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Hemodynamics through progressive symmetric shaped stenosis

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Abstract
The deposition of fatty particles leading to atherosclerosis may occur in arteries, which alters hemodynamics. This effect becomes more pronounced when a stenosis thickness increases continuously over time. Since atherosclerotic plaque formation is a major cause of cardiovascular disease, understanding its influence on blood flow characteristics is of significant clinical importance. In this study, hemodynamic behaviors due to increasing stenosis is analyzed. A proper increasing rate can estimate the time for complete occlusion and the cardiovascular disease can be cured previously without reaching to the alarming situation. A new mathematical model is developed by incorporating a non-dimensional temporal term in the geometry of the symmetric shaped stenosis and used it in the Navier–Stokes equations in cylindrical coordinates system. The equation is then solved for analytical solution under certain boundary conditions. Analytical expressions for velocity distribution, volumetric flow rate, pressure drop, pressure drop ratio, shear stress and shear stress ratio are derived and evaluated using computational tools. The results indicate that velocity and volumetric flow rate decrease with increasing time and stenotic thickness, while pressure drop and pressure drop ratio increase. The study further demonstrates that the shear stress ratio decreases as stenosis thickness progresses over time. By comparing these theoretical predictions with clinical measurements of plaque growth rates, the approximate time to complete arterial occlusion can be estimated. This modeling framework overcomes limitations associated with symmetric stenosis assumptions and provides a more realistic description of progressive arterial narrowing. The findings offer valuable insights for early diagnosis, prediction of disease progression, and timely clinical intervention in cardiovascular disorders.

Keywords
Temporal term, increasing stenosis, impedance, hemodynamic parameters, aortic stenosis.

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1 Introduction

The study of blood flow through a stenosed artery is of great importance for understanding abnormalities in the cardiovascular system. Blood is a complex suspension consisting of plasma and cellular components. Plasma behaves as a Newtonian fluid, whereas the suspended blood cells exhibit non-Newtonian and deformable characteristics [1, 2]. These suspended constituents include red blood cells (RBCs), white blood cells (WBCs), platelets, proteins, organic molecules, and salts. Among them, RBCs occupy the largest volume fraction, accounting for approximately 45% of the total blood volume [3]. Blood flow in small arteries and capillaries strongly depends on the deformability of hematocrits, which increases apparent viscosity and creates favorable conditions for particle deposition [4–7]. Stenosis is commonly formed due to irregular intravascular growth, which reduces the arterial lumen and obstructs blood flow [8]. As stenosis progresses, the arterial radius decreases gradually, leading to serious circulatory disorders [9]. Regions of low velocity and low wall shear stress, such as arterial bifurcations, are particularly prone to plaque accumulation, which may eventually lead to ischemic stroke [10]. Stenosis often increases gradually with time and may ultimately result in complete arterial occlusion [8].

Antonova et al. [11] investigated the deformability, elasticity, and adhesion forces of erythrocytes during capillary flow and reported that erythrocyte elasticity increases significantly in diabetic patients compared to healthy individuals. Gautam and Kafle [12] have used the centrifugal force to measure the effect of curvature, a major factor to influence the flow parameters. The effect of curvature on flow parameters can be shown appropriately by using two-layer curvature model [13]. Under increased shear stress, normal RBCs tend to align with the direction of flow and exhibit tank-treading motion, deforming into elliptical shapes [14]. This obstruction significantly alters hemodynamic parameters and can lead to severe cardiovascular complications. Patients suffering from both cancer and aortic stenosis experience a particularly high risk of mortality, as stenosis in such cases progresses rapidly [8]. Mechanical factors play a crucial role in the initiation and progression of stenosis, and low mean or oscillatory wall shear stress has been identified as a key contributor to plaque formation [15].

Young [16] introduced the concept of progressively increasing stenosis with time to evaluate pressure drop and its ratio. There is no noticeable difference in pressure, velocity, or WSS if the stenosis occupies less than 50% of the lumen. When the stenosis increases from 60% to 70%, there are sudden and significant changes in hemodynamics and their values. Additionally, there is a significant pressure difference, WSS, and blood flow velocity in the core if the stenosis occupies more than 70% of the lumen [17]. Antonova et al. [18] performed a hemodynamic analysis of carotid arteries with and without stenosis and evaluated velocity profiles and wall shear stress under various conditions. Young et al. [19] conducted a comparative study of flow parameters in stenosed and non-stenosed arteries. Khan et al. [20] examined the effects of time-dependent stenosis while incorporating slip velocity at the arterial wall.

As stenosis increases, blood pressure rises to maintain adequate blood supply. This results in elevated shear stress, with the maximum pressure gradient occurring at the point of maximum stenosis thickness [21]. The curvature in the artery reduces the velocity and affected all other flow parameters significantly, specially it helps to reduce the pressure [13]. Due to cardiac activity, arteries experience complex wall motions that significantly influence blood transport and oxygen delivery [22]. Lee and Fung [23] analyzed steady blood flow through a constricted tube at low Reynolds number and derived expressions for flow variables. Kafle et al. [24] discussed velocity, volumetric flow rate, and pressure drop in stenosed arteries and derived corresponding analytical expressions. Padmanabhan and Jayaraman [7] investigated shear stress and flow resistance due to stenosis and concluded that arterial walls exhibit elastic behavior and are less sensitive to pressure gradients. Dash et al. [25] studied blood flow characteristics in stenosed arteries with inserted catheters and found that increasing catheter size leads to higher pressure drop, impedance, and shear stress.

Bravo-Jaimes et al. [8] examined patients with both cancer and aortic stenosis and reported a high rate of stenosis progression is found in such cases. Gautam et al. [26] have used the temporal term to calculate the increasing rate of the stenosis and calculated all the fundamental flow parameters. The above literature clearly indicates that blood flow in stenosed arteries has a significant impact on hemodynamics and the progression of cardiovascular disease. Uniformly increasing stenosis reduces the arterial lumen and alters the ratio of radii between stenosed and normal regions, thereby affecting flow parameters. In the present work, a dimensionless temporal term $1 - e^{-t/T}$ is incorporated into the stenosis geometry to represent the progressive reduction in arterial radius. This formulation was previously employed by Young [16] for calculating pressure drop using a Newtonian fluid model. In the expression $1 - e^{-t/T}$, T is used to make the term dimensionless and we have fixed it at $T = 12$. As the time t measured in years increases gradually, the term $e^{-t/T}$ decreases and the term $1-e^{-t/T}$ increases, which increases the thickness of the stenosis. More improved result can be obtained if we find the clinical data for the increasing rate and adjusting the value of T accordingly. When $T = 12$ and $t = 1$ the whole expression becomes 0.0796 which indicates nearly 8.0% increment in a year, which can be refined if we get exact value from the clinical data. The dimensionless parameter varies between 0 and 1 and allows the rate of stenosis progression to be controlled by adjusting the time t for a fixed characteristic time T. After modifying the geometry accordingly, the flow variables are evaluated and analyzed in detail.

2 Geometry of the blood flow model for progressive stenosis

The geometry of stenosis is defined as [27]

$$\frac{R}{R_0} = \begin{cases} 1 - \frac{2\delta_0}{R_0}\left[1 + \cos\left(\frac{\pi z}{z_0}\right)\right]\left(1 - e^{-t/T}\right), & \text{for } -z_0 \le z \le z_0 \\ 1, & \text{for } |z| > z_0. \end{cases} \tag{1}$$

Here R and R_0 are the radii in the stenosed and non-stenosed region, respectively. The ratio $\frac{R}{R_0}$ becomes unity when the radii are equal in the stenosis-free region. In this study, we investigate the effect of continuously increasing stenosis over time on blood flow parameters. To address this issue effectively, we have incorporated a dimensionless temporal term $\left(1 - e^{-t/T}\right)$ into the stenosis geometry, which represents the progressive reduction in arterial radius over time.To address this issue effectively, we have incorporated a dimensionless temporal term in the geometry of the stenosis. Different ways are possible to measure the thickness of the stenosis. In our previous work we have used the temporal term which helps to measure the reducing rate of the ratio of the radii in the stenotic and non-stenotic region [26]. In this case the temporal term directly measures the thickness increasing rate of the stenosis. After using this temporal term we have calculated the analytical solutions for velocity profile, volumetric flow rate, shear stress and pressure drop, and the results are analyzed.

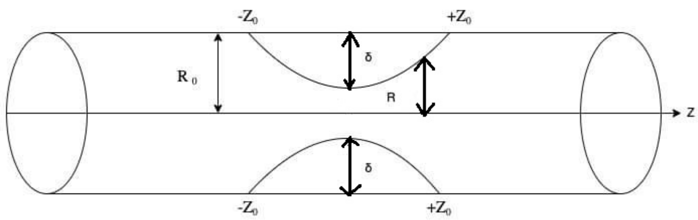

Figure 1: Part of the artery with symmetric shaped increasing stenosis

In this mathematical model, it is assumed that the flow is laminar and axi-symmetric in the stenosed region of an artery. We also assume that the flow is completely developed. The radius of the stenosed artery is $R(z,t)$, and the radius in the non-stenosed region is R_0. Velocity is a function of r, and r varies from 0 to R in the stenosed region. Pressure is denoted by P, where

$$\frac{\partial p}{\partial z} = P. \tag{2}$$

Thickness of increasing stenosis over time is addressed by using a non-dimensional temporal term $e^{-t/T}$. In this term, we have used a fixed characteristic time T to make the temporal factor dimensionless, which can be chosen in an appropriate way. The time t (in years) is used to measure the rate of annual increment.

In the geometry of stenosis, we have taken a cosine function which is symmetric in nature. Consequently, the ratio $\frac{R}{R_0}$ decreases naturally for the interval from $-z_0$ to 0. However, after 0, the value of the cosine function decreases and the ratio $\frac{R}{R_0}$ increases, which is opposite to the real physical situation. To solve this problem, we are using this temporal term in the geometry of the stenosis. When we use an appropriate value of the time t, we can address the problem effectively in the interval from 0 to z_0, because this factor will help to measure the decreasing ratio of $\frac{R}{R_0}$.

3 Extended blood flow curvature model in quadratic form

In this mathematical model, it is assumed that the flow is laminar and axi-symmetric in the stenosed region of a curved artery with stenosis increasing over time. It is also assumed that the flow is completely developed. Radius of the stenosed artery is R, radius in non-stenosed region is R_0, r is the variable radius and p is pressure. Velocity is the function of r and the three velocity components are v^r, v^θ and v^z along r, θ and z direction respectively. The continuity and momentum balance equation in cylindrical polar coordinate are [27]

$$\frac{1}{r}\frac{\partial(rv^r)}{\partial r}+\frac{1}{r}\frac{\partial v^\theta}{\partial \theta}+\frac{\partial v^z}{\partial z}=0 \tag{3}$$

$$\rho\left(\frac{\partial v^r}{\partial t}+v^r\frac{\partial v^r}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^r}{\partial \theta}-\frac{(v^\theta)^2}{r}+v^z\frac{\partial v^r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v^r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v^r}{\partial \theta^2}+\frac{\partial^2 v^r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v^\theta}{\partial \theta}-\frac{v^r}{r^2}\right) \tag{4}$$

$$\rho\left(\frac{\partial v^\theta}{\partial t}+v^r\frac{\partial v^\theta}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^\theta}{\partial \theta}+v^z\frac{\partial v^\theta}{\partial z}+\frac{v^\theta v^r}{r}\right)=-\frac{\partial p}{\partial \theta}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v^\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v^\theta}{\partial \theta^2}+\frac{\partial^2 v^\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v^r}{\partial \theta}-\frac{v^\theta}{r^2}\right) \tag{5}$$

$$\rho\left(\frac{\partial v^z}{\partial t}+v^r\frac{\partial v^z}{\partial r}+\frac{v^\theta}{r}\frac{\partial v^z}{\partial \theta}+v^z\frac{\partial v^z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v^z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v^z}{\partial \theta^2}+\frac{\partial^2 v^z}{\partial z^2}\right) \tag{6}$$

Where, v^r, v^θ, and v^z are the velocities along r, θ and z direction respectively. Again t is time, ρ is blood density and μ is viscosity of the blood. Since the flow is steady, fully developed, and axi-symmetric only, v^r and v^θ are both zero and it is assumed that the component $v^z = v$, and $-\frac{\partial p}{\partial z} = P$ then the z component of N-S equation reduces to

$$-P\frac{r}{\mu}=\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right), \tag{7}$$

We integrate twice the Equation (6), with boundary conditions $\frac{\partial v}{\partial r} = 0$, at $r = 0$, and $v = 0$ at $r = R$, where R is the radius in stenotic region.

$$v=\frac{P}{4\mu}(R^2-r^2) \tag{8}$$

We have considered R_0 as the radius in stenosis free region. Now, right hand side of (7) is multiplied and divided by R_0^2, and after using the temporal term in the symmetric shaped geometry of the stenosis, the velocity in terms of pressure gradient, radius and temporal term becomes

$$v=\frac{PR_0^2}{4\mu}\left\{\left(\frac{R}{R_0}\right)^2-\left(\frac{r}{R_0}\right)^2\right\} \tag{9}$$

where $\frac{R}{R_0}$ in the stenotic region is

$$\frac{R}{R_0}=\left\{1-\frac{\delta}{2R_0}(1-exp(-t/T))\left(1+cos\frac{\pi z}{z_0}\right)\right\} \tag{10}$$

Here, $\left(1+\cos\frac{\pi z}{z_0}\right)$ is the symmetric shaped stenosis and the $(1-exp(-t/T))$ is the temporal term which represents increasing rate of the stenosis. Finally using the geometry in (8), the blood flow velocity in the stenotic region is,

$$v=\frac{PR_0^2}{4\mu}\left\{a^2-\left(\frac{r}{R_0}\right)^2\right\} \tag{11}$$

where $a=\left\{1-\frac{\delta}{2R_0}(1-exp(-t/T))\left(1+\cos\frac{\pi z}{z_0}\right)\right\}$

3.1 Volumetric flow rate with time effect

To calculate the volumetric flow rate Q, we use

$$Q = \int_0^R 2\pi r v dr.$$

After substituting the expression for v from (7),

$$Q = 2\pi \left(\frac{P}{4\mu}\right) \int_0^R (R^2 - r^2) r dr \tag{12}$$

after integrating and simplification,

$$Q = \frac{\pi P}{8\mu} R^4. \tag{13}$$

finally after using the geometry, as defined in case of velocity

$$Q = \frac{\pi P}{8\mu} R_0^4 a^4. \tag{14}$$

3.2 Effect of increasing stenosis over time on pressure gradient

From Equation (12),

$$P = \frac{8\mu Q}{\pi R^4}. \tag{15}$$

The P used in (14) is in fact a pressure gradient $-\frac{\partial p}{\partial z}$, so to get the pressure drop we integrate this P with respect to z from $z = -1$ to $z = 1$ after substituting the geometry with temporal term which contains z to indicate the shape of the geometry. To reduce the index 4 raised over $\frac{R}{R_0}$, we have taken first three terms of the binomial expansion. This is the integrated pressure between the starting and ending point of the stenotic region called pressure drop and is denoted by ΔP.

$$\Delta P = \frac{8\mu Q}{\pi R_0^4} \left\{ 2 + 4\left(\frac{\delta}{R_0}\right)(1 - exp(-t/T)) + \frac{9}{2}\left(\frac{\delta}{R_0}\right)^2 (1 - exp(-t/T))^2 \right\}. \tag{16}$$

In absence of stenosis, both δ and t are 0. Then the pressure drop $(\Delta P)_0$ becomes

$$(\Delta P)_0 = \frac{16\mu Q}{\pi R_0^4}. \tag{17}$$

From equation (15) and (16), the pressure drop ratio can be expressed as

$$\frac{\Delta P}{(\Delta P)_0} = \left\{ 1 + 2\left(\frac{\delta}{R_0}\right)(1 - exp(-t/T)) + \frac{9}{4}\left(\frac{\delta}{R_0}\right)^2 (1 - exp(-t/T))^2 \right\}. \tag{18}$$

3.3 Shear stress and its ratio of maximum and minimum

The shear stress increases with the distance from the center, so it is greater at the stenosis free wall than in the inner wall of the stenotic region.

$$\tau_{\min} = \frac{PR}{2}. \tag{19}$$

Similarly, the shear stress at the wall of the normal artery (without stenosis) becomes maximum because its distance from the center of the artery is more than the distance in the stenotic region.

$$\tau_{\max} = \frac{PR_0}{2}. \tag{20}$$

So, the shear stress ratio is expressed as

$$\frac{\tau_{\min}}{\tau_{\max}} = \left\{ 1 - \frac{\delta}{2R_0}(1 - exp(-t/T))\left(1 + \cos\frac{\pi z}{z_0}\right) \right\} \tag{21}$$

4 Results and discussion

Here, we analyze the results obtained from the analytical solutions of the hemodynamic parameters after incorporating the temporal term. Before describing the results, here we have mentioned some values we have considered with respective sources. The artery we have considered has a size of 0.0015 m, which is a medium-sized artery and approximately equal in size to the left anterior descending artery [28]. Arteries of the same category are classified as large-sized by Kapur [27]. The pressure in this artery is (6.4 ± 4.43) mm Hg, which is approximately equivalent to 1000 Pa [29]. The volume flow rate is about 400 $mm^3 s^{-1}$, and the range of peak central line velocity is 0 to 60 $cm\,s^{-1}$. The pressure drop at rest in such arteries is about 19 mm Hg, which increases to 35–37 mm Hg under stress [30].

Nearest to these given values and taking a suitable average, we have chosen the parametric values as follows: artery size 1.5 mm, height of the stenosis taken from 0.75 mm to 1.2 mm (which is 80% of the radius), time varies from 3.0 years to 12.0 years, and blood viscosity is 4.5 cP. All these quantities are in the

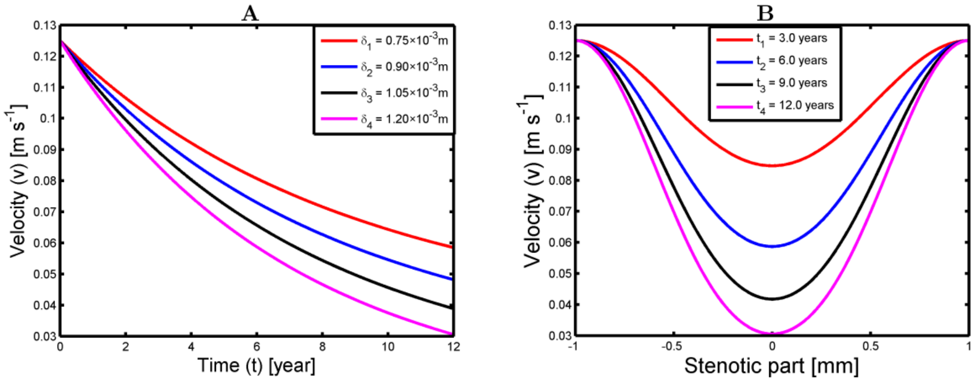

Figure 2: Effect of stenosis on flow velocity: A variation of velocity with time for different stenotic thicknesses; B variation of velocity with progressive stenosis for different time.

3 Effect of time and stenotic height on velocity

Figure 2A shows the effect of increasing stenosis over time on velocity, when the average blood viscosity is constant. A significant change in flow parameters is seen when the size of the stenosis exceeds 70% of the lumen radius; therefore, we have considered stenotic thickness from 50% to 80% of the lumen radius [17]. Initially when $t = 0.0$, the velocity is 0.125 $m\,s^{-1}$. After 12 years, the velocity reduces to 0.0585 $m\,s^{-1}$. Similarly, the velocities are 0.0482 $m\,s^{-1}$, 0.0388 $m\,s^{-1}$, and 0.0301 $m\,s^{-1}$ for stenotic thicknesses of 0.75×10^{-3} m, 0.90×10^{-3} m, 1.05×10^{-3} m, and 1.20×10^{-3} m, respectively. The percentage decrease in the first case is 53.20%, and in the last case, when the stenosis occupies 80.0% of the lumen, the decreasing percentage is 75.60%. This shows that the velocity decreases gradually as the stenosis increases over time and remains only about one fourth of the normal value after 12.0 years, when about one fifth of the lumen is occupied by the stenosis.

Figure 2B shows the relation between blood velocity and stenotic height for different time periods measured in years. All the flow parameter values are the same as in Figure 2A. Initially, just before the starting point of the stenosis, the velocity is 0.125 $m\,s^{-1}$, which decreases to 0.084 $m\,s^{-1}$, 0.058 $m\,s^{-1}$, 0.041 $m\,s^{-1}$, and 0.030 $m\,s^{-1}$ for 3.0 years, 6.0 years, 9.0 years, and 12.0 years, respectively. The velocities decrease gradually with increasing time, and the decreasing amount and percentage reduce with increasing time. However, the aggregate decrement for 80% occupation is again 75.60%. From these two figures, we conclude that the normal velocity decreases by three fourths of its initial value when 80% of the lumen is occupied by the stenosis.

3.1 Effect of time and stenotic height on volumetric flow rate

Figure 3A describes the relation between volumetric flow rate and time-increasing thickness of the stenosis, keeping blood viscosity constant. The value of t is taken up to 12.0 years. The height of the stenosis is considered from 50% to 80%. When 50% of the lumen is occupied by the stenosis, the volumetric flow rate decreases by 38.636% over 12.0 years. Over the same time interval, the volumetric flow rate decreases by 94.029% when the stenosis occupies 80% of the lumen. It is not possible to take a fixed t for all cases, because the rate of increment of stenosis is different for different age groups. The volumetric flow rate does not change rapidly before the ratio $\frac{\delta_0}{R_0}$ reaches 0.50, indicating that there is no significant change in the volumetric flow rate for mild stenosis. As the ratio $\frac{\delta_0}{R_0}$ increases to 80.0%, the volumetric flow rate decreases rapidly as time increases, which shows the significance of time on volumetric flow rate.

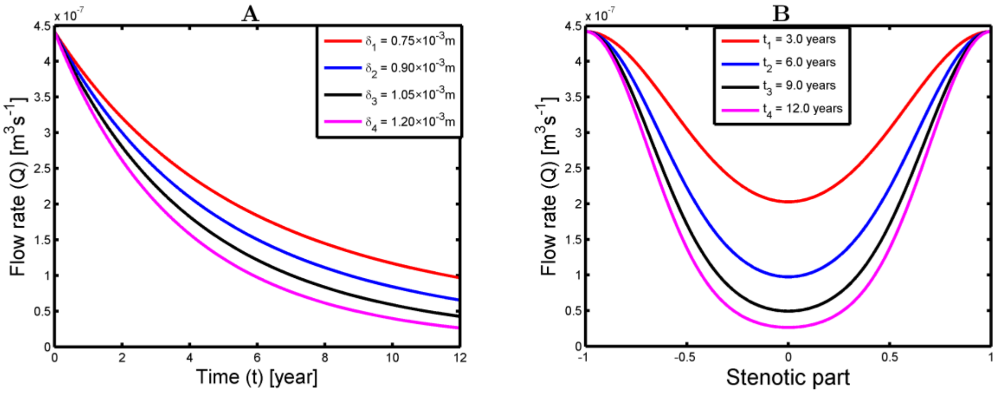

Figure 3: Effect of stenosis on volume flow rate: A variation of volume flow rate with time for different stenotic thicknesses; (B) variation of volume flow rate with progressive stenosis for different time.

Figure 3B describes the relation between volumetric flow rate (Q) and gradually increasing thickness of the stenosis for different time period. All the parameters are same as in Fig. 3A. Volumetric flow rate is measured just before the starting of the stenosis and at the peak of the stenosis. The percentage decrease in volumetric flow rate are 54.109%, 77.948%, 88.840% and 94.027% for 3.0 years, 6.0 years, 9.0 years and 12.0 years respectively. This result shows that if the stenosis increases gradually and occupies 80% of the lumen in 12.0 years time, the volume flow rate is reduced by 94.027%. Our body requires equal amount of blood, so to maintain the amount naturally the pressure increases.

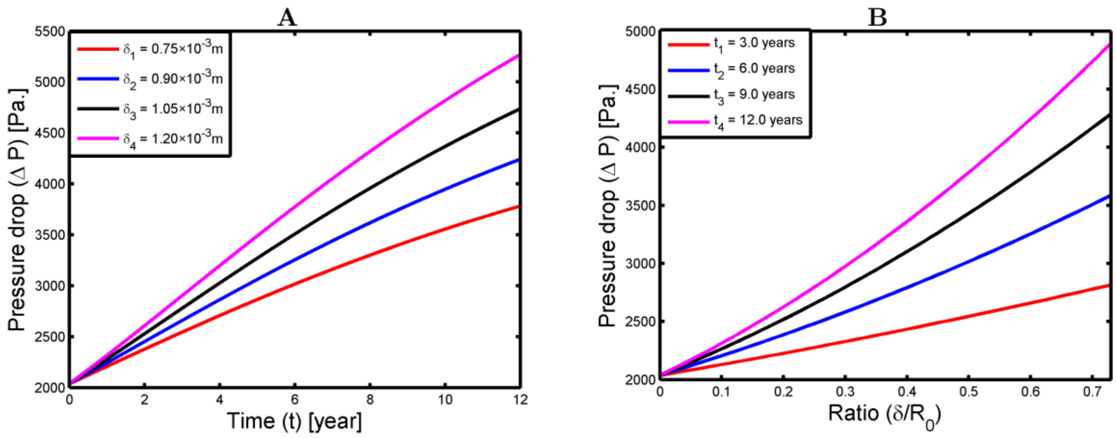

Figure 4: Effect of stenosis on Pressure drop: A variation of Pressure drop with time for different stenotic thicknesses; B variation of Pressure drop with progressive stenosis for different time.

3.2 Effect of time and stenotic height on pressure drop

Figure 4A describes the relationship between pressure drop and increasing stenosis over time. As time increases, the stenotic thickness also increases, and pressure increases as a result. At the beginning, the pressure drop is 2040 Pa, which increases to 3783 Pa over 12.0 years when the stenotic height is 0.75×10^{-3} m. The pressure increases to 4242 Pa, 4737 Pa, and 5270 Pa when the stenotic heights are 0.90×10^{-3} m, 1.05×10^{-3} m, and 1.20×10^{-3} m, respectively. In conclusion, if the stenotic height increases by 0.45×10^{-3} m while keeping time fixed, the pressure rises by 39.30%. If both time and stenotic height increase to their maximum values, the pressure rises by nearly 160.0% of its initial value over 12.0 years.

Figure 4B describes the relationship between pressure drop and the ratio $\frac{\delta_0}{R_0}$ for different time periods. Pressure is measured when the ratio is 0.75 for the times 3.0 years, 6.0 years, 9.0 years, and 12.0 years. The corresponding pressure drop values are 2815 Pa, 3586 Pa, 4287 Pa, and 4893 Pa, respectively. When we take 12.0 years and the maximum ratio of 0.75, the pressure increases from its minimum value of 2040 Pa to 4893 Pa, which is nearly 140% increment from the initial value.

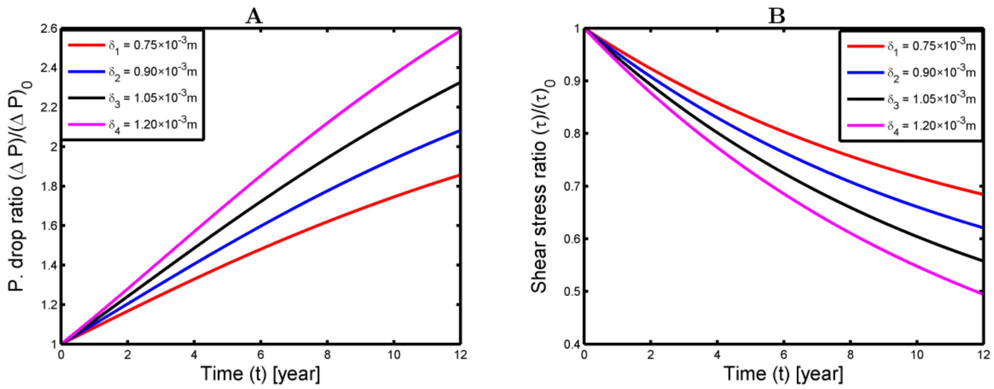

Figure 5: Relationship between pressure drop and time for different stenotic heights

3.3 Effect of time and stenotic height on pressure drop ratio and shear stress ratio

Figure 5A describes the relation between pressure drop ratio and time for different stenotic heights. The pressure drop ratio is measured when the stenotic height occupies more than 50.0% of the lumen and up to 80.0% of the lumen. The pressure drop ratio increases with time as well as with stenotic height. In this case, the pressure drop ratio increases from 1 to 1.857 over 12.0 years when the stenotic thickness is 0.75×10^{-3} m, representing an 85.7% increment from the initial value. Similarly, the increasing percentages are 108.20%, 132.60%, and 158.70% for stenotic thicknesses of 0.90×10^{-3} m, 1.05×10^{-3} m, and 1.20×10^{-3} m, respectively. This result shows that the pressure drop ratio is heavily influenced by stenotic height. The ratio increases by more than two and a half times the initial ratio over twelve years when 80.0% of the lumen is occupied by the stenosis. This may be an interesting and useful finding for further research.

Figure 5B describes the relation between shear stress ratio and time for different stenotic thicknesses. All parametric values are as above. The figure shows that the shear stress ratio decreases gradually with increasing thickness of the stenosis. The shear stress ratio decreases from 1 to 0.6839 over 12.0 years when the stenotic thickness is 0.75×10^{-3} m, representing a 32.61% decrement from the initial value. Similarly, the decreasing percentages are 37.93%, 44.25%, and 55.57% for stenotic thicknesses of 0.90×10^{-3} m, 1.05×10^{-3} m, and 1.20×10^{-3} m, respectively. Thus, we see that over twelve years the shear stress ratio decreases by nearly half.

4 Conclusion

We studied the blood flow parameters in an artery with progressive stenosis. A temporal term was used to measure the increasing rate of the stenosis. An artery with radius 1.5×10^{-3} m was considered, with stenosis that occupies 50% of the lumen at the beginning and increases to 80.0% at the end of twelve years. The temporal term was added to the geometry of the stenosis and used in the

Navier-Stokes equation. The equation was then solved with suitable boundary conditions to obtain analytical solutions for the blood flow parameters. The blood flow parametric relations are expressed in terms of time, pressure gradient, volume, velocity, and viscosity as the stenosis increases.

When the results were analyzed, they showed that velocity and volumetric flow rate decrease as time increases. The pressure drop and its ratio increase with time. The shear stress ratio decreases with increasing time. The results demonstrate the effect of progressively increasing stenosis on flow parameters over different time periods. This study may be useful in the field of medical science for predicting the hemodynamic effects of cardiovascular stenosis.

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