible by 2, otherwise not. Here, X_1 is the last digit (digit of the unit's place) of the given number. X_1 is divisible by 2 if it is an even number (i.e. 0, 2, 4, 6, 8).

Example: 21415<u>6</u> is divisible by 2.

34563<u>7</u> is not divisible by 2.

3. Division by 3

Let the number be

$... + 1000\ X_4 + 100\ X_3 + 10\ X_2 + X_1.$

$= ... + 999\ X_4 + 99\ X_3 + 9\ X_2$

$+ ... + X_4 + X_3 + X_2 + X_1.$

This is divisible by 3, if $(... + X_4 + X_3 + X_2 + X_1)$ is divisible by 3, because the upper expression is always divisible by 3 since 9 and integer multiples of 9 are always divisible by 3.

Example: 235078, Sum = 25, not divisible by 3.

153762, Sum = 24, divisible by 3.

4. Division by 4

Any number $... + 1000\ X_4 + 100\ X_3 + 10\ X_2 + X_1$ can be written as $(... + 1000\ X_4 + 100\ X_3) + 10\ X_2 + X_1.$

The terms within the parenthesis are always divisible by 4, because they are the integer multiples of 100 (i.e. 25 x 4). So if $10 X_2 + X_1$ is divisible by 4, then the given number is divisible by 4, otherwise not.

Example: 214574 is not divisible by 4.

367948 is divisible by 4.

5. Division by 5

Any number $\ldots + 1000 X_4 + 100 X_3 + 10 X_2 + X_1$ can be written as $\ldots (\ldots + 1000 X_4 + 100 X_3 + 10 X_2) + X_1$.

The terms within the parenthesis are always divisible by 5 because they are the integer multiples of 10 (i.e. 5 x 2). So a number divisible by 5, if X_1 (the digit of the unit's place) is divisible by 5. It will be so when X_1 is 0 or 5.

Example: 324675 is divisible by 5.

768543 is not divisible by 5.

6. Division by 6

Any number divisible by 6 must be divisible both by 2 and 3. So from above, we can infer that if the number is even and the sum of all its digits is divisible by 3, then the number is divisible by 6.

Example: 823341 is not divisible by 6 because it is an odd number.

832314 is divisible by 6 because it is an even number and the sum of the digits is divisible by 3.

7. Division by 7

The most difficult task to check whether a number is divisible or not is by 7. Any number

$$\ldots 10000 X_5 + 1000 X_4 + 100 X_3 + 10 X_2 + X_1$$

can be written as

$$\ldots 7000 X_5 + 700 X_4 + 70 X_3 + 7 X_2 + 3000 X_5 + 300 X_4 + 30 X_3 + 3 X_2 + X_1$$

or as

$$\begin{aligned} &\ldots 7000 X_5 + 700 X_4 + 70 X_3 + 7 X_2 \\ &\ldots + 2800 X_5 + 280 X_4 + 28 X_3 \\ &\ldots + 140 X_5 + 14 X_4 \\ &\ldots + 56 X_5 \\ &\ldots + X_7 + 5 X_6 + 4 X_5 + 6 X_4 + 2 X_3 + 3 X_2 + X_1. \end{aligned}$$

The first four rows are always divisible by 7. The given number will be divisible by 7 if the last row is divisible by 7. It will be so when $\ldots + X_7 + 5X_6 + 4X_5 + 6X_4 + 2X_3 + 3X_2 + X_1$ is divisible by 7.

Example: 2354 Here 6x2+2x3+3x5+4 = 37 which is not divisible by 7. So the given number also is not divisible by 7.

1897 Here 6x1+2x8+3x9+7 = 56 which is divisible by 7, so the given number also is divisible by 7.

Is not the ordinary trial and error method much easier than this method ?

8. Division by 8

A number

$$\ldots 10000\,X_5 + 1000\,X_4 + 100\,X_3 + 10\,X_2 + X_1$$

can be written as $(\ldots 10000\,X_5 + 1000\,X_4) + 100\,X_3 + 10\,X_2 + X_1$.
The terms within the parenthesis are always divisible by 1000 so by 8 also (because 1000 = 8x125) 8. So if $(100\,X_3 + 10\,X_2 + X_1)$ is divisible by 8, then the number is divisible by 8.

Example: 23456 is divisible by 8.

24546 is not divisible by 8.

9. Division by 9

A number $\ldots 1000\,X_4 + 100\,X_3 + 10\,X_2 + X_1$ can be written as $(\ldots 999\,X_4 + 99\,X_3 + 9\,X_2) + (\ldots X_4 + X_3 + X_2 + X_1)$. The number within the first parenthesis is always divisible by 9. If the expression in the second parenthesis is divisible by 9 then the whole expression (the given number) is divisible by 9, otherwise not. By iteration, this we can infer that any number is divisible by 9 if the sum (of the sum ...) of all the digits of the given number is 9. The last number obtained by adding all the digits will be the remainder when divided by 9.

Example: 2547 is divisible by 9 because the sum of all the digits of this number, 18, is divisible by 9 because 1+8 = 9 is divisible by 9.

2387 is not divisible by 9 because the sum, 20, is not divisible by 9 because 2+0 = 2 is not divisible by 9. This 2 is the remainder when we divide the given number 2387 by 9.

10. Division by 10

Any number ... $1000\ X_4 + 100\ X_3 + 10\ X_2 + X_1$ is divisible by 10 if X_1 is divisible by 10 because it is obvious that all the other terms are divisible by 10. X_1 is divisible by 10 if it is zero. So any number ending with zero is divisible by 10, otherwise not.

Example: 9870 is divisible by 10, whereas.

4809 is not.

This can be proved by the rule of 2 and 5. Because if a number is divisible by 10, it must be divisible by 2 and 5. From the rule of 2 and 5, the only common case when both 2 and 5 satisfy is the case when the number ends up with 0.

11. Division by 11

Any number ... $10000\ X_5 + 1000\ X_4 + 100\ X_3 + 10\ X_2 + X_1$ can be written as $(\ldots\ 9999\ X_5 + 1001\ X_4 + 99\ X_3 + 11\ X_2) + (\ldots + X_5 - X_4 + X_3 - X_2 + X_1)$.

The number will be divisible by 11 if the number within the second parenthesis is divisible by 11 because the expression within the first parenthesis is always divisible by 11 being the integer multiples of 11. The number within the second parenthesis can be written as

$$(\ldots + X_5 + X_3 + X_1) - (\ldots + X_4 + X_2).$$

It means that if the difference of the sums of the digits of alternate places is divisible by 11, then only the given number is divisible by 11.

Example: 57981 is divisible by 11, because (5+9+1) - (7+8) = 0 is divisible by 11.

59718 is not divisible by 11, because (5+7+8) - (9+1) = 10 is not divisible by 11.

12. Division by 12

Any number to be divisible by 12 must be divisible by 3 and 4. So applying the rule of 3 and 4, we can say that if the sum of all the digits of the given number is divisible by 3 and the number with the last two digits is divisible by 4, then the number is divisible by 12, otherwise not.

Example: 17892 is divisible by 3 and 4, so also by 12.

17829 is not divisible by 4, so not also by 12.

If we have to find out a number that is exactly divisible by all the numbers from 1 to 12, we have to find out the LCM of all these numbers. This gives us that the integer multiples of 27,720 is divisible by any of them.

CONCLUSION

In this paper it is attempted to present some devices for immediate check of the results obtained from the four simple operational rules of arithmetic, and also to check whether any number is exactly divisible by any of the first one dozen natural numbers or not. It is hoped that if we can apply those techniques mentioned above, it will be very helpful for us to test the accuracy of our results, and will save our time and labour as well.

BOOK REVIEW

Amatya, D.B. (1986): Nepal's Fiscal Issues: New Challenges (New Delhi: Sterling Publishers), pp. xiv + 205, price: Rs. 100.00 I.C.

Public Finance or Fiscal Policy, once a dull and unimaginative subject, has been one of the most exciting areas of study in modern economics. In the last thirty years or so, eminent economists like Nicholas Kaldor, James Buchanan, John F. Due, R.A. Musgrave, R.M. Bird, Vito Tanzi and others have been making significant contributions to Government Finance. Their work have become classic and have been an effective instrument to deal with the problem of stabilisation in developed as well as developing countries. Unfortunately, Professor Nicholas Kaldor passed away this year creating a void in the field. However, it is remarkable that the coveted Nobel Prize (1986) for Economics has gone to Professor James Buchanan, for his scholarly contribution to the field of Government Finance.

In Nepal, Government Finance has received considerable attention in recent years. At present this country is facing persisting problems of resource gap with limited exports and unlimited imports, adverse balance of payments situation, increasing pressure on foreign exchange reserves, excessively rising public expenditure, increasing internal and external borrowings. These factors have chiefly been responsible for Nepal's increasingly difficult and cumbersome economic situation. It is high time for Nepali Scholars and researchers interested in the study of Nepalese Government Finance to concentrate on these problems. The present book Nepal's Fiscal Issues is an outcome of this growing realisation.

In this book, the author has been critical of fiscal development in Nepal. Analysing the major issues he writes: "Nepal's fiscal policies since 1960 have been directed mainly to revenue and more revenue, sidetracking other important fiscal objectives such as growth, equity, redistribution and allocation ..... The emphasis on revenue aspect alone has led to misdirection of resources into unproductive lives, vulnerability to Indian political and economic pressure and concentration of income among non-residents leading to capital flight while increasing development expenditure has generated double digit inflation, the fixed income groups and traders of indigenous origin have suffered miserably." He further states that the country's fiscal vitality has so far been sustained by an upward revision of indirect taxes and since this has reached its peak, any further taxation will have adverse consequences.

The author has been very honest in tracing the historical facts with sufficient data and information. The book also contains methodology for Re-Exports Estimation, BOP viability projects and fiscal viability. In addition to the chapters related to fiscal policies the writer has also touched upon industrual development, foreign trade and political climate of the economy.

In an environment where there is paucity of authentic literature on Nepalese economy with Nepali writers sublishing very little, Dr. Amatya has done a commendable job by producing a book dealing with the various aspects of fiscal development in Nepal. This publication certainly is an asset to students, researchers and policy makers in the field of Government Finance in Nepal.

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