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Application of Control Charts to Pasteurised Milk Production in Nepal

(a case study of Dairy Development Corporation, Kathmandu)

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Summary :

The objective of this paper is to examine how far quality control has been maintained in the pasteurised milk produced by Dairy Development Corporation, Kathmandu, Nepal. The study has used the methods of statistical quality control which was first developed by walter A. Snewhart of the bell Telephone Laboratories in... The characteristics under study were *percentage of fat present* and *solid not fat* (SNF) present in milk samples. The control charts for averages and ranges were constructed to find out whether the production process was running under control or not. The process capability was then computed to findout what percentages of the produced milk meet the specifications maintained by Food Laws and Regulations of His Majesty's Government.

The operating characteristic functions where then derived. These functions help in revealing the probability of points falling within control limits when the process has actually changed as to either its central tendency or its variability.

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Introduction :

It is well realised fact that milk and milk products are the most important diet of human beings. The main supplier of milk are cows, buffaloes and in some countries goats, yaks and reinders etc. Since the source of milk from these mammals cannot fulfil the need of people growing so fast it has became essential to develop the dairy industry in every country. Both governments and dairies have developed techniques for checking the purity of milk both from the chemical and bacteriological sides. In our country too, Food Research Section of Department of Food and Agriculture Marketing Service, has given special attention in checking the quality of milk together with other food commodities. Here the quality of milk is checked by finding the percentage of fat and SNF present in milk samples and by phosphate test. However, the result obtained by phosphate test has not been dealt here.

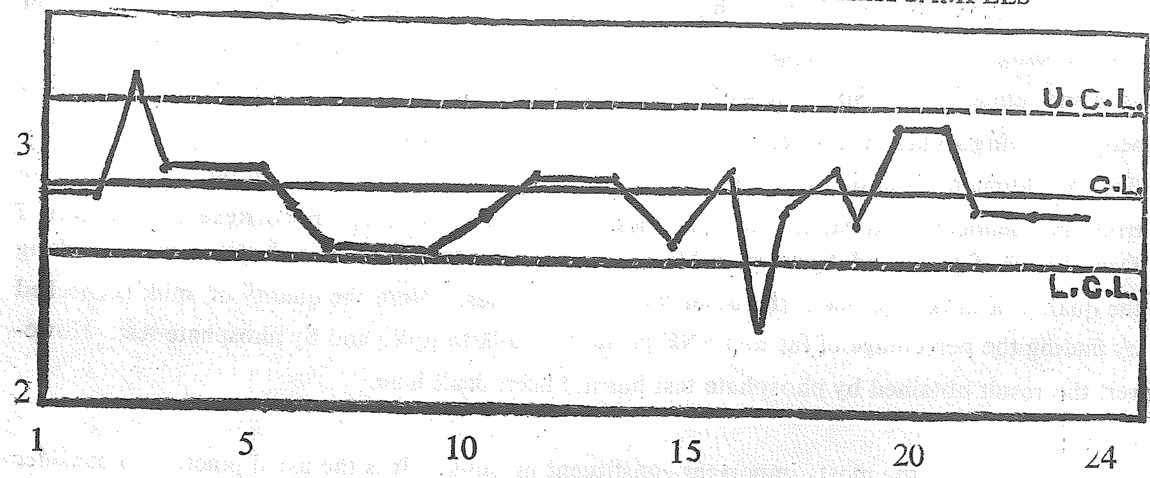
Fat is the most important constituent of milk. It is the usual practice to consider the quality of milk on the basis of its fat content.

Solid not fat means all the milk solids except the amount of fat present where milk solids include all the compounds present in milk except water.

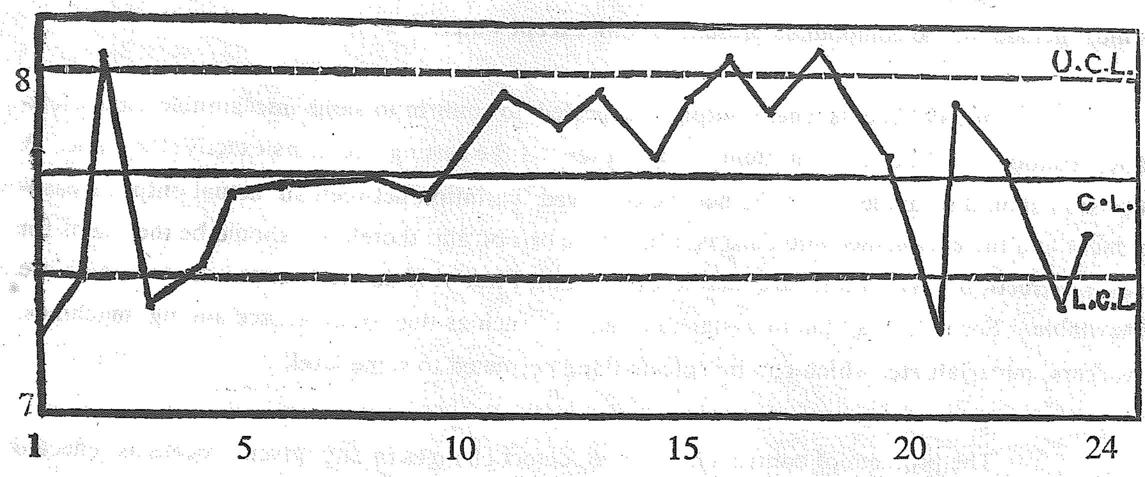
In any process where output is expected to confirm to same measurable characteristic variation will occur from item to item even if the setting remains exactly the same. A decision should be made as to whether the observed variation between an actual output measurement and the established standard is acceptable or not and therefore should be the basis for some correction. Variations are due to two causes. First it is due to chance causes which are inevitable. Second, it is due to assignable causes such as due to difference among machines, workers, materials etc. which can be detected and corrected to some level.

The problem of control charts is to detect changes in any given process as reflected by observations of output variation. It should be understood that control chart cannot turn a basically poor process into a good one. It is intended to provide a means for ensuring that a process remains in control. In statistical quality control a manufacturing or production process is said to be in control when the systematic variations in quality are traced out and eliminated or reduced to an acceptable level leaving behind the system of chance causes alone. These charts enable to ascertain sudden or gradual departure from permitted tolerances thus eliminating or at least minimising the amount of unacceptable work produced.

$\bar{X}$ CHART FOR PERCENTAGE OF FAT PRESENT IN MILK SAMPLES



$\bar{X}$ CHART FOR PARCENTAGE OF SNF PRESENT IN MILK SAMPLES



Application :

The data regarding the results of percentage of fat & SNF present in milk samples tested in Food Research section from Kartik 2038 to Chaitra 2038 were collected. The data were then classified into 24 sub groups each sub group consisting of five values. Control limits were then computed for both the characteristics from the corresponding averages and ranges of the 24 subgroups.

Table 1

Data representing the averages & ranges

Sub group No.	Percentage of fat		Percentage of S N F	
	Average	Range	Average	Range
1	2.9	1.0	7.28	1.2
2	2.9	0.5	7.46	0.7
3	3.2	0.3	8.14	0.7
4	3.0	0.2	7.40	0.5
5	3.0	0.1	7.48	0.7
6	3.0	0	7.73	0.8
7	2.9	0.2	7.78	0.6
8	2.8	0.2	7.63	0.1
9	2.8	0.2	7.73	0
10	2.8	0.5	7.71	0.1
11	2.9	0.3	8.03	0.3
12	3.0	0.2	7.90	0.3
13	3.0	0.8	8.03	0.1
14	2.8	0.3	7.80	0.3
15	3.0	0.1	8.08	1.0
16	2.4	2.5	8.12	1.6
17	2.9	0.1	8.06	0.3
18	3.0	0.1	8.13	0.2
19	2.9	0.2	7.88	0.7
20	3.1	0.1	7,25	0.3
21	3.1	0.3	8.08	0.2
22	2.9	0.2	7.89	0.1
23	2.9	0.1	7.40	0.4
24	2.9	0.1	7.60	0.3

Compulation of Control Limits

Characteristic 1. (Percentage of fat)

For $\bar{X}$ Chart

Central line $\bar{\bar{X}}=2.95$, Average range $\bar{R}=.36$

Upper Control limit $= \bar{\bar{X}} + A_2 \bar{R} = 3.16$

Lower Control limit $= \bar{\bar{X}} - A_2 R = 2.74$

For R Chart

Upper control limit $= D_4 \bar{R} = .76$

Lower control limit $D_3 \bar{R} = 0$

Characteristic 2 (Percentage of SNF)

For $\bar{X}$ Chart

Central line $\bar{\bar{X}} = 7.77$ $\bar{R} = .51$

Upper control limit $= 8.06$

Lower control limit $= 7.47$

For R Chart

Upper Control Limit $= 1.07$

Lower Control Limit $= 0$

The coefficients A_2 D_3 and D_4 were taken from table C -

Grant & Leavenworth [2]

The significance of considering 3 sigma limits as control limits is that under the normal assumption 99.73% of the population falls within these limits if chance causes alone are at work in the production process. Hence when there is maximum control only 27 points out of 10,000 points will fall out side the control limits.

While studying the control charts for the above characteristics it was found that corresponding to characteristic 1, Two points were outside the control limits in case of $\bar{X}$ Chart suggesting the presence of assignable cause of variation. In case of R Chart all points were below the upper control limit.

In case of $\bar{\bar{X}}$ Chart for characteristic 2 even though seven points were found to be out side the control limits, five points were very near to the control lines. Hence only two points corresponding to sub group on. 1 & subgroup no. 20 seem to be due to the presence of assignable cause. In case of R chart two points were outside the upper control limit showing that the process was not under control at the level with respect to the variability.

The process capability was then computed. According to the standards maintained by HMG. Food Laws and Regulations, the percentage of milk fat should not be less than 3% and S N F should not be less than 8% or at least 7.86%

Under the assumption of normality the probability for any point meeting the specification in case of characteristic 1 is given by

$$P\{ x \geqslant 3 \}$$
$$P\left[z \geqslant \frac{x_1 - 4}{\sigma^1} \right]$$
$$P\left[2 \geqslant \frac{3-2\ 95}{.1547} \right]$$
$$P[z \geqslant .32] = .3745$$

Where $\sigma^1 = \frac{\bar{\bar{R}}}{d_2}$

$d_2 = 2.326$

The proportion not meeting the specification is

$$.5 - .3745 = .1255$$

that is 12% of the milk samples do not meet the standard maintained by HMG.

Similarly in case of SNF when the standard maintained is 8%

Probability for any point meeting the specification is

$$P[x \geqslant 8]$$
$$P[z \geqslant 1.0492]$$
$$= .1492$$

Hence the proportion not meeting the specification is

$$0.5 - .1492 = .3508$$

Thus with respect to characteristic 2. 35% of the milk samples do not meet the standard maintained by HMG of the percentage of SNF is maintained as 7.86% then the proportion not meeting the specification will be 15% only.

Operating Characteristic Functions

The operating characteristic curves for both the $\bar{\bar{X}}$ & R Charts were computed for both the characteristics. These OC functions give the probability of not catching on a single sample a specified change in the process when the process has actually changed with respect to its central tendency or variability or both. In deriving the OC function the basic assumption that is to be made is that the distribution of the process is normal. This limitation is not a serious one since measurable quality characteristics are often normally distributed or at least approximately so. Under the assumption of normality the OC function of the R Chart was derived which is independent of the mean. It is assumed that the process standard deviation was originally equal to the standard value of 6” but subsequently increase by 5% to 50% giving the new values of 6’. In order to compute the probabilities that is

$$P[R \leq UCL] \approx P[W \leq UCL\ 1\ \sigma^1] \quad \text{Where } W = \frac{R}{\sigma^1}$$

Table D I Appendix II Duncan [1] was referred.

The required probabilities were then computed by interpulation method.

For characteristic 1

6” = .1547

UCL = .76

For characteristic 2

6” = .2192

UCL = 1.07

Table No. 2

Computation of OC function for R chart

Percentage increase in 6"	Increased Value of 6		UCL/6		$P\left\{w \leq \frac{UCL}{6}\right\}$	
	Fat	SNF	Fat.	SNF	Fat.	SNF
5	.16268	.23016	4.674	4.675	.991	.991
10	.17043	.24112	4.461	4.462	.984	.984
15	.17818	25.208	4.267	4.268	.977	.977
20	.18592	.26304	4.090	4.091	.966	.966
25	,19367	.27400	3.926	3.927	.954	.954
30	.20142	.28496	3.774	3.776	.943	.943
35	.20917	.29592	3.635	3.636	.876	.877
40	.21691	.30688	3.605	3.506	.836	.836
45	.22466	.31784	3.384	3.385	.798	.798
50	.23241	.32880	3.271	3.272	.763	.763

In order to compute the OC function for $\bar{\bar{x}}$ chart it was assumed that 6' is constant. The points of the OC curve were obtained by finding the probability of the points falling within the control limits when the process average $\bar{x}^1$ shifts to $\bar{\bar{x}} + k\,6_{\bar{x}}$ where $\bar{\bar{x}}^1 = \bar{\bar{x}}$ and $k\,6_{\bar{x}} = \frac{6}{\sqrt{n}}$ & k is a constant. Actually this probability is the probability of a normally s:ı buted variable above $\bar{\bar{x}} + k\,6_{\bar{x}} - \bar{\bar{x}} - 3\,6_{\bar{x}} = (k - 3)\,6_{\bar{x}}$. Those probabilities for different values of k were computed as follow.

Table 3

Computation of QC function for $\bar{x}$ chart

k	Probability for any point to fall within control limits when the process average					
	$\bar{x}^1$ = 2.95 shift to			$\bar{x}^1$ = 7.77 shift to		
1	3.02759	=	.9712	7.868	=	.9712
2	3.09688	=	.8413	7.966	=	.8413
3	3.16617	=	.5000	8.064	=	.5000
4	3.23546	=	.1587	8.162	=	.1587
5	3.30475	=	.0228	8.260	=	.0228
6	3.37404	=	.0013	8.358	=	.0013

Concluding remarks

The $\bar{x}$ and R charts for both the characteristics sugggests the presence of assignable cause of variation. These may be due to the negligence of workmen or due to the fact that the machines may not be working properly or may be any other causes. These causes may be eliminated or reduced to certain level if proper attention is given to the production process.

However, the percentage of milk samples not meeting the specifications as shown by the computation of process capability should not be taken seriously since these represent only crude measures.

The OC function for R chart shows that as the standard deviation increases from 5% to 50% the chance of a sample point to fall outside the control limits increases from. 001 to .237. The OC function for $\bar{x}$ chart shows that as the process average increases by k $6_{\bar{x}}$ where k = 1 to 6 the chance for any point falling outside the control limits increases from .03 to .99.

References

1. Duncan Acheson J. quality Control and Industrial Statistics Richard D. Irwin Inc. Homewood Illinois, Third Edition.

2. Grant – Leavenworth – Statistical Quality Control International Student edition– Fourth edition.

3. Halpern Siegmund – The Assurance Sciences – An Introduction to Quality Control and Reliability – Prentice Hall of India Private Limited, New Delhi.

4. Meyer-Food Chemistry. Reinhold Publishing Corporation.