The Economic Monthly
Vol. 1. No. 3. March-Apr. 1978 (T. U.)

Simple Inventory System and its Optimization*

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Businessmen frequently arrange for market surveys to estimate how much of their product they will be able to sell in the next year or a some period in the future. In the basis of such estimations it decides how much is to put in the inventory of stock. There may be a wide set of choices open to the decision makes, anyone of which may permit him to stay in business or even to prosper. A shop-keeper can hardly be expected to have a large inventory of stock when his forecast of future demand is negligible. A large stationery store may well find it profitable to derive an optimal inventory policy for stocking its supply of pencils where as you would find it silly to derive such a policy for the supply of pencils in your desk drawer. Here what I mean to say is that the concept of optimization arises in the business behaviour and business problems in which the cost in associated in keeping inventory. The application of inventory models are more wide spread in manufacturing and wholesale companies we find scientifically based inventory replenishment rules employed in large manufacturing plants to control hundreds and of ten thousands of items of raw materials, intermediate goods, and finished products. In this situations a very few men are responsible for all the inventory replenishment decisions.

* This paper was presented in the seminar (1976) organized by Economic Society, Economics instruction committee, Kirtipur Campus.

The controlling of inventory arises whenever cost is associated in keeping the goods in hand, in maintaining them in proper way. All the goods at hand will not be sold at once. We need to keep the goods for sometime so as to finish them. We have to order according to the demand. And the demand being an unknown factor we can not order a large number of items at a time. Even if the demand is fixed or supposed to be fixed we can not order a large number of items at a time. The items may chemically change, or they may go on deterioration or after some time they may expire. In such situations we have to make an optimal decision with respect to the inventory system. In after words an inventory problem is concerned with the making of decisions which minimizer the total cost of an inventory system. But making of decision is not an easy job. When one cost in decreased the other cost may increase. There is thus the problem of controlling the costs so that their sum will be the lowest. The inventory problem is thus a problem of making optimal decisions with respect to cost. And the inventory models are prepared in which the resultant optimal solutions are implementable. In most inventory situations the solutions are simply in terms of time and quantity:

i) When should the inventory replemished ?

ii) How much should be added to the inventory ?

The time element and the quantity element are the variables which are subject to control in the inventory system. No matter how complex the mathematical model underlying the replenishment rules, the description of the policy in terms of (i) and (ii) is always easy to understand. The description of the policy in terms of (i) and (ii) is nothing but it is the cost minimization problem. The cost minimization problem is not so much easy. There are various costs associated in keeping goods in inventory. The following three types of costs are significant.

1. The cost of carrying inventories

This cost includes the cost of maintaining the inventories is the rent for the space, interest on money locked-up, handing cost (which includes the cost of labour, transportation charges etc.) cost of clerical staffs required to keep the record, insurance, depreciation, deterioration etc.

2. The cost of incurring shortages

This cost includes the cost of lost sales, loss of good will, over-time payments, the cost of special administrative effort (telephone calls, memos, letters etc.)

3. The cost of replenishing inventories

It includes the cost of machine set-ups for production runs, cost of preparing orders etc.

The problems of inventory arise due to the fact that these costs are interrelated. For example, if we increase the inventory the shortages and hence the shortages costs decrease, but the carrying cost will increase. Also in this case set-up cost per unit time will decrease, as the stock will survive for a long period. Similarly if we decrease the inventory, the carrying cost will decrease while the shortages and set-up costs per unit time will increase. There is thus the problem of balancing the costs so that their sum will be the lowest. One illustration may give an idea of balancing the cost.

Illustration

A customer orders soaps from a manufacturing company at a uniform rate of 50,000 soaps per year. The company makes any number of soaps, at a time. The cost of setting-up the machine for a production run is Rs. 75. The cost of carrying inventory is Rs. 0.30 per soap per year. Shortages are not allowed. The manufacturing company wants to find how many soaps should be produced for each production run.

Solution:

Let company producer q soaps for each production run. This variable q is reffered to as the lot size.

Let q be chosen to be 1000. Then on the average there will be 500 soaps in inventory during the year.

Then the inventory carrying cost $=$ Rs. $500 \times 0.30 =$ Rs. 150.

And if $q=1000$, there will be $\frac{50,000}{1,000}=50$ set-ups during the year. These will cost $50 \times 75 =$ Rs. 3750.

In this case the total cost $=$ Rs. $(3750+150) =$ Rs. 3900.

Similarly the total cost when $q=2000$, 3000, and so on will be as shown in the following table:

Lot-size	Average inventory	Inventory Carrying cost per year	No. of set-ups per year	Cost of set-ups per year	Total cost per year
1000	500	150	50	3750	3900
2000	1000	300	25	1875	2175
3000	1500	450	$16\frac{2}{3}$	1250	1700
4000	2000	600	$12\frac{1}{2}$	937.5	1537.5
5000	2500	750	10	750	1500
6000	3000	900	$8\frac{1}{3}$	625	1525
7000	3500	1050	$7\frac{1}{7}$	507.04	1557.04
8000	4000	1200	$6\frac{1}{4}$	447.75	1647.75
9000	4500	1350	$5\frac{5}{9}$	394.44	1744.44
10000	5000	1500	5	350	1850

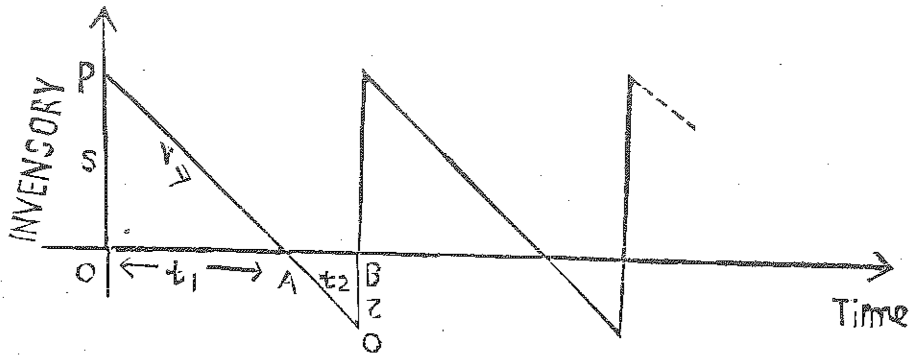
Thus the total cost becomes minimum when the lot-size is taken as 5000. And the two decisions as to when to replenish and by how much are answered as follows: replenish every $\frac{12}{10}=1.2$ month (36 days) whenever the inventory reaches to a zero level and make a lot size of 5000 soaps. This discusion will result in a minimum total yearly cost of Rs. 1500.

Such type of hit and trial method is generally not the best method. Because it is not sure that the best alternative is actually included in the list of alternatives. And also it is lengthy and time consuming process. So a mathematical model describing the system is obtained and optimum decision rules are derived from the model.

To get the model the following assumptions are made:

1. The demand is in uniform rate and is at r units of quantity per unit of time. So far this assumption is concerned, the estimation can be made with the past records in similar situations, and hence it may be known completely or it may be known in terms of probabilities.
2. The inventory carrying cost is C_1 per unit of quantity per unit of time.
3. The cost of setting up a machine before production, or cost of placing an order is C_3.
4. The shortages are allowed and the shortage cost is C_2 per unit of quantity per unit of time.

Let q units of quantity can be produced at a time and s units of quantity is kept in the inventory. Then the time inventory graph is as follows:



Since the demand is at a rate of r units of quantity per unit of time, the inventory finishes after time t_1 so that $s=rt_1$. But shortages are allowed the businessman goes on keeping orders till the shortage becomes z units of quantity. Then the goods are reordered to keep in inventory. Thus one cycles finishes and another similar cycles starts.

The inventory carrying cost for one cycle

$$= c_1 \times \text{area of the } \triangleright \text{ OPA}$$

$$= \frac{sc_1t_1}{2}$$

The shortage cost for one cycle,

$$= c_2 \times \text{area of the } \triangle\text{ABC}$$

$$= \frac{zc_2t_2}{2}.$$

Set-up cost for this cycle = c_3.

Hence the total cost for one cycle=Inventory Carrying Cost+Shortage Cost +Set-up Cost.

$$\text{i.e. } c=\tfrac{1}{2}sc_1t_1+\tfrac{1}{2}zc_2t_2+c_3$$

But we have, $s=rt_1$ i.e. $t_1 = s/r$.

$z=rt_2$ i.e. $t_2 = z/r$.

and $t_1+t_2 = t$ (time for cycle)

Total cost per unit time $= \frac{c}{t} = \frac{c_1s^2}{2rt} + \frac{c_2z^2}{2rt} + \frac{c_3}{t}$.

After time period t_1 the businessman has to order $z+s=q$ units of quantity 50 q is the quantity of goods that is demanded in time period t.

$$\text{So, } q = rt \text{ an } z = q-s = rt-s.$$

Hence the cost function is

$$c(s_1 t) = \frac{c_1 s^2}{2rt} + \frac{c_2 (rt-s)^2}{2rt} + \frac{c_3}{t} \quad \cdots \cdots \cdots (1)$$

This is the function of two variables s and t. The optimum value of s and t are obtained by solving

$$\left.\frac{\delta c}{\delta s}\right]_{s=\hat{s}} = 0 \text{ and } \left.\frac{\delta c}{\delta t}\right]_{t=\hat{t}} = 0.$$

Then optional solutions are,

$$\hat{t} = \sqrt{\frac{2(c_1+c_2)c_3}{c_1 c_2 r}} \quad \cdots \cdots \cdots (2)$$

$$\hat{s} = \sqrt{\frac{2rc_2c_3}{c_1(c_1+c_2)}} \quad \cdots \cdots \cdots (3)$$

$$\hat{q} = r\hat{t} = \sqrt{\frac{2rc_3(c_1+c_2)}{c_1 c_2}} \quad \cdots \cdots \cdots (4)$$

Many managers state that they do not allow any shortages to occur. Their statement imply that they assume that the unit cost of shortage is infinite. Such a unit cost does not seem realistic. What the managers are really trying to say is that the unit shortage cost is relatively high and that they therefore argue not to have any shortages. It also can be said that the unit cost of shortage can not be measured. And it is true that this is very difficult to measure. This, however, does not mean that the unit does not have specific value. Shortages do occur in most inventory systems. If the decisions are good oner, it is quite evident that a value on the uait cost of shortage is placed.

In case when shortages are not allowed i. e. $c_2 = \alpha$ then

$$\hat{t} = \sqrt{\frac{2c_3}{c_1 r}} \quad \cdots \cdots \cdots \cdots (5)$$

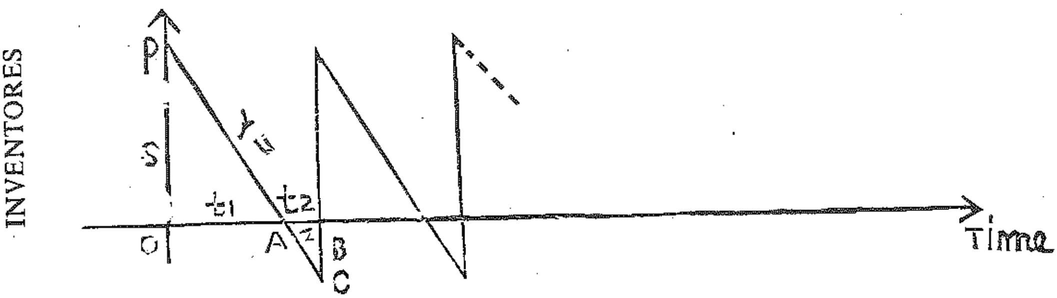
$$\hat{s}=\hat{q}=\sqrt{\frac{2rc_3}{c_1}} \quad \ldots\ldots\ldots\ldots \quad (6)$$

In the above illustration, the optimum lot-size may be fixed by using 6 i. e.

$$\hat{s}=\sqrt{\frac{2\times 50\,000\times 75}{0.3}} = 5000 \text{ soaps per production run.}$$

$$\hat{t}=\sqrt{\frac{2\times 75}{0.3\times 50{,}000}}=\sqrt{\frac{1}{100}}=\frac{1}{10} \text{ year} = 1.2 \text{ months.}$$

We can also derive a model in which the time of replenishment is fixed. In this case one may be interested in finding the level of shortages. Because, time t being fixed, the uniform demand rate r being known the quantity of inventory $q=rt$ is known exactly. In this case the time-inventory graph will be as follows:



Here we are to find upto what level the shortage is advisable.

Now, $s=rt_1$ i. e. $t_1={}^{s}/_{r}$.

$z=rt_2$ i. e. $t_2={}^{z}/_{r}=\frac{q-s}{r}=\frac{rt-s}{r}$.

Inventory carrying cost for one cycle $=\frac{st_1c_1}{z}=\frac{c_1s^2}{zr}$.

Shortage cost for one cycle $=\frac{zt_2c_2}{z}=\frac{c_2z^2}{zr}=\frac{c_2(rt-s)^2}{zr}$.

Set-up cost for one cycle $=c_3$.

Total cost $= c = \frac{c_1s^2}{zr}+\frac{c_2(rt-s)^2}{zr}+c_3$.

$$\text{Total cost per unit time} = \frac{c}{t} = c(s) = \frac{c_1 s^2}{zrt} + \frac{c_2(rt-s)^2}{zrt} + \frac{c_3}{t}.$$

$$\text{Optimal value of S is } \left.\frac{dc}{ds}.\right]_{s=\hat{s}} = 0$$

$$\text{This implies that, } \hat{s} = rt \frac{c_2}{c_1+c_2} \quad \ldots\ldots\ldots\ldots(7)$$

This model does not consider the set-up cost. Because it is fixed as the time of one run is fixed.

Similarly other mother may also be found out according to the demand pattern.

When the methodological procedures for replenishing inventoried items are established, the scientific method frees the decision makes to concentrate his talents on discretionary and exceptional situations where his experience is of greatest benefit. Their an effective scientific inventory system really requires a harmonium combination of human judgement and mathematical formulas.

References

1. Baumol, William J. – Economic theory and operations analysis.
2. Naddor, Eliezer – "Elements of Inventory System" Operations Research and System Engineering - Edited by Charles D. Flagle, William H. Huggins, Robert H. Roy.
3. Wagner, Harvey M. – Principles of operations research, with applications to managerial decisions.
4. Goel, B. S. and Mittal, S. K. – Operations research.