RELATION BETWEEN PRIME AND COMPOSITE NUMBER

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An integer P which is not 0 or ± 1 & is divisible by no integers except ± 1 & ± P is called Prime number but an integer that has two or more than two prime factors as 4, 6, 10 in destination to ± 1 is called composite number.

Now it is cleared that the relation between prime and composite numbers depend upon their factors.

There are infinite numbers of prime numbers but no general formula for prime. It is very easy to find the factors of small number but very difficult for large numbers.

Three century ago, French mathematician Fermat found that 112303 & 898423 are factors of one large number 100895598169 but neither of these two small numbers (112303 & 898423) could be factored.

PARTICULAR TEST FOR PRIMES

(i) The number N = 2 is a prime because N – 1 = 1, 1 + 1= 2,2/2

(ii) The number N = 3 is a prime because N – 1 = 2, 1× 2 +1 = 3, 3/3

(iii) The number N= 4 is not a prime because N – 1 = 3, (1×2×3) + 1 = 7, 4 does not divide 7.

Similarly we can apply this test on other number also. But it becomes very difficult to carry out this test increasingly. So only for limited number Wilson's theorem is highly effected.

GENERAL TEST

By Euclid (300 B.C), it can be cleared that if P be a prime then the product of all primes up to P gives 2 × 3 × 5 × .... × P

If P = 2 then we get 2

If P = 3 then we get 2 × 3 = 6

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If P = 5 then we get 2 × 3 × 5 = 30 and so on.

Lets assume N = (2 × 3 × 5 × 7 × .... × P) + 1

$\Rightarrow N > P$

Then we have two possibilities (a) N is a prime (b) is not a prime but divisible by a prime. On the hand, we found that a prime number greater than P (N itself) but on the other hand N is divisible by prime. If N is divisible by 3, it divides 2 × 5 × 7 ×........... × P times and leaves a reminder 1 and it is true for any prime 2, 3, 5, 7 up to P which establishes the first possibility. But from second condition it follows that there must be a prime number which is greater than P and N is divisible by some prime.

From both conditions we get a conclusion that a prime is greater than P.

No one can say about specific prime numbers. Probably the largest prime no. is $2^{3217}-1$.

Mathematician Wilson states that a number N is a prime iff it divides the number $1\times2\times3\times4\times5\times.................\times(N-1)+1$.

PARTICULAR TEST FOR COMPOSITE NUMBER

The test for composite number depends upon its divisor.

Some illustrations:

(i) A composite number has one factor 2 iff its last digit is divisible by 2.

(ii) A composite number has one factor 3 iff sum of its digit is divisible by 3.

(iii) A composite number has one factor 4 iff its unit digit plus twice of its tens digit is divisible by 4.

(iv) A composite number has one factor 5 iff its unit digit is divisible by 5 (ends at 0 or 5)

(v) A composite number has one factor 6 iff its unit digit is even and sum of its digits is divisible by 3.

(vi) A composite number has one factor 7 iff its 3 times unit digit plus 2 times ten's digit minus 1 time hundred's digit minus 3 times thousand's digit minus 2 times ten thousand's digit plus 1 time hundred thousand's digit is divisible by seven.

(vii) A composite number has one factor 8 iff its units' digit plus 2 times ten's digit plus 4 times thousand's digit is divisible by 8.

(viii) A composite number has one factor 9 iff the sum of its digits is divisible by 9.

(ix) A composite number has one factor 10 iff its last digit is 0.

GENERAL TEST

Let us consider that N be the three digit numbers a, b, c with divisor 7 then it can be written as $N = 100a + 10b + c$.

If S be the sum of these numbers then

$$S = -1 \times a + 2 \times b + 3 \times c$$

$$\Rightarrow 2S = -2 \times a + 4 \times b + 6 \times c \quad \text{and} \quad N + 2S = 98 \times a + 14 \times b + 7 \times c = 7(14a + 2b + c)$$

It implies that sum of $N+2S$ is a multiple of 7 (say 7R) and if N is a multiple of 7 (say 7P) then $2S = 7R - 7P = 7(R - P)$

It makes clear that S must be divisible by 7.

Conversely if S is multiple of 7(say 7L) then $N = 7R - 2 \times 7L = 7(R - 2L)$, which shows that N must be multiple of 7.

Similarly we can develop other test for other composite numbers also.

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