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BEHAVIOR OF POSITIVE INTEGERS IN PARTITION

Raju Ram Thapa*

The representation of number as sum of other numbers is called partition of the number. Euler defined the theory in 1740 AD for partition. According to professor Hardy, Ramenujan was the first mathematician who studied the behavior of positive integers in partition.

Partition of a positive integer is a representation of the integer as sum of any number of positive integers. These integers are termed as summands & the order of parts of numbers is irrelevant. The number of partitions of N is defined by P(N).

Thus these can be written as $P(0)=1, P(1)=1, P(3)=3, P(4)=5, P(5)=6$ etc, which sows that the number 0 & 1 have only one partition, similarly the number 5 has 6 partitions ; whose parts are 5, 4 + 1, 3 + 2, 3 + 1 + 1, 2 + 1 + 1 + 1, 1 + 1 + 1 + 1+ 1. & the number 6 has 11 partitions, with parts 6, 5 + 1, 4 + 2, 4 + 1 + 1, 3 + 3, 3 + 2 + 1, 3 + 1 + 1 + 1, 2 + 2 + 2, 2 + 2 + 1 + 1, 2 + 1 + 1 + 1 + 1, 1 + 1 + 1 + 1 + 1 + 1.

Euler's theorem for partition states that the number of partitions of an integer N, in which all parts are odd equal to the number of partitions of N in which all parts are distinct.

If $P^o(N)$ & $P^d(N)$ represents the number of partitions of N into odd and distinct parts respectively then we will get $P^d(6)=4$ & $P^o(6)=4$. For N = 6, the partition of 6 with odd parts are 5 + 1, 3 + 3, 3 + 1 + 1 + 1, 1 + 1 + 1 + 1 + 1 + 1.

$\Rightarrow P^o(6)=4$ and the partitions of 6 with distinct parts are 6, 5 + 1, 4 + 2, 3 + 2 + 1.

$\Rightarrow P^d(6)=4$

Therefore, we get a conclusion that for any integer N, $P^d(N)=P^o(N)$.

Ferror used graph for partition. His graph has a great role in partition. By using it, a partition of a number is denoted by horizontal rows of nodes (dots). The

* Lecturer, Dept. of Mathematics, P. G. Campus, Biratnagar, T.U.

left side nodes of each row lies on a vertical line & distance between dots must be same. The graphical picture of number 10 & 11 can be shown as follows. :

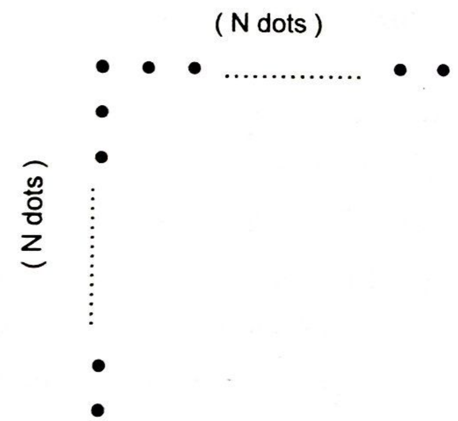
For number 10, the number of dots in rows represents the partition 4 + 3 + 2 + 1 but in column it shows the partition 4 + 3 + 2 + 1.

But for number 11, the number of dots in rows represents partition 5 + 3 + 2 + 1 but in column it. shows the partition 4 + 3 + 2 + 1. The conjugate partition of 5 + 3 + 2 + 1 is 4 + 3 + 2 + 1 + 1 & vice versa.

So a partition is said to be conjugate of partition (dots in rows) if it will obtain by number of dots in successive columns of graphical representation.

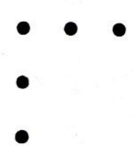
If partition is identical with its conjugate, it will be self-conjugate partition. 4 + 3 + 2 + 1 is a self conjugate partition of number 10.

But every odd number has self conjugate partition. If 2N + 1 be the odd number, it can be partitioned into (N + 1) + 1 + 1 + ...+ 1.



Self conjugate partition of (N+1) + 1 .... + 1 is (N + 1) + 1 + ... + 1 because number of dots in rows & columns are (N + 1) + 1 .... + 1.

e.g. odd number 5 has self conjugate partition which can be shown graphically as :



Furthermore, if a number has a self-conjugate partition then there must be at least one partition into distinct odd parts & conversely if a number has a partition into distinct odd parts, then it must has a self conjugate partition.

Hence for integer N, the sequence of positive integers 1, 2. 3 .... r ; $1 \leq 2 \leq .... \leq r$ form a partition of N if $N = 1 + 2 + ... + r$. G. H. Hardy & S. Ramanujan established a formula for partition of number N, first few terms of the formula are given by

$$\frac{1}{2\pi\sqrt{2}}\frac{d}{dx}\left(\frac{\exp\frac{2\pi}{\sqrt{6}}\sqrt{N-\frac{1}{24}}}{\sqrt{N-\frac{1}{24}}}\right)$$

The actual formula is an asymptotic series in which small error appears in using finite number of terms.

REFERENCES :

Herstein, I.N; Topics in Algebra

Ninven, J. & zuckermann, H.S; An introduction to theory of Numbers.

Ranganathan, S.R; The man & Mathematics.