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Effective action approach to the Leggett's mode in two-gap superconductors

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ABSTRACT
When electrons of two electronic bands participate in superconducting phenomena, it is said to be two gap superconductor. There are two set of cooper pairs in different energy gap with different energy. The observation of Leggett's mode in two band superconductor provides an additional information about superconductor. By using the effective action, the thermodynamic potential in the case of neutral and charged two gap superconductor are calculated. Using phase dependent action, we investigate a collective excitation (Leggett's mode) corresponding to small fluctuations of the relative phase of two condensates in two band superconductor. We consider the possibility of observing Leggett's mode in MgB_2 superconductor and conclude that for the known values of two band model parameters for MgB_2, Leggett's mode rises above the two particle threshold.

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1. Introduction

Superconductivity was first discovered by Dutch Physicist H .Kamerlingh Onnes , three years after he liquefied helium. He found that the resistance of mercury dropped to almost zero when the sample was sufficiently cooled to low temperature. Cooper pairs are responsible for the phenomenon of superconductivity. The electrons with opposite momentum and spin undergo Bose-Einstein condensation to form cooper pair. Exchange of phonon between electrons seems to have an attraction between electrons thus forming cooper pairs [1].

In the presence of weak uniform magnetic field, number of cooper pairs and their internal structure is unaltered. It leads to the vanishing of magnetic field in the interior of bulk superconductor. A superconductor in an external magnetic field carries an electric current near its surface. This current is of magnitude such that it cancels the external magnetic field. Thus there is no field inside superconductor [2]. This is called Meissner effect.

If the electrons of single electronic band are participating for superconducting state, material is said to be one gap superconductor. Energy required to break cooper pairs is same if all the pairs are formed with same energy and hence shows only one gap. If the electrons of two electronic bands are participating for the superconducting state, material is said to be two gap superconductor. There are two set of cooper pairs in different energy bands and energy required to break the pairs is also different. Interestingly cooper pairs of both bands are created at same critical temperature. The Josephson effect occurs if two superconductors are separated by a thin insulator. The tunneling of cooper pairs through the insulator was first introduced by Josephson [3, 4, 5].

The study of multiband superconductors started from the works of Moskalenko, Suhl, Peretti and Kando, as a generalization of Bardeen-Cooper-Schrieffer (BCS) theory to a multi gap superconductors. In the case of multi gap superconductors, coulomb repulsive interaction turns the one plasma mode into a gapped plasma mode. These modes are massive due to Josephson interactions. There is a possibility that some of these modes become massless Nambu-Goldstone modes when the Josephson couplings are frustrated. The Josephson couplings between different bands will bring about attractive phenomena: they are time reversal symmetry breaking and existence of gapless modes. The phase difference mode between two gaps is called Leggett's mode [6].This mode yields new excitation modes in multi-gap superconductors. The Leggett's mode is realized as a Josephson Plasma oscillation in layered superconductors.

The fluctuation of the inter band phase difference in the multi-gap superconductor is Leggett's mode. This fluctuation can elevate the superconducting transition temperature. According to conventional superconducting microscopic BCS theory, the Leggett's mode is not implemented and their entropy is not taken into account. The formation of pair means the loosing of entropy. The competition between the gain of the energy due to gap evolution and the cost due to missing entropy determines if BCS gap opens or not. If the pair still has entropy after the formation, the cost due to missing entropy is reduced. This reduction assists the evolution of gap. The entropy, which Leggett's mode has corresponds to entropy of the pair [7].

2. Theory

Microscopic BCS theory for development of Hamiltonian of the system

We consider two electron system in Fermi sea which aren't interacting with each other. The electrons have equal and opposite spin so that the lowest energy state have total momentum zero [4]. The Hamiltonian gives the total energy of the system. Hamiltonian can be expressed as

$$\hat{H} = \sum_{k=1}^{N} T(x_k) + \frac{1}{2}\sum_{k=l=1}^{n} V(x_k, x_l)$$

where T is kinetic energy and V is potential energy of interaction between particles , x_k describes the coordinate of kth particle. Similarly, x_l denotes the co-ordinate of lth particle.

In case of two gap superconductor, Hamiltonian is ,

$$\hat{H} = \sum_{l} \hat{H}_{TB,l} + \hat{H}_T$$

where, $\hat{H}_{TB,l}$ is Hamiltonian for two band superconductor in ith layer and Hamiltonian $\hat{H}_T$ describes the electron tunneling between two adjacent S layers through the insulator.

This can be expressed as,

$$\hat{H}_T = \sum_{i,j,\sigma} (T_{ij} c^{i\dagger}_{\sigma,1} c^{j}_{\sigma,2} + h.c.)$$

where, T_{ij} is the tunneling matrix element for an electron from i to j band. Also, $c^{i\dagger}_{\sigma,l}$ and $c^{i}_{\sigma,l}$ denote the operator which create and destroy an electron with spin σ in the i-band. In the absence of magnetic field,

$$\hat{H}_{TB,l} = \sum_{i=s,d} E^i c^{i\dagger}_{\sigma,l} c^{i}_{\sigma,l} + \hat{H}^{pair}_l$$

where, Ei is the energy of electron in i-band (i = s or d band) about Fermi energy. H^{pair}_l is Hamiltonian for interaction between electrons.

According to Leggett, BCS wave function in terms of pairing operator can be expressed as,

$$\psi_l^i = c_{\uparrow,l}^{i\dagger} c_{\downarrow,l}^{i\dagger}$$

By using this concept, Pairing Hamiltonian can be written as

$$\hat{H}_l^{pair} = -V_{ss} c_{\uparrow,l}^{s\dagger} c_{\downarrow,l}^{s\dagger} c_{\downarrow,l}^{s} c_{\uparrow,l}^{s} - V_{dd} c_{\uparrow,l}^{d\dagger} c_{\downarrow,l}^{d\dagger} c_{\downarrow,l}^{d} c_{\uparrow,l}^{d} - V_{sd}(c_{\uparrow,l}^{s\dagger} c_{\downarrow,l}^{s\dagger} c_{\downarrow,l}^{d} c_{\uparrow,l}^{d} + \text{h.c.})$$

Here, V_{ij} is the strength of pairing interaction potential. Interband pairing interaction between two electrons in s and d band is described by the Hamiltonian $H_{inter,l}^{pair}$ which is the last term of the above relation. This can be expressed as,

$$\hat{H}_{inter,l}^{pair} = -V_{sd} \sum_{k,k'} c_{k\uparrow,l}^{i\dagger} c_{-k\downarrow,l}^{i\dagger} c_{-k'\downarrow,l}^{j} c_{k'\uparrow,l}^{j}$$

Here, i and j can take same value. By using this Leggett concept, total Hamiltonian for our system [5, 9] will be

$$\hat{H} = \sum_{k,\sigma} E_k^s c_{k,\sigma}^\dagger c_{k,\sigma} + \sum_{k,\sigma} E_k^d c_{k,\sigma}^\dagger c_{k,\sigma} - \sum_{k,k'} V_{k,k'}^{ss} c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{-k'\downarrow} c_{k'\uparrow} - \sum_{k,k'} V_{k,k'}^{dd} d_{k\uparrow}^\dagger d_{-k\downarrow}^\dagger d_{-k'\downarrow} d_{k'\uparrow} - \sum_{k,k'} V_{k,k'}^{sd} \left(c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger d_{-k'\downarrow} d_{k'\uparrow} + d_{k\uparrow}^\dagger d_{-k\downarrow}^\dagger d_{-k'\downarrow} d_{k'\uparrow}\right)$$

Collective Excitation

Bogolyubov and Anderson discovered that density oscillation can couple for oscillation of the phase of superconducting order parameter through pairing action. In neutral system, these collective sound like oscillation are known as Bogolyubov Anderson Goldstone (BAG mode). In charged system, the frequency of the mode is pushed into plasma frequency due to coulomb interaction [8]. A main idea beyond this approach is rather simple since the collective modes present low energy degree of freedom.

Physically, Leggett's mode is a collective excitation corresponding to a small fluctuation of the relative phase of two band superconductor. Leggett's mode is obtained using the modulus of phase variables in the path integral formalism.

The action integral is given by,

$$S = \int_0^\beta d\tau\left[\sum_{i,\sigma,k} c_{k,\sigma}^i\, \partial_t c_{k,\sigma}^i + \hat{H}(c)\right]$$

The effective action can be written as,

$$S = S_{pair} + S_{coulomb}$$

Using Hubbard - Stratonovich transformation and Nambu notation, the effective action becomes,

$$S = \int_0^\beta \left\{ \sum_{\vec{k}\vec{k'}} \left[\frac{\phi_{\vec{k}}^{s\dagger} \phi_{\vec{k'}}^{s}}{g_{ss}} + \frac{\phi_{\vec{k}}^{d\dagger} \phi_{\vec{k'}}^{d}}{g_{dd}} - \frac{g_{sd}}{g_{ss} g_{dd}} (\phi_{\vec{k}}^{s\dagger} \phi_{\vec{k'}}^{s}) \right] - \text{Trln}G_s^{-1} - \text{Trln}G_d^{-1} \right\}$$

Now, the thermodynamic potential can be written as,

$$\Omega = \frac{1}{\beta} \int_0^\beta d\tau \left[\frac{|\Delta k^s|^2}{g_{ss}} + \frac{|\Delta k^d|^2}{g_{dd}} - 2\frac{g_{sd}}{g_{ss} g_{dd}} |\Delta k^s||\Delta k^d| \cos(\theta^s - \theta^d) \right] - \frac{1}{\beta}(\text{Trln}G_s^{-1} - \text{Trln}G_d^{-1})$$

Here, Ω can be written as the sum of Ω_{kin} and Ω_{pot} as,

$$\Omega(\Delta_i, \theta_i, \phi) = \Omega_{kin}(\Delta_i, \theta_i, \phi) + \Omega_{pot}(\Delta_i, \theta_i, \phi)$$

where, Ω_{kin} is the sum of energies of phase fluctuations in each band and Ω_{kin} is responsible for the appearance of Leggett's mode term in the Josephson coupling energy of the condensates in two bands. This term explicitly depends on relative phase ($\theta_1 - \theta_2$) of two condensates.

If we minimize Ω with respect to $\theta^s - \theta^d$, we get

$$\frac{d\Omega}{d(\theta^s - \theta^d)} = \frac{1}{\beta} \int_0^\beta d\tau \sum \frac{2 g_{sd}}{g_{ss} g_{dd}} |\Delta k^s||\Delta k^d| \sin(\theta^s - \theta^d) = 0$$

From this we obtain,

$$\Delta^s - \frac{g_{sd}}{g_{dd}}\Delta^d - g_{ss}\Delta^s N_1 F(\delta_1) = 0$$

and,

$$\Delta^d - \frac{g_{sd}}{g_{ss}}\Delta^s - g_{dd}\Delta^d N_2 F(\delta_2) = 0.$$

where, $N_i = \frac{m_i p f_i}{2\pi^2}$ is the density of states in ith band.

In case of neutral superconductor, the terms with electric potential disappear from the equations above, and we can get $\omega^2 = \omega_0^2 + v^2k^2$ for positive solution and $\omega^2 = c^2k^2$ for negative solution where $c^2 = \frac{N_1C_1^2 + N_2C_2^2}{N_1+N_2}$ and $v^2 = \frac{N_1C_2^2 + N_2C_1^2}{N_1+N_2}$.

The positive solution corresponds to Leggett's mode whereas negative solution corresponds to BAG mode. The collective mode is only possible if $\omega_0^2 > 0$ since $V_{12} > 0$ (H. Goldstein et al. 2011). This implies that Leggett's mode exists for $V_{11}V_{22} - V_{12}^2 > 0$.

But in case of charged superconductor, long distance coulomb interaction has a drastic influence on BAG mode transforming in the plasma mode. Here we get,

$$\omega^2 = \omega_0^2 + v^2k^2$$

$$\text{where, } v = \frac{(N_1+N_2)C_1^2C_2^2}{N_1C_1^2 + N_2C_2^2}$$

This represents that the equation for collective mode has only solution describing Leggett's mode.

3. Results and Discussion

Recently discovered MgB_2 superconductor can be described by the classical two gap model which convincingly fits the specific heat and penetration depth measurement. To be observed experimentally, Leggett's mode should have the value of ω_0 in a well separated from two particle threshold given by smallest gap δ_1. Here we estimate the value of ω_0 using recently suggested values of the coupling constants, introducing the dimensionless coupling constants, $\lambda_{ij} = N_iV_{ij}$ that are often used for description of two band model.

We may rewrite equation of ω in the form as,

$$\omega^2 = \frac{4(\lambda_{12} + \lambda_{21})\Delta_1\Delta_2}{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}}$$

For specific value of coupling constants $\lambda_{11} = 0.96$, $\lambda_{22} = 0.28$, $\lambda_{12} = 0.16$, $\lambda_{21} = 0.22$ making $\Delta_1 = 1.8$ MeV fixed we get, $\omega_0 = 3.42\sqrt{\Delta_2}$

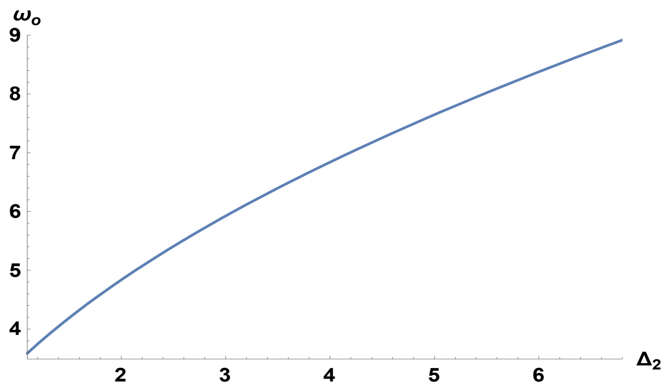

Fig. 1: Variation of ω as a function of gap parameter Δ2 for $\lambda_{11} = 0.96$ $\lambda_{22} = 0.28$, $\lambda_{12} = 0.16$, $\lambda_{21} = 0.22$ and varying Δ_2 from 1.11 mev

The Fig. 2 represents a parabola with vertex at origin. Here, $\Delta_1 = 1.8$ MeV so $2\Delta_1 = 3.6$ MeV. If $\Delta_2 = 1$ MeV, $\omega_0 = 3.42$ Hz, which in turn implies that the ratio $\frac{\omega_0}{2\Delta_1} > 1$. This is the reason why we exclude $\Delta_2 = 1$ MeV and Leggett's mode is unlikely to be observed in MgB_2.

Making $\Delta_2 = 8$ MeV, we get $\omega_0 = 4.26\sqrt{\Delta_1}$ and the graph is plotted as,

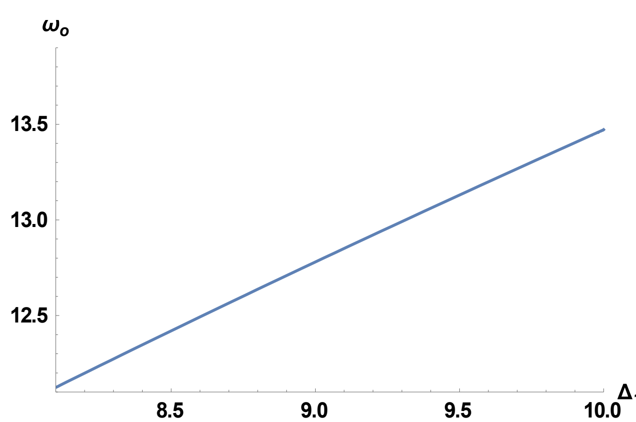

Fig. 3: Variation of frequency ω as a function of gap parameter Δ_1 for $\lambda_{11} = 2$ $\lambda_{22} = 2$, $\lambda_{12} = 1$, $\lambda_{12} = 1$ and varying Δ_1 from 8 mev.

Here we fix $\Delta_2 = 8$ MeV, the nature of the curve is a straight line. If $\Delta_1 = 8$ MeV, $\omega_0 = 13.07$ Hz, which implies $\frac{\omega_0}{2\Delta_1} < 1$ and explains that Leggett's mode is likely to be observed in MgB_2.

The results suggest that for the values of two band model parameters known at present for the two band model of MgB_2, Leggett's mode arises above the two particle threshold and unlikely to be observed.

We don't exclude however, that Leggett's mode can be observed in MgB_2 if the values of coupling constants λ_{12} and λ_{21} would become smaller. The observation of Leggett's mode provides an additional insight to the underlying physics of such a superconductor.

4. Conclusion

Leggett's mode is a collective excitation corresponding to a small fluctuation of the relative phase of two band superconductor. Leggett's mode is obtained using the modulus of phase variables in the path integral formalism. This work presents the study of validity of Leggett's mode in the two-gap superconductor like magnesium-diboride. Starting from the microscopic BCS Hamiltonian of the system we derived effective action of the system and thermodynamic potential. We obtained the condition if the ratio $\frac{\omega_0}{2\Delta_1} < 1$ Leggett's mode is likely to be observed on the other hand when $\frac{\omega_0}{2\Delta_1} > 1$ Leggett's mode is unlikely to be observed in MgB_2.

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